Consider solving

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}.$$

This system has the exact solution $x_1 = x_2 = 1$. Equivalently we can write the system as

$$\begin{cases} 3x_1 + 2x_2 = 5 \\ x_1 + 4x_2 = 5 \end{cases}$$

This implies that

$$\begin{cases} x_1 = \frac{1}{3}(5 - 2x_2) \\ x_2 = \frac{1}{4}(5 - x_1) \end{cases}$$

A naive idea is to solve the system by

$$\begin{cases} x_1^{(k)} = \frac{1}{3}(5 - 2x_2^{(k-1)}) \\ x_2^{(k)} = \frac{1}{4}(5 - x_1^{(k-1)}) \end{cases}$$

that is, to use the iterative formulation

$$\begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1^{(k-1)} \\ x_2^{(k-1)} \end{bmatrix} \right)$$

If we choose the initial guess $x_1^{(0)} = x_2^{(0)} = 0$, we would obtain

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1.6667 \\ 1.2500 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1.6667 \\ 1.2500 \end{bmatrix} \right) = \begin{bmatrix} 0.8333 \\ 0.8333 \end{bmatrix}$$

By repeating the process, we have the following table

k	3	4	5	6	7
$x_1^{(k)}$	1.1111	0.9722	1.0185	0.9954	1.0031
$x_2^{(k)}$	1.0417	0.9722	1.0000	0.9954	1.0012

From this example, we observe that the basic idea is to split the coefficient matrix A into

$$A = M - (M - A),$$

for some matrix M, which is called the splitting matrix. Here we assume that A and M are both nonsingular. Then the original problem is rewritten in the equivalent form

$$Mx = (M - A)x + b.$$

This suggests an iterative process

$$x^{(k)} = (I - M^{-1}A)x^{(k-1)} + M^{-1}b \equiv Tx^{(k-1)} + c,$$

where T is usually called the iteration matrix.