Diagnostics for leverage and influence

1. The hat matrix $H$ is the projection of the data points onto the space spanned by $X$

2. leverage($h$): The diagonal elements of $H$, denote by $h_{ii}$, $i = 1, \ldots, n$ reflect the influence of the $i$th data point on the model fit.

Cutoff point for $h_{ii}$. If $h_{ii} \geq 2p/n$, then point $i$ is judged as having an unusual value.

```R
> X=matrix(c(rep(1,4), 1:4), nrow=4)
> Y=as.matrix(c(1.127,1.541,1.846, 2.407))
> beta.hat=solve(t(X) %%*% X) %%*% t(X) %%*% Y
> H=X %%*% solve(t(X) %%*% X) %%*% t(X)
> y.hat=H %%*% Y
> X

[,1] [,2]
[1,]  1  1
[2,]  1  2
[3,]  1  3
[4,]  1  4

> Y

[,1]
[1,] 1.127
[2,] 1.541
[3,] 1.846
[4,] 2.407

> beta.hat

[,1]
[1,] 0.6940
[2,] 0.4145

> H

[1,]  0.7  0.4  0.1 -0.2
[2,]  0.4  0.3  0.2  0.1
[3,]  0.1  0.2  0.3  0.4
[4,] -0.2  0.1  0.4  0.7

> y.hat
```

1
Some common used residuals:

- standardized residuals:
- studentized residuals (internally studentized residuals):
- deleted residuals

Rstudentized residuals (externally studentized residuals, Jackknifed residuals):

\[ d_i = \frac{e_i}{\sqrt{\hat{\sigma}^2(1 - h_{ii})}} \]

\[ e_{(i)} = \frac{e_i}{1 - h_{ii}} \]

\[ t_i = \frac{e_{(i)}}{\sqrt{\text{Var}(e_{(i)})}} = \frac{e_i}{\sqrt{\hat{\sigma}^2(1 - h_{ii})}} \]

Example: delivery time

```r
> library(xtable)
> delivery[1:5,]

<table>
<thead>
<tr>
<th></th>
<th>TIM</th>
<th>CAS</th>
<th>DIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.68</td>
<td>7</td>
<td>560</td>
</tr>
<tr>
<td>2</td>
<td>11.50</td>
<td>3</td>
<td>220</td>
</tr>
<tr>
<td>3</td>
<td>12.03</td>
<td>3</td>
<td>340</td>
</tr>
<tr>
<td>4</td>
<td>14.88</td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>13.75</td>
<td>6</td>
<td>150</td>
</tr>
</tbody>
</table>
```

```r
> y=delivery$TIM
> x1=delivery$CAS
> x2=delivery$DIS
> f=lm(y~x1+x2)
> anova(f)
```

Analysis of Variance Table

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>1</td>
<td>5382</td>
<td>5382</td>
<td>506.6</td>
<td>&lt; 2e-16 ***</td>
</tr>
</tbody>
</table>
> summary(f)

Call:
  lm(formula = y ~ x1 + x2)

Residuals:
     Min      1Q  Median      3Q     Max
-5.7880 -0.6630  0.4360  1.1570  7.4200

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 2.341232   1.09673  2.130 0.04417 *
x1          1.615909   0.17073  9.460 3.3e-09 ***
x2          0.014377   0.00361  3.978 0.00063 ***

---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.26 on 22 degrees of freedom
Multiple R-squared: 0.96,   Adjusted R-squared: 0.956
F-statistic: 261 on 2 and 22 DF,  p-value: 4.69e-16

> y.hat=fitted(f)
> ei=residuals(f)
> di=rstandard(f)
> ti=rstudent(f)
> hii=hatvalues(f)
> deleted.r=ei/(1-hii)
> press=sum(deleted.r^2)
> press

[1] 459

> xtable(cbind(y, y.hat, ei, di, ti, deleted.r, hii))

<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th>y.hat</th>
<th>ei</th>
<th>di</th>
<th>ti</th>
<th>deleted.r</th>
<th>hii</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3.079</td>
<td>0.788</td>
<td>0.436</td>
<td>1.157</td>
<td>0.788</td>
<td>0.788</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4.187</td>
<td>0.663</td>
<td>0.363</td>
<td>0.663</td>
<td>0.663</td>
<td>0.663</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>5.295</td>
<td>0.436</td>
<td>0.236</td>
<td>0.436</td>
<td>0.436</td>
<td>0.436</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6.403</td>
<td>0.214</td>
<td>0.114</td>
<td>0.214</td>
<td>0.214</td>
<td>0.214</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>7.511</td>
<td>0.094</td>
<td>0.094</td>
<td>0.094</td>
<td>0.094</td>
<td>0.094</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>8.619</td>
<td>-0.094</td>
<td>-0.094</td>
<td>-0.094</td>
<td>-0.094</td>
<td>-0.094</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>9.727</td>
<td>-0.214</td>
<td>-0.214</td>
<td>-0.214</td>
<td>-0.214</td>
<td>-0.214</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>10.835</td>
<td>-0.436</td>
<td>-0.436</td>
<td>-0.436</td>
<td>-0.436</td>
<td>-0.436</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>11.943</td>
<td>-0.663</td>
<td>-0.663</td>
<td>-0.663</td>
<td>-0.663</td>
<td>-0.663</td>
</tr>
</tbody>
</table>

> outlier.test(f)

max|rstudent| = 4.311, degrees of freedom = 21, unadjusted p = 0.000309, Bonferroni p = 0.007726

Observation: 9
> library(car)
> par(mfrow=c(1,2))
> plot(ti)
> abline(h=c(0,-2,2), lty=c(1,2,2))
> qqnorm(ti); qqline(ti) #qq.plot(f, simulate=T)  #identify few possible outliers
\begin{tabular}{cccccccc}
\hline
y & y.hat & ei & di & ti & deleted.r & hii \\
\hline
1 & 16.68 & 21.71 & -5.03 & -1.63 & -1.70 & -5.60 & 0.10 \\
2 & 11.50 & 10.35 & 1.15 & 0.36 & 0.36 & 1.23 & 0.07 \\
3 & 12.03 & 12.08 & -0.05 & -0.02 & -0.02 & -0.06 & 0.10 \\
4 & 14.88 & 9.96 & 4.92 & 1.58 & 1.64 & 5.38 & 0.09 \\
5 & 13.75 & 14.19 & -0.44 & -0.14 & -0.14 & -0.48 & 0.08 \\
6 & 18.11 & 18.40 & -0.29 & -0.09 & -0.09 & -0.30 & 0.04 \\
7 & 8.00 & 7.16 & 0.84 & 0.27 & 0.26 & 0.92 & 0.08 \\
8 & 17.83 & 16.67 & 1.16 & 0.37 & 0.36 & 1.24 & 0.06 \\
9 & 79.24 & 71.82 & 7.42 & 3.21 & 4.31 & 14.79 & 0.50 \\
10 & 21.50 & 19.12 & 2.38 & 0.81 & 0.81 & 2.96 & 0.20 \\
11 & 40.33 & 38.09 & 2.24 & 0.72 & 0.71 & 2.45 & 0.09 \\
12 & 21.00 & 21.59 & -0.59 & -0.19 & -0.19 & -0.67 & 0.11 \\
13 & 13.50 & 12.47 & 1.03 & 0.33 & 0.32 & 1.09 & 0.06 \\
14 & 19.75 & 18.68 & 1.07 & 0.34 & 0.33 & 1.16 & 0.08 \\
15 & 24.00 & 23.33 & 0.67 & 0.21 & 0.21 & 0.70 & 0.04 \\
16 & 29.00 & 29.66 & -0.66 & -0.22 & -0.22 & -0.79 & 0.17 \\
17 & 15.35 & 14.91 & 0.44 & 0.14 & 0.13 & 0.46 & 0.06 \\
18 & 19.00 & 15.55 & 3.45 & 1.11 & 1.12 & 3.82 & 0.10 \\
19 & 9.50 & 7.71 & 1.79 & 0.58 & 0.57 & 1.98 & 0.10 \\
20 & 35.10 & 40.89 & -5.79 & -1.87 & -2.00 & -6.44 & 0.10 \\
21 & 17.90 & 20.51 & -2.61 & -0.88 & -0.87 & -3.13 & 0.17 \\
22 & 52.32 & 56.01 & -3.69 & -1.45 & -1.49 & -6.06 & 0.39 \\
23 & 18.75 & 23.36 & -4.61 & -1.44 & -1.48 & -4.81 & 0.04 \\
24 & 19.83 & 24.40 & -4.57 & -1.50 & -1.54 & -5.20 & 0.12 \\
25 & 10.75 & 10.96 & -0.21 & -0.07 & -0.07 & -0.23 & 0.07 \\
\hline
\end{tabular}

\begin{verbatim}
> plot(hii, type="h") # type for vertical line
> points(hii)
> abline(h=c(2, 3)*3/25, lty=2)
\end{verbatim}
> influencePlot(f)

[1] 9 22

3. DEFITS residuals: The deletion influence of the $i$th obs on the predicted or
fitted values
\[ \frac{\hat{y}_i - \hat{y}_{(i)}}{\sqrt{\hat{\sigma}^2_{(i)} h_{ii}}} \]

\[ \text{Var}(\hat{y}) = \sigma^2 h_{ii} \]

Cutoff point for DEFITS. If \( \text{DEFIT}_i > 2\sqrt{p/n} \), then point \( i \) is judged as having an unusual value.

> \text{dffits}(f)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.570850</td>
<td>0.098619</td>
<td>-0.005204</td>
<td>0.500802</td>
<td>-0.039459</td>
<td>-0.018779</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.078990</td>
<td>0.093761</td>
<td>4.296081</td>
<td>0.398713</td>
<td>0.217953</td>
<td>-0.067670</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>-0.017626</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Cook’s distance: The sum of squares of the deletion influences of each of the \( i \)th obs on the predicted or fitted values

\[ \sum_{j=1}^{n} (\hat{y}_j - \hat{y}_{j(i)})^2 \]

\[ \frac{p\hat{\sigma}^2}{\hat{\sigma}^2} \]

Cutoff point for Cook distance. If \( D_i > qf(0.5, p, n - p) \), then point \( i \) is judged as having an unusual value.

The \( D_i \) can be rewritten as

\[ D_i = \frac{e_i^2}{p\hat{\sigma}^2} \frac{h_{ii}}{1 - h_{ii}} \]

Another way to interpret Cook’s distance is that it is the squared Euclidean distance (apart from \( p\hat{\sigma}^2 \)) that the vector of fitted values moves when the \( i \)th observation is deleted.

> \text{cookd}(f)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.001e-01</td>
<td>3.376e-03</td>
<td>9.456e-06</td>
<td>7.765e-02</td>
<td>5.432e-04</td>
<td>1.231e-04</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2.172e-03</td>
<td>3.051e-03</td>
<td>3.419e+00</td>
<td>5.385e-02</td>
<td>1.620e-02</td>
<td>1.596e-03</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1.192e-02</td>
<td>1.324e-01</td>
<td>5.086e-02</td>
<td>4.510e-01</td>
<td>2.990e-02</td>
<td>1.023e-01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
> cooks.distance(f)

1    2    3    4    5    6
1.001e-01 3.376e-03 9.456e-06 7.765e-02 5.432e-04 1.231e-04
7    8    9   10   11   12
2.172e-03 3.051e-03 3.419e+00 5.385e-02 1.620e-02 1.596e-03
13   14   15   16   17   18
19   20   21   22   23   24
1.192e-02 1.324e-01 5.086e-02 4.510e-01 2.990e-02 1.023e-01
25
1.085e-04

> plot(cookd(f), type="h", pch=12)
> points(cookd(f))  #add point in the plot

5. DFBETAS: how much the regression coefficient $\hat{\beta}_j$ changes if the, in standard deviation units, $i$ obs were deleted.

> dfbs=dfbeta(f)
> head(dfbs)

(Intercept)    x1    x2
1  -0.197151  0.0674112 -1.508e-03
2   0.100491 -0.0083216  5.314e-05
3  -0.003946  0.0006900 -1.053e-05
> plot(dfbs[,c(2,3)])

6. COVRATIO: ratio for the precision of estimation when \( i \)th obs were deleted.

\[
COVRATIO_i = \frac{|(X'X)^{-1}\hat{\sigma}^2|}{|(X'X)^{-1}\hat{\sigma}^2|}
\]

> influence.measures(f)

Influence measures of
\[ \text{lm(formula = y ~ x1 + x2)} \] :

<table>
<thead>
<tr>
<th>dfb.1_</th>
<th>dfb.x1</th>
<th>dfb.x2</th>
<th>dffit</th>
<th>cov.r</th>
<th>cook.d</th>
<th>hat</th>
<th>inf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.18727 0.41131</td>
<td>-0.43486</td>
<td>-0.5709</td>
<td>0.871</td>
<td>1.00e-01 0.1018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.08979</td>
<td>-0.04776</td>
<td>0.01441</td>
<td>0.0986</td>
<td>1.215</td>
<td>3.38e-03 0.0707</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.00352 0.00395</td>
<td>-0.00235</td>
<td>-0.0052</td>
<td>1.276</td>
<td>9.46e-06 0.0987</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.45196 0.08828</td>
<td>-0.27337</td>
<td>0.5008</td>
<td>0.876</td>
<td>7.76e-02 0.0854</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.03167</td>
<td>-0.01330</td>
<td>0.02424</td>
<td>-0.0395</td>
<td>1.240</td>
<td>5.43e-04 0.0750</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.01468 0.00179</td>
<td>0.00108</td>
<td>-0.0188</td>
<td>1.200</td>
<td>1.23e-04 0.0429</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.07807</td>
<td>-0.02228</td>
<td>-0.01102</td>
<td>0.0790</td>
<td>1.240  2.17e-03 0.0818</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.07120</td>
<td>0.03338</td>
<td>-0.05382</td>
<td>0.0938</td>
<td>1.206</td>
<td>3.05e-03 0.0637</td>
<td></td>
</tr>
</tbody>
</table>
| 9      | -2.57574 | 0.92874 | 1.50755 | 4.2961 | 0.342  3.42e+00 0.4983 | *
10  0.10792 -0.33816  0.34133  0.3987  1.305  5.38e-02  0.1963
11  -0.03427  0.09253 -0.00269  0.2180  1.172  1.62e-02  0.0861
12  -0.03027 -0.04867  0.05397 -0.0677  1.291  1.60e-03  0.1137
13   0.07237 -0.03562  0.01134  0.0813  1.207  2.29e-03  0.0611
14   0.04952 -0.06709  0.06182  0.0974  1.228  3.29e-03  0.0782
15   0.02228 -0.00479  0.00684  0.0426  1.192  6.32e-04  0.0411
16  -0.00269  0.06442 -0.08419 -0.0972  1.369  3.29e-03  0.1659
17   0.02886  0.00649 -0.01570  0.0339  1.219  4.01e-04  0.0594
18   0.24856  0.18973 -0.27243  0.3653  1.069  4.40e-02  0.0963
19   0.17256  0.02357 -0.09897  0.1862  1.215  1.19e-02  0.0964
20   0.16804 -0.21500 -0.09292 -0.6718  0.760  1.32e-01  0.1017
21  -0.16193 -0.29718  0.33641 -0.3885  1.238  5.09e-02  0.1653
22   0.39857 -1.02541  0.57314 -1.1950  1.398  4.51e-01  0.3916 *
23  -0.15985  0.03729 -0.05265 -0.3075  0.890  2.99e-02  0.0413
24  -0.11972  0.40462 -0.46545 -0.5711  0.948  1.02e-01  0.1206
25  -0.01682  0.00085  0.00559 -0.0176  1.231  1.08e-04  0.0666

possible outliers 9 & 22

> cal.press=function(f){
+   ei=residuals(f)
+   hii=hatvalues(f)
+   deleted.r=ei/(1-hii)
+   press=sum(deleted.r^2)
+   (press)
+ }

> b=matrix(NA, nrow=4, ncol=6) #4*6 matrix
> head(delivery)

    TIM  CAS  DIS
   1  16.68   7  560
   2  11.50   3  220
   3  12.03   3  340
   4  14.88   4  80
   5  13.75   6  150
   6  18.11   7  330

> f=lm(TIM~CAS+DIS, data=delivery)
> summary(f)

Call:
  lm(formula = TIM ~ CAS + DIS, data = delivery)

Residuals:
    Min     1Q   Median     3Q    Max
  -0.3382  -0.0842   -0.0972  -0.0972  5.38e-02  6.32e-04
Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | 2.34123 | 1.09673 | 2.13     | 0.04417 * |
| CAS     | 1.61591 | 0.17073 | 9.46     | 3.3e-09 *** |
| DIS     | 0.01438 | 0.00361 | 3.98     | 0.00063 *** |

---

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.26 on 22 degrees of freedom
Multiple R-squared: 0.96, Adjusted R-squared: 0.956
F-statistic: 261 on 2 and 22 DF, p-value: 4.69e-16

```r
> SSres=sum(residuals(f)^2)
> p=length(coef(f))
> n=p+df.residual(f)
> SSto=(n-1)*var(delivery$TIM)
> MSres=SSres/df.residual(f)
> R2=1-(SSres/SSto)
> (sigma2=ls.diag(f)$std.dev^2)
> b[1,]=c(coef(f), MSres, R2, cal.press(f))
> del.9=delivery[-9,]
> f=lm(TIM~CAS+DIS, data=del.9)
> summary(f)

Call:
  lm(formula = TIM ~ CAS + DIS, data = del.9)

Residuals:
     Min      1Q  Median      3Q     Max
-4.0325 -1.2331  0.0199  1.4730  4.8167

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | 4.44724 | 0.95247 | 4.67     | 0.00013 *** |
| CAS     | 1.49769 | 0.13021 | 11.50    | 1.6e-10 *** |
| DIS     | 0.01032 | 0.00285 | 3.62     | 0.00161 ** |

---

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.43 on 21 degrees of freedom
Multiple R-squared: 0.949, Adjusted R-squared: 0.944
F-statistic: 194 on 2 and 21 DF, p-value: 2.86e-14
```
> SSres=sum(residuals(f)^2)
> p=length(coef(f))
> n=p+df.residual(f)
> SSSto=(n-1)*var(del.9$TIM)
> MSres=SSres/df.residual(f)
> R2=1-(SSres/SSSto)
> (sigma2=ls.diag(f)$std.dev^2)

[1] 5.905

> b[2,]=c(coef(f), MSres, R2, cal.press(f))
> del.22=delivery[-22,]
> f=lm(TIM~CAS+DIS, data=del.22)
> summary(f)

Call:
  lm(formula = TIM ~ CAS + DIS, data = del.22)

Residuals:
     Min      1Q  Median      3Q     Max
-6.7070 -0.9139  0.5079  1.4264  5.6756

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.91574   1.10511   1.73  0.0977 .
CAS         1.78632   0.20176   8.85  1.6e-08 ***
DIS         0.01237   0.00377   3.28  0.0036 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.17 on 21 degrees of freedom
Multiple R-squared: 0.956,  Adjusted R-squared: 0.952
F-statistic: 230 on 2 and 21 DF,  p-value: 5.15e-15

> SSres=sum(residuals(f)^2)
> p=length(coef(f))
> n=p+df.residual(f)
> SSSto=(n-1)*var(del.22$TIM)
> MSres=SSres/df.residual(f)
> R2=1-(SSres/SSSto)
> (sigma2=ls.diag(f)$std.dev^2)

[1] 10.07

> b[3,]=c(coef(f), MSres, R2, cal.press(f))
> del.both=delivery[-c(9,22),]
> f=lm(TIM~CAS+DIS, data=del.both)
> summary(f)
Call:
`lm(formula = TIM ~ CAS + DIS, data = del.both)`

Residuals:
```
  Min 1Q Median 3Q Max
-4.060 -1.253 -0.136 1.515 5.140
```

Coefficients:
```
            Estimate Std. Error t value  Pr(>|t|) 
(Intercept)  4.64269    1.12598   4.120  0.00053 ***
    CAS          1.45561    0.18048   8.067 1.00e-07 ***
    DIS          0.01055    0.00299   3.530  0.00209 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 2.48 on 20 degrees of freedom
Multiple R-squared:  0.907, Adjusted R-squared:  0.898
F-statistic: 97.8 on 2 and 20 DF, p-value: 4.74e-11

```r
> SSres=sum(residuals(f)^2)
> p=length(coef(f))
> n=p+df.residual(f)
> SSto=(n-1)*var(del.both$TIM)
> MSres=SSres/df.residual(f)
> R2=1-(SSres/SSto)
> (sigma2=ls.diag(f)$std.dev^2)

[1] 6.163
```

```r
> b[4,]=c(coef(f), MSres, R2, cal.press(f))
> library(xtable)
> rownames(b)=c("all", "-9", "-22", "-c(9,22)")
> colnames(b)=c("b0","b1","b2","MSres","R2","PRESS")
> xtable(b)
```

<table>
<thead>
<tr>
<th></th>
<th>b0</th>
<th>b1</th>
<th>b2</th>
<th>MSres</th>
<th>R2</th>
<th>PRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>2.34</td>
<td>1.62</td>
<td>0.01</td>
<td>10.62</td>
<td>0.96</td>
<td>459.04</td>
</tr>
<tr>
<td>-9</td>
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<td>1.50</td>
<td>0.01</td>
<td>5.90</td>
<td>0.95</td>
<td>165.25</td>
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<tr>
<td>-22</td>
<td>1.92</td>
<td>1.79</td>
<td>0.01</td>
<td>10.07</td>
<td>0.96</td>
<td>474.22</td>
</tr>
<tr>
<td>-c(9,22)</td>
<td>4.64</td>
<td>1.46</td>
<td>0.01</td>
<td>6.16</td>
<td>0.91</td>
<td>186.73</td>
</tr>
</tbody>
</table>