3rd Workshop of Numerical Algebra and Applications
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Convergence analysis of structure-preserving doubling algorithm for the nonlinear matrix equation $X + A^T X^{-1} A = Q$

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Abstract

In 2006, Lin and Xu proposed a structure-preserving doubling algorithm (SDA) for solving the nonlinear matrix equation $X + A^T X^{-1} A = Q$ (with Q>0). The SDA was proven to be convergent quadratically to the maximal solution X_+ when all eigenvalues of $X_+^{-1}A$ lie inside the unit circle. In this talk, we will show that SDA converges linearly to a symmetric solution X_+ when all eigenvalues of $X_+^{-1}A$ are inside or on the unit circle and the partial multiplicity of each unimodular eigenvalue is half of the partial multiplicity of the corresponding unimodular eigenvalue of the associated symplectic pencil.

A structured doubling algorithm for nonsymmetric algebraic Riccati equations (a singular case)

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Abstract

In this talk, we propose a structured doubling algorithm (SDA) for the computation of the minimal nonnegative solutions to the nonsymmetric algebraic Riccati equation (NARE) and its dual equation simultaneously, for a singular case. Similar to the Newton's method we establish a global and linear convergence for SDA under the singular condition, using only elementary matrix theory. Numerical experiments show that the SDA algorithm is feasible and effective, outperforms Newton's method for NARE. Furthermore, SDA algorithm can easily be applied to solving the quadratic matrix equation, arising form quasi-birth-death (QBD) processes, which is different from the existing Latouchu-Ramaswami (LR) algorithm. The convergence of SDA is shown to be linear at least with rate $\frac{1}{2}$ when QBD is null recurrent.

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Balanced Realization of Periodic Descriptor Systems with Time-Varying Dimensions

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Abstract

From the necessary and sufficient conditions for complete reachability and observability of periodic descriptor systems with time-varying dimensions, the symmetric positive semi-definite reachability/observability Gramians are defined. These Gramians can be shown to satisfy some projected generalized discrete-time periodic Lyapunov equations. We propose a numerical method for solving these projected Lyapunov equations, and an illustrative numerical example is given. As an application of our results, the balanced realization of periodic descriptor systems is discussed.

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Transversal homoclinic points in the measure-preserving Henon map

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Abstract

Using shadowing techniques and computer-assisted methods, we prove the area and orientation preservingHenon map (corresponding to b=-1) $H_{a,b}(x,y)=(a-x^2+by,x)$ admits a transversal homoclinic point for a set of parameter values which is broad. The best result is that fixed a≥ -0.9786, then there exists transversal homoclinic points for $H_{a,b}$. This supports an old conjecture due to Devaney and Nitecki dating back to 1979.

Quadratic pencil solutions to the quadratic inverse eigenvalue problem

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Abstract

We are interested in the study of solvability of the quadratic inverse eigenvalue problem (QIEP) of dimension n. Let $k_* = (1+\sqrt{1+8n})/2$ and $0 \le k < k_*$, and for $m := n+k$ prescribed eigenpairs $\{(\lambda_{j,j})\}_{j=1}^{m}$, we prove that, generically, there is a constructible nonsingular symmetric quadratic pencil solution $Q(\lambda) \equiv \lambda^2 M + \lambda C + K$ to the QIEP such that $Q(\lambda_j)_j = 0 \quad (j = 1, \cdots, m)$. If $k_* \le k \le n$, we show that, generically, all symmetric quadratic pencil solutions are singular. We also derive the dimension of the solution subspace of the QIEP for both cases. Furthermore, we develop an algorithm for finding a symmetric positive definite M for the QIEP if it exists.

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Structure preserving methods for palindromic quadratic eigenvalue problems

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Abstract

We consider the palindromic quadratic eigenvalues problem $(\lambda^2 A_1^T + \lambda A_0 + A_1)x = 0$ with $A_1, A_0 \in \mathbb{C}^{n\times n}$ and $A_0^T = A_0$, which arises in the vibration analysis of fast trains and rail tracks. The eigenvalues of such palindromic QEP have symplectic structure, which means that they appear in pair $(\lambda, \frac{1}{\lambda})$. We propose two methods for finding all the eigenvalues and eigenvectors of the palindromic QEP, which preserve the special symplectic structure, and use unitary transformations whenever possible, and hence are numerically reliable. Numerical examples illustrate the good behavior of the methods.

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Adaptive continuation algorithms for computing energy levels of the rotating Bose-Einstein condensates

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Abstract

We describe adaptive continuation algorithms for computing energy levels of the Bose-Einstein condensates (BEC) with emphasis on the rotating BEC. We show that the rotating BEC in the complex plane is governed by special two-coupled nonlinear Schrödinger equations (NLS) in the real domain. Moreover, the energy levels of the discrete rotating BEC are twofold degenerate, which agree with those of an electron in quantum mechanics. A predictor-corrector continuation method is used to trace solution curves of the rotating BEC defined in square domains. The wave functions of the rotating BEC can be easily obtained whenever the solution curves of the two-coupled NLS are numerically traced. The superfluid densities we obtain on the first solution branch resemble the experimental results of Anglin and Ketterle [8]. We also obtain superfluid densities on the other solution branches, which to the best of our knowledge, have never shown up in the literatures.

Numerical Study of Two-Component Bose-Einstein Condensates

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Abstract

We develop a continuation method for the computation of positive bound states of time-independent, coupled Gross-Pitaevskii equations(CGPEs) which describe a multi-component Bose-Einstein condensate(BEC). A discretization of the CGPEs leads to a nonlinear algebraic eigenvalue problem(NAEP). The solution curve with respect to some parameter of the NAEP is then followed by the proposed method. For a single-component BEC, we prove that there exists a unique global minimizer (the ground state) which is represented by an ordinary differential equation with the initial value. For a two-component BEC, we prove that 2 identical ground/bound states will bifurcate in to 2 different ground/bound states at a finite repulsive inter-component scattering length. Numerical results of bifurcation diagrams and the forms of ground/positive bound state solutions for a two-component BEC with various inter-component scattering length and trap potentials are presented.

A Numerical Study of m-coupled Nonlinear Schrödinger Equations

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Abstract

We develop numerical schemes to solve the m-coupled discrete nonlinear Schrödinger equations (DNLSE) that are used to model nonlinear optics and multi-component Bose-Einstein condensates. These equations have abundant solution characteristics that are not fully understood due to the complexities of the systems. Our approaches include an iterative method for finding the ground state solution of the one-component DNLSE; and a modified continuation method that overcomes the stable and efficient problems due to the simulation of unbounded solution domain. The methods are analyzed theoretically and then implemented to conduct numerical experiments. Computational results of 3-coupled DNLSE demonstrate the versatility of the numerical solutions and a lower energy non-radially symmetric solution that is tricky to find.

(This is a joint work with Yuen-Cheng Kuo, Wen-Wei Lin, and Shih-Feng Shieh)

The measure theoretic entropy and eigenvalue estimates

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Abstract

In this talk, we will introduce the measure theoretic entropy for Markov chains in the dynamical system. The next, we will generalize the Parry's Theorem to provide a lower bound for the dominant eigenvalue of a nonnegative matrix. We will also show that this bound is better that the bound obtained from the Rayleigh quotient.

Application of the Self-adjusted Relaxation Factor in Computation of Turbulent Flow Through Parallel Plates with Streamwise Periodic Ribs

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Key words: relaxation factors, Bi-CGSTAB method, Nusselt number, faster convergence

ABSTRACT

In this study, a faster converged numerical method is applied to simulate the flow and forced-convection characteristics of turbulent flow through plates with streamwise periodic ribs. The method based on self-adjusted under relaxation factors and which is motivated from the concept of Bi-CGSTAB algorithm, thus the under relaxation factors for each control volume can be determined and hence the faster convergence for the numerical simulation can be achieved. In the flow-field, the upper wall is assumed thermally insulated, whereas the bottom plate with streamwise rectangular periodic ribs is provided with a uniform heat flux. The Durbin's $k-\varepsilon-v^2$ low Reynold's number model was applied to predict the flow field and heat transfer characteristics, the numerical approach is based on the finite volume technique with staggered grid arrangement, and the PISO algorithm is adopted to obtain the pressure and the velocities. The local Nusselt number distribution along the bottom wall is calculated and compared with the experiment results concluded by Lorenz et al. The proposed method provides the accurate flow-field and the forced-convection characteristics.

A Numerical Study on Reconstruction of Discontiuity

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Coupling Interface Method for Elliptic Interface Problem

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Abstract

We propose a coupling interface method (CIM) under Cartesian grid for solving elliptic *complex* interface problems in arbitrary dimensions, where the coefficients, the source terms, and the solutions may be discontinuous or singular across the interfaces. It consists of a first-order version (CIM1) and a second-order version (CIM2).
In one dimension, the CIM1 is derived from a linear approximation on both sides of the interface. The method is extended to high dimensions through a dimension-by-dimension approach. To connect information from each dimension, a coupled equation for the first-order derivatives is derived through the jump conditions in each coordinate direction. The resulting stencil uses the standard 5 grid points in two dimensions and 7 grid points in three dimensions. Similarly, the CIM2 is derived from a quadratic approximation in each dimension. In high dimensions, a coupled equation for the principal second-order derivatives $u_{x_{kl} x_k}$ is derived through the jump conditions in each coordinate direction. The cross derivatives are approximated by one-side interpolation. This approach reduces the number of grid points needed for one-side interpolation. The resulting stencil involves 8 grid points in two dimensions and 12-14 grid points in three dimensions.
A numerical study for the condition number of the resulting linear system of the CIM2 in one dimension has been performed. It is shown that the condition number has the same behavior as that of the discrete Laplacian, independent of the relative location of the interface in a grid cell Further, we also give a proof of the solvability of the coupling equations, provided the curvature κ of the interface satisfies $\kappa h \le Const.$, where h is the mesh size.

The CIM1 requires that the interface intersects each grid segment (the segment connecting two adjacent grid points)at most once. This is a very mild restriction and is always achievable by refining meshes. The CIM2 requires basically that the interface does not intersect two adjacent grid segments simultaneously. In practice, we classify the underlying Cartesian grid points into interiors, normal on-fronts, and exceptionals, where a standard central finite difference method, the CIM2,and the CIM1 are

adopted, respectively. This hybrid CIM maintains second order accuracy in most applications due to the fact that usually in d dimensions, the number of normal on-front grid points is $O(h^{1-d})$ and the number of the exceptional points is $O(1)$. Numerical convergence tests for the CIM1 and CIM2 are performed. A comparison study with other interface methods is also reported. Algebraic multigrid method is employed to solve the resulting linear system. Numerical tests demonstrate that CIM1 and CIM2 are respectively first order and second order in the maximal norm with less error as compared with other methods. In addition, this hybrid CIM passes many tests of complex interface problems in two and three dimensions. Therefore, we believe that it is a competitive method for complex interface problems.