Program one

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Let

$$d = 0.01,\ \Delta r = \frac{2}{2n+1} \text{ and } \Delta\theta = \frac{2\pi}{m}.$$

Define

$$\mu_i = \frac{1}{2i-1},\ \beta_i = \frac{1}{(i-1/2)^2(\Delta\theta)^2} \text{ for } i = 1, \ldots, n,$$
$$\delta = -2 - \frac{(\Delta r)^2}{d},$$
$$\eta = -1 + \mu_n - \frac{(\Delta r)^2}{d},$$

$$\Psi = \begin{bmatrix} 2 & -1 & & -1 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ -1 & & -1 & 2 \end{bmatrix} \in \mathbb{R}^{m\times m}, \tag{1}$$

and

$$A = \begin{bmatrix} \delta I - \beta_1\Psi & (1+\mu_1)I & & & \\ (1-\mu_2)I & \delta I - \beta_2\Psi & (1+\mu_2)I & & \\ & \ddots & \ddots & \ddots & \\ & & (1-\mu_{n-1})I & \delta I - \beta_{n-1}\Psi & (1+\mu_{n-1})I \\ & & & (1-\mu_n)I & \eta I - \beta_n\Psi \end{bmatrix}. \tag{2}$$

Problem: Solve the linear system

$$Ax = b. \tag{3}$$

(1) 使用module 的指令, 將n and m 建立一模組

(2) 利用allocate 的指令, 宣告A and b 爲可變動大小之array.

(3) 撰寫subroutine 建立A and b, when $b = [1, \cdots, 1]^T$.

(4) 將以下algorithm 撰寫成subroutine

Algorithm 1 (Gauss-Seidel Method)

Give an initial guess $x^{(0)}$ *and stop criteria* M *and* ε. *Set* $k = 0$

While $(k < M)$ *or* $(\| Ax^{(k)} - b \|_2 \geq \varepsilon)$ *do*

For $i = 1, 2, \ldots, n$

$$x_i^{(k)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k-1)} \right) \Big/ a_{ii}$$

End for

$k = k + 1$

End while

(5) Use (4) to solve the linear system $Ax = b$.

(6) Output $\| Ax - b \|_2$.