The second program

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Let

$$d = 0.01,\ \Delta r = \frac{2}{2n+1} \ \text{ and } \ \Delta\theta = \frac{2\pi}{m}.$$

Define

$$\mu_i = \frac{1}{2i-1},\ \beta_i = \frac{1}{(i-1/2)^2(\Delta\theta)^2} \ \text{ for } i = 1, \ldots, n,$$
$$\delta = -2 - \frac{(\Delta r)^2}{d},$$
$$\eta = -1 + \mu_n - \frac{(\Delta r)^2}{d},$$

$$\Psi = \begin{bmatrix} 2 & -1 & & -1 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ -1 & & -1 & 2 \end{bmatrix} \in \mathbb{R}^{m\times m}, \tag{1}$$

and

$$A = \begin{bmatrix} \delta I - \beta_1\Psi & (1+\mu_1)I & & & \\ (1-\mu_2)I & \delta I - \beta_2\Psi & (1+\mu_2)I & & \\ & \ddots & \ddots & \ddots & \\ & & (1-\mu_{n-1})I & \delta I - \beta_{n-1}\Psi & (1+\mu_{n-1})I \\ & & & (1-\mu_n)I & \eta I - \beta_n\Psi \end{bmatrix}. \tag{2}$$

Problem: Solve the linear system

$$Ax = b. \tag{3}$$

(1) 使用module 的指令, 將n and m 建立一模組

(2) 利用allocate 的指令, 宣告A and b 爲可變動大小之array.

(3) 撰寫subroutine 建立A and b, when $b = [1, \cdots, 1]^T$.

(4) 將以下algorithm 撰寫成subroutine

Algorithm 1 (Gaussian elimination) *Given $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^n$, this algorithm implements the Gaussian elimination procedure to reduce A to upper triangular and modify the entries of b accordingly.*

for $k = 1, \ldots, n-1$ **do**
  for $i = k+1, \ldots, n$ **do**
    $t = A(i,k)/A(k,k)$
    $A(i,k) = 0$
    $b(i) = b(i) - t * b(k)$
    for $j = k+1, \ldots, n$ **do**
      $A(i,j) = A(i,j) - t * A(k,j)$
    end for
  end for
end for

(5) 將以下algorithm 撰寫成subroutine

Algorithm 2 (Back Substitution) *Suppose that $U \in \mathbb{R}^{n\times n}$ is nonsingular upper triangular and $b \in \mathbb{R}^n$. This algorithm computes the solution of $Ux = b$ using row-oriented procedure.*

$x(n) = b(n)/U(n,n)$
for $i = (n-1), \ldots, 1$ **do**
  $tmp = 0.0$
  for $j = i+1 : n$ **do**
    $tmp = tmp + U(i,j) * x(j)$
  end for
  $x(i) = (b(i) - tmp)/U(i,i)$
end for

(6) Use Algorithm 1 and Algorithm 2 to solve the linear system $Ax = b$.

(7) Output $\| Ax - b \|_2$.