

$$\begin{aligned} a_{11}x_1^{(k)} + a_{12}x_2^{(k-1)} + a_{13}x_3^{(k-1)} + \cdots + a_{1n}x_n^{(k-1)} &= b_1 \\ a_{21}x_1^{(k-1)} + a_{22}x_2^{(k)} + a_{23}x_3^{(k-1)} + \cdots + a_{2n}x_n^{(k-1)} &= b_2 \\ &\vdots \\ a_{n1}x_1^{(k-1)} + a_{n2}x_2^{(k-1)} + a_{n3}x_3^{(k-1)} + \cdots + a_{nn}x_n^{(k)} &= b_n. \end{aligned}$$



If we decompose the coefficient matrix A as

$$A = L + D + U,$$

where D is the diagonal part, L is the strictly lower triangular part, and U is the strictly upper triangular part, of A , and choose $M = D$, then we derive the iterative formulation for Jacobi method:

$$x^{(k)} = -D^{-1}(L + U)x^{(k-1)} + D^{-1}b.$$

With this method, the iteration matrix $T = -D^{-1}(L + U)$ and $c = D^{-1}b$. Each component $x_i^{(k)}$ can be computed by

$$x_i^{(k)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k-1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k-1)} \right) / a_{ii}.$$



Algorithm (Jacobi Method)

For $k = 1, 2, \dots$

For $i = 1, 2, \dots, n$

$$x_i^{(k)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k-1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k-1)} \right) / a_{ii}$$

End for

End for

Only the components of $x^{(k-1)}$ are used to compute $x^{(k)}$.

$\Rightarrow x_i^{(k)}, i = 1, \dots, n$, can be computed in parallel at each iteration k .

