When computing $x_i^{(k)}$ for $i > 1$, $x_1^{(k)}, \ldots, x_{i-1}^{(k)}$ have already been computed and are likely to be better approximations to the exact $x_1, \ldots, x_{i-1}$ than $x_1^{(k-1)}, \ldots, x_{i-1}^{(k-1)}$. It seems reasonable to compute $x_i^{(k)}$ using these most recently computed values. That is

$$\begin{aligned}
a_{11}x_1^{(k)} + a_{12}x_2^{(k-1)} + a_{13}x_3^{(k-1)} + \cdots + a_{1n}x_n^{(k-1)} &= b_1 \\
a_{21}x_1^{(k)} + a_{22}x_2^{(k)} + a_{23}x_3^{(k-1)} + \cdots + a_{2n}x_n^{(k-1)} &= b_2 \\
a_{31}x_1^{(k)} + a_{32}x_2^{(k)} + a_{33}x_3^{(k)} + \cdots + a_{3n}x_n^{(k-1)} &= b_3 \\
&\vdots \\
a_{n1}x_1^{(k-1)} + a_{n2}x_2^{(k-1)} + a_{n3}x_3^{(k-1)} + \cdots + a_{nn}x_n^{(k)} &= b_n.
\end{aligned}$$

This improvement induce the Gauss-Seidel method.

The Gauss-Seidel method sets $M = D + L$ and defines the iteration as

$$x^{(k)} = -(D+L)^{-1}Ux^{(k-1)} + (D+L)^{-1}b.$$

That is, Gauss-Seidel method uses $T = -(D+L)^{-1}U$ as the iteration matrix. The formulation above can be rewritten as

$$x^{(k)} = -D^{-1}\left(Lx^{(k)} + Ux^{(k-1)} - b\right).$$

Hence each component $x_i^{(k)}$ can be computed by

$$x_i^{(k)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k-1)}\right) \Bigg/ a_{ii}.$$

Algorithm (Gauss-Seidel Method)

For $k = 1, 2, \ldots$

For $i = 1, 2, \ldots, n$

$$x_i^{(k)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k-1)} \right) \Bigg/ a_{ii}$$

End for

End for

At each iteration, since $x_i^{(k)}$ can not be computed until $x_1^{(k)}, \ldots, x_{i-1}^{(k)}$ are available, Gauss-Seidel method is not a parallel algorithm in nature.