

Chapter 9

Vectors and the Geometry of Space

(向量與空間幾何)

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9.0 Definitions and Preliminaries

9.6 Surfaces in Space

9.7 Cylindrical and Spherical Coordinates



Section 9.0

Definitions and Preliminaries

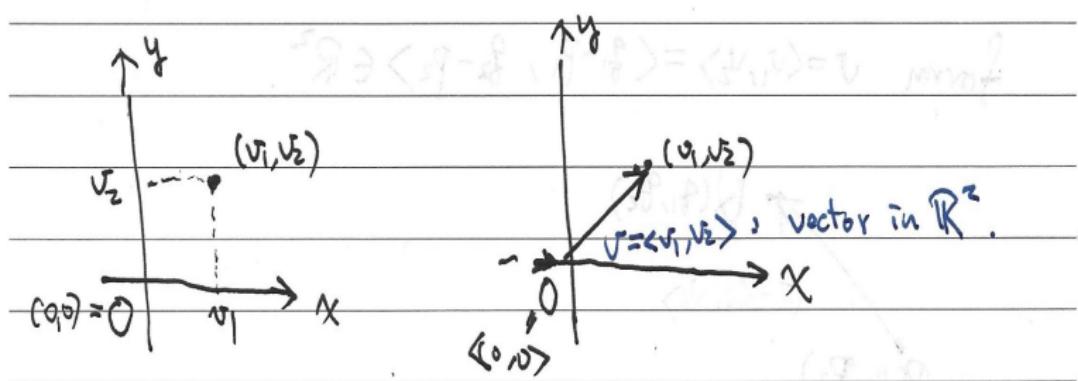
(定義和預備知識)



Vectors in the Plane (平面向量)

Two-Dimensional Euclidean (Vector) Space

$$\begin{aligned}\mathbb{R}^2 &= \{(v_1, v_2) \mid v_1, v_2 \in \mathbb{R}\} \\ &= \{v = \langle v_1, v_2 \rangle \mid v \text{ is a vector (向量)}\}\end{aligned}$$



- Vector Addition (向量加法):

$$v + u = \langle v_1, v_2 \rangle + \langle u_1, u_2 \rangle = \langle v_1 + u_1, v_2 + u_2 \rangle \quad \forall v, u \in \mathbb{R}^2.$$

- Scalar Multiplication (純量乘法):

$$cv = c\langle v_1, v_2 \rangle = \langle cv_1, cv_2 \rangle \quad \forall c \in \mathbb{R} \text{ and } v \in \mathbb{R}^2.$$

- Length or Norm (範數) of a Vector:

$$\|v\| = \|\langle v_1, v_2 \rangle\| = \sqrt{v_1^2 + v_2^2} \geq 0 \quad \forall v \in \mathbb{R}^2.$$

- $v \in \mathbb{R}^2$ is a unit vector (單位向量) if $\|v\| = 1$.



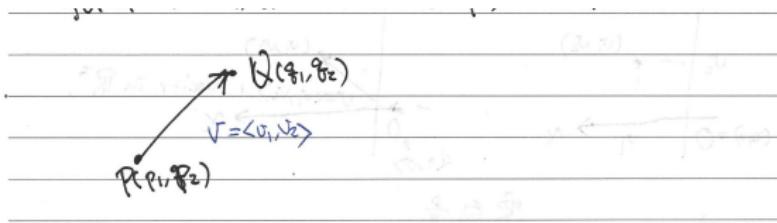
Definitions and Notations (2/2)

- If $i = \langle 1, 0 \rangle$ and $j = \langle 0, 1 \rangle$ are standard unit vectors in \mathbb{R}^2 , then

$$v = \langle v_1, v_2 \rangle = v_1 i + v_2 j \quad \forall v \in \mathbb{R}^2.$$

- If v is represented by the directed line segment from $P(p_1, p_2)$ to $Q(q_1, q_2)$, then it has the component form (分量形式)

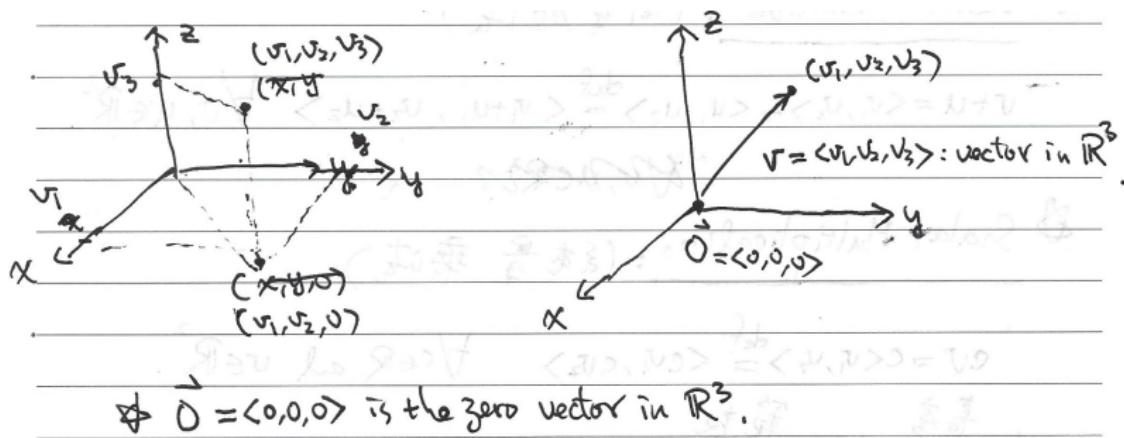
$$v = \langle v_1, v_2 \rangle = \langle q_1 - p_1, q_2 - p_2 \rangle \in \mathbb{R}^2.$$



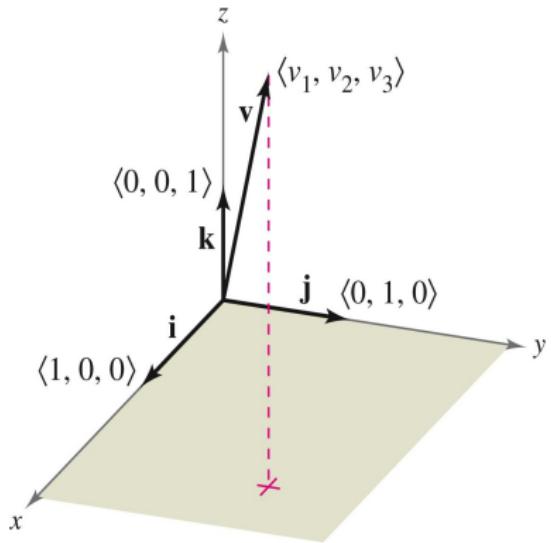
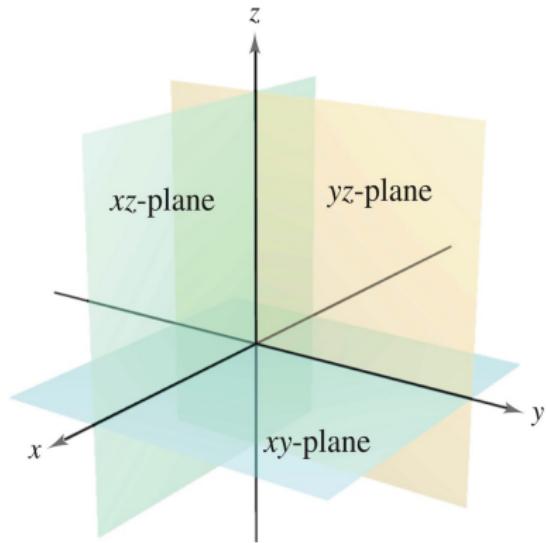
Vectors in Space (空間向量)

Three-Dimensional Euclidean (Vector) Space

$$\begin{aligned}\mathbb{R}^3 &= \{(v_1, v_2, v_3) \mid v_1, v_2, v_3 \in \mathbb{R}\} \\ &= \{v = \langle v_1, v_2, v_3 \rangle \mid v \text{ is a vector in space}\}\end{aligned}$$



Vectors in Space



- Vector Addition (向量加法):

$$\begin{aligned}v + u &= \langle v_1, v_2, v_3 \rangle + \langle u_1, u_2, u_3 \rangle \\&= \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle \quad \forall v, u \in \mathbb{R}^3.\end{aligned}$$

- Scalar Multiplication (純量乘法):

$$cv = c\langle v_1, v_2, v_3 \rangle = \langle cv_1, cv_2, cv_3 \rangle \quad \forall c \in \mathbb{R} \text{ and } v \in \mathbb{R}^3.$$

- Length or Norm of a Vector:

$$\|v\| = \|\langle v_1, v_2, v_3 \rangle\| = \sqrt{v_1^2 + v_2^2 + v_3^2} \geq 0 \quad \forall v \in \mathbb{R}^3.$$

- $v \in \mathbb{R}^3$ is a unit vector if $\|v\| = 1$.



Definitions in \mathbb{R}^3 (2/2)

- If $i = \langle 1, 0, 0 \rangle$, $j = \langle 0, 1, 0 \rangle$ and $k = \langle 0, 0, 1 \rangle$ are standard unit vectors in \mathbb{R}^3 , then

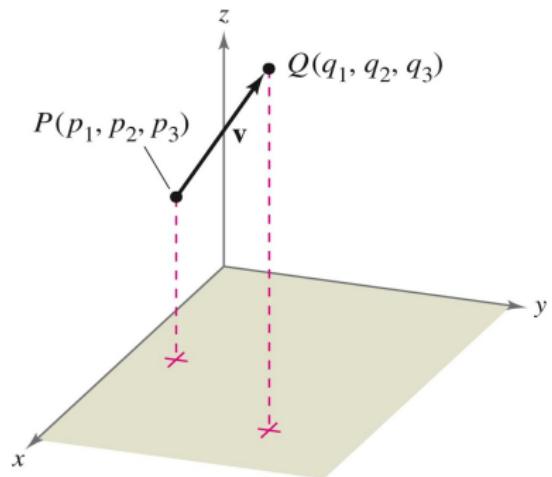
$$v = \langle v_1, v_2, v_3 \rangle = v_1 i + v_2 j + v_3 k \quad \forall v \in \mathbb{R}^3.$$

- If v is represented by the directed line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$, then it has the component form

$$v = \langle v_1, v_2, v_3 \rangle = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle \in \mathbb{R}^3.$$



向量 \overrightarrow{PQ} 的示意圖 (承上頁)



$$\mathbf{v} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle$$



* Properties of Vector Operations :

Thm: Let $u, v, w \in \mathbb{R}^2$ or \mathbb{R}^3 and $c, d \in \mathbb{R}$. Then

$$\textcircled{1} \quad u+v = v+u.$$

$$\textcircled{2} \quad (u+v)+w = u+(v+w).$$

$$\textcircled{3} \quad u+\vec{0} = \vec{0}+u = u \quad \textcircled{4} \quad u+(-u) = \vec{0}.$$

$$\textcircled{5} \quad c(d u) = (cd) u = d(c u)$$

$$\textcircled{6} \quad (c+d) u = cu + du.$$

$$\textcircled{7} \quad c(u+v) = cu + cv$$

$$\textcircled{8} \quad (1) u = u, (2) u = \vec{0}.$$

$$\textcircled{9} \quad \|cu\| = |c| \cdot \|u\|$$

$$\textcircled{10} \quad \|u\| = 0 \Leftrightarrow u = \vec{0}.$$



Def. (向量內積的定義)

- (1) The dot product of $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$ is

$$u \bullet v = u_1 v_1 + u_2 v_2 \in \mathbb{R}.$$

- (2) The dot product of $u = \langle u_1, u_2, u_3 \rangle$ and $v = \langle v_1, v_2, v_3 \rangle$ is

$$u \bullet v = u_1 v_1 + u_2 v_2 + u_3 v_3 \in \mathbb{R}.$$

- (3) In some textbooks, the dot product is also called the inner product of vectors.



Thm (Properties of Dot Product)

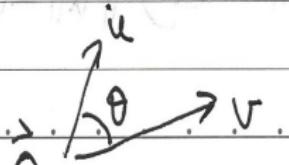
Let $u, v, w \in \mathbb{R}^2$ or \mathbb{R}^3 , and $c \in \mathbb{R}$.

$$\textcircled{1} \quad u \cdot v = v \cdot u. \quad \textcircled{2} \quad u \cdot (v+w) = u \cdot v + u \cdot w.$$

$$\textcircled{3} \quad c(u \cdot v) = (cu) \cdot v = u \cdot (cv) \quad \textcircled{4} \quad \vec{0} \cdot v = 0$$

$$\textcircled{5} \quad v \cdot v = \|v\|^2. \quad \textcircled{6} \quad \cos \theta = \frac{u \cdot v}{\|u\| \|v\|} \quad \text{for } 0 \leq \theta \leq \pi,$$

where θ is the angle between vectors u and v .



Some Special Vectors

Let u and v be vectors in \mathbb{R}^2 or \mathbb{R}^3 .

- $\frac{v}{\|v\|}$ is the unit vector in the direction of $v \neq 0$.
(沿著 v 方向的單位向量)
- u and v are parallel vectors (平行向量) if $\exists c \in \mathbb{R}$ s.t. $u = cv$.
- u and v are orthogonal vectors (垂直向量) if $u \bullet v = 0$.



Example 2, p. 571: (求向量的夾角)

For $u = \langle 3, -1, 2 \rangle$, $v = \langle -4, 0, 2 \rangle$, $w = \langle 1, -1, -2 \rangle$

and $z = \langle 2, 0, -1 \rangle$, find the angle between

- (a) u and v (b) u and w (c) v and z .

Sol:

$$(a) \cos\theta = \frac{u \cdot v}{\|u\|\|v\|} = \frac{-12 + 0 + 4}{\sqrt{14}\sqrt{20}} = \frac{-8}{2\sqrt{14}\sqrt{5}} = \frac{-4}{\sqrt{70}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-4}{\sqrt{70}}\right) \approx 2.069 \text{ radians.}$$

$$(b) \cos\theta = \frac{u \cdot w}{\|u\|\|w\|} = \frac{3 + 1 - 4}{\sqrt{14}\sqrt{6}} = 0 \Rightarrow \theta = \frac{\pi}{2}.$$

So, u and w are orthogonal vectors.

$$(c) \cos\theta = \frac{v \cdot z}{\|v\|\|z\|} = \frac{-8 + 0 - 2}{\sqrt{20}\sqrt{5}} = \frac{-10}{\sqrt{100}} = -1 \Rightarrow \theta = \pi.$$

So, v and z are parallel vectors with $v = (-2)z$.



Cross Product of Vectors in \mathbb{R}^3

Def. (空間向量的外積)

The cross product of $u = \langle u_1, u_2, u_3 \rangle$ and $v = \langle v_1, v_2, v_3 \rangle$ is

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \quad (\text{對第一列作行列式降階!})$$

$$= \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} i - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} j + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} k.$$

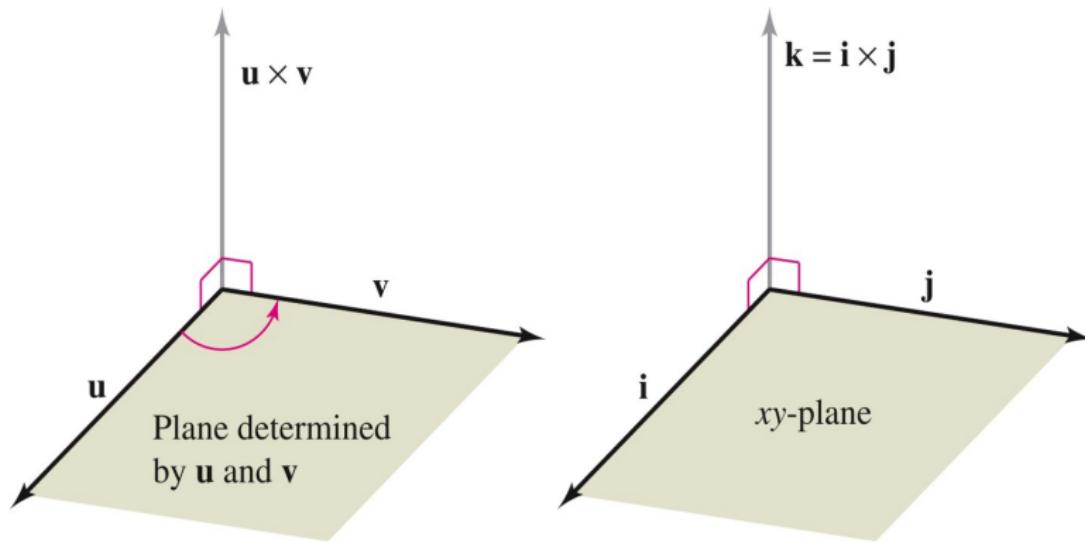


Notes

- $u \times v$ is a vector in \mathbb{R}^3 , but $u \bullet v$ is a scalar.
- $(u \times v) \bullet u = 0 = (u \times v) \bullet v$, i.e., the vector $u \times v$ is orthogonal to u and v , respectively.
- $v \times u = -(u \times v)$, i.e., they are parallel vectors, but in the opposite directions.



外積的示意圖 (承上頁)



Thm (Properties of Cross Products)

Let $u, v, w \in \mathbb{R}^3$ and $c \in \mathbb{R}$. Then

$$\textcircled{1} \quad u \times v = -(v \times u) \quad \textcircled{2} \quad u \times (v + w) = u \times v + u \times w.$$

$$\textcircled{3} \quad c(u \times v) = (cu) \times v = u \times (cv) \quad \textcircled{4} \quad u \times \vec{0} = \vec{0} \times u = \vec{0}.$$

$$\textcircled{5} \quad u \times u = \vec{0} \quad \textcircled{6} \quad u \bullet (v \times w) = (u \times v) \bullet w.$$



Example: Find $U \times V$ of vectors $U = \langle 1, -2, 1 \rangle$ and $V = \langle 3, 1, -2 \rangle$.

Sol:

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 3 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -2 \\ 1 & -2 \\ 3 & -2 \end{vmatrix} i + \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} j + \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} k$$

$$= 3i + 5j + 7k = \langle 3, 5, 7 \rangle \in \mathbb{R}^3$$



Section 9.6

Surfaces in Space

(空間中的曲面)



Type I: Cylindrical Surfaces (柱狀曲面或是柱面)

If the line L is not parallel to the plane containing a curve C , then

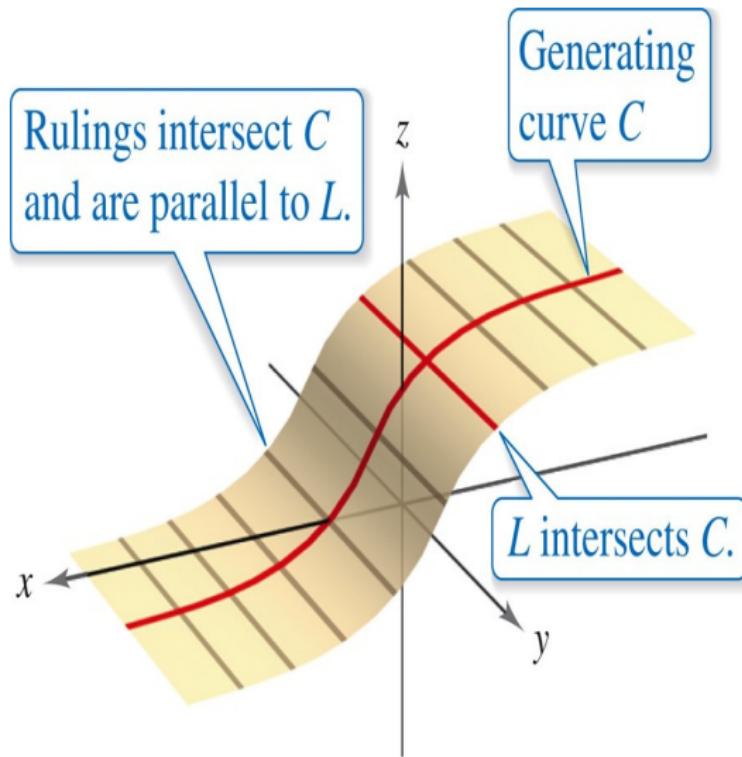
$$\mathcal{S} = \{\ell \mid \ell \text{ is a line parallel to } L \text{ and intersecting } C\}$$

is a cylindrical surface, or simply a cylinder.

- C is the **generating curve** of \mathcal{S} .
- The lines parallel to L are **rulings**.



柱面的示意圖 (承上頁)



Example

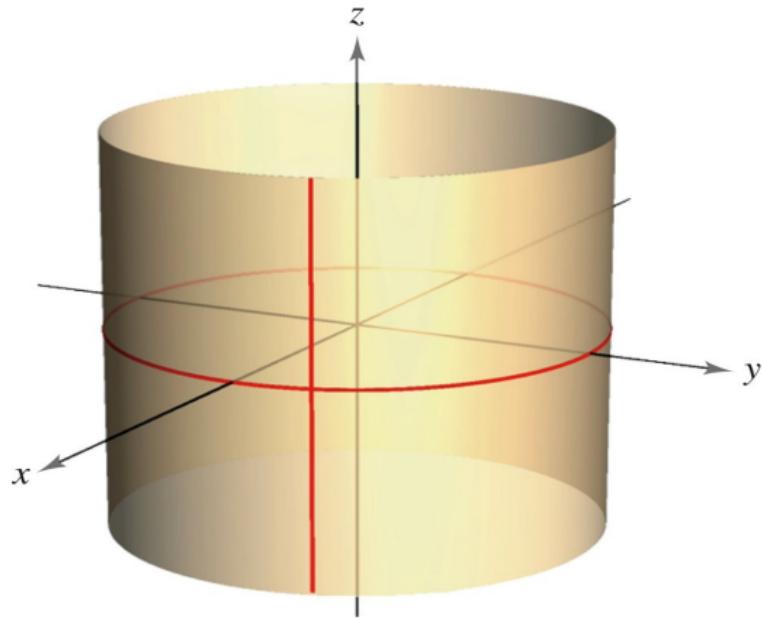
The right circular cylinder (圓柱面) with radius $a > 0$ is defined by

$$\mathcal{S} = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = a^2\}.$$

Then the generating curve of the cylinder \mathcal{S} lying in the xy -plane is
 $C: x^2 + y^2 = a^2$.



圓柱曲面的示意圖 (承上頁)



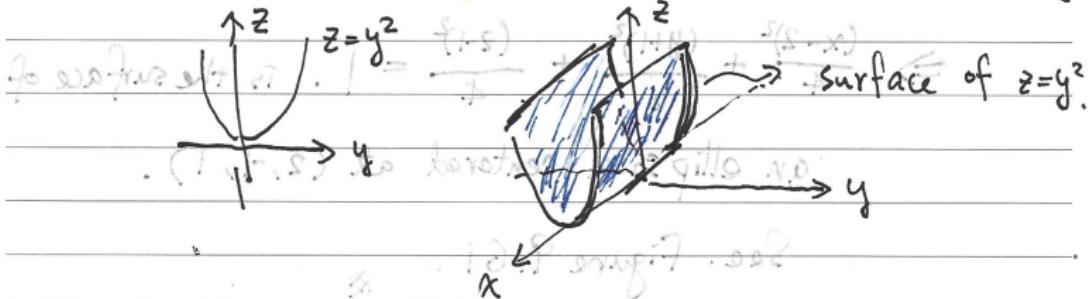
Right circular cylinder:
 $x^2 + y^2 = a^2$



Example 1: Sketch the surfaces of the cylinders.

(a) $z = y^2$ and $x \in \mathbb{R}$

\Rightarrow the generating curve lying on the yz -plane is $z = y^2$.



(b) $z = \sin x$ for $0 \leq x \leq 2\pi$ and $y \in \mathbb{R}$

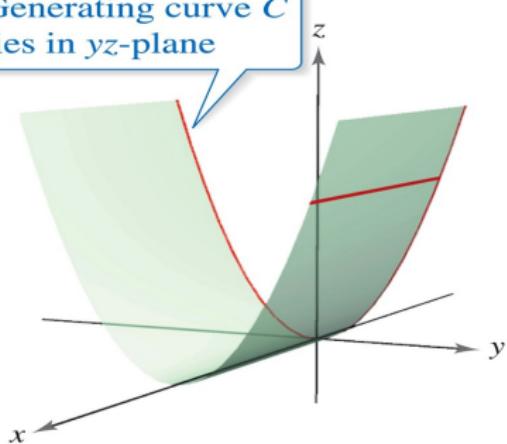
\Rightarrow the generating curve lying on the xz -plane is

$z = \sin x$ for $0 \leq x \leq 2\pi$



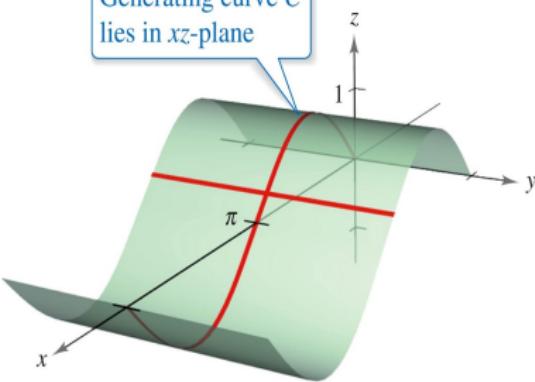
Example 1 的示意圖

Generating curve C
lies in yz -plane



$$\text{Cylinder: } z = y^2$$

Generating curve C
lies in xz -plane



$$\text{Cylinder: } z = \sin x$$



Type II: Quadratic Surfaces (二次曲面)

The general equation of a quadratic surface is

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0,$$

where the coefficients A, B, \dots, J are real numbers.



Type II: Quadratic Surfaces

(1) Ellipsoid: (橢圓曲面)

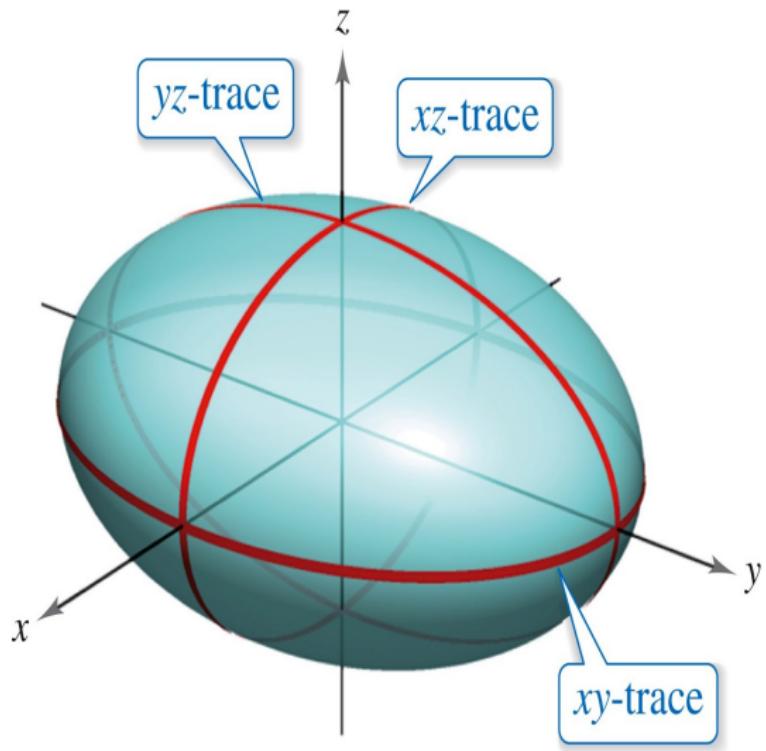
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{with } a, b, c > 0.$$

Then the xy -trace, xz -trace and yz -trace of the surface are

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \frac{x^2}{a^2} + \frac{z^2}{c^2} = 1 \quad \text{and} \quad \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$



Ellipsoid 的示意圖 (承上頁)



Example 4:

Sketch the surface $x^2 + 2y^2 + z^2 - 4x + 4y - 2z + 3 = 0$.

Sol: $x^2 + 2y^2 + z^2 - 4x + 4y - 2z + 3 = 0$

$$\Rightarrow (x^2 - 4x + 4) + 2(y^2 + 2y + 1) + (z^2 - 2z + 1) - 4 = 0.$$

$$\Rightarrow (x-2)^2 + 2(y+1)^2 + (z-1)^2 = 4$$

$$\Rightarrow \frac{(x-2)^2}{4} + \frac{(y+1)^2}{2} + \frac{(z-1)^2}{4} = 1. \text{ is the surface of}$$

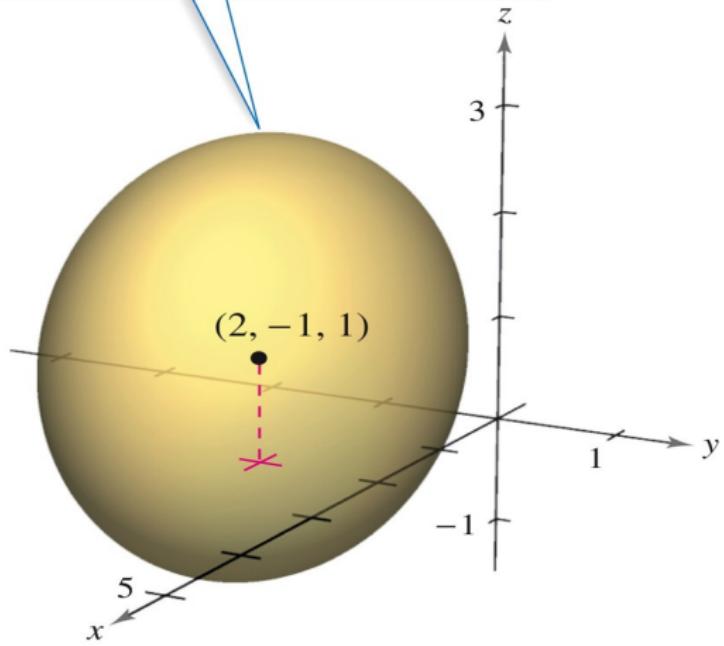
an ellipsoid centered at $(2, -1, 1)$.

See Figure 9.61.



Example 4 的示意圖

$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{2} + \frac{(z-1)^2}{4} = 1$$



Type II: Quadratic Surfaces

(2) Hyperboloid of One Sheet: (單片雙曲面)

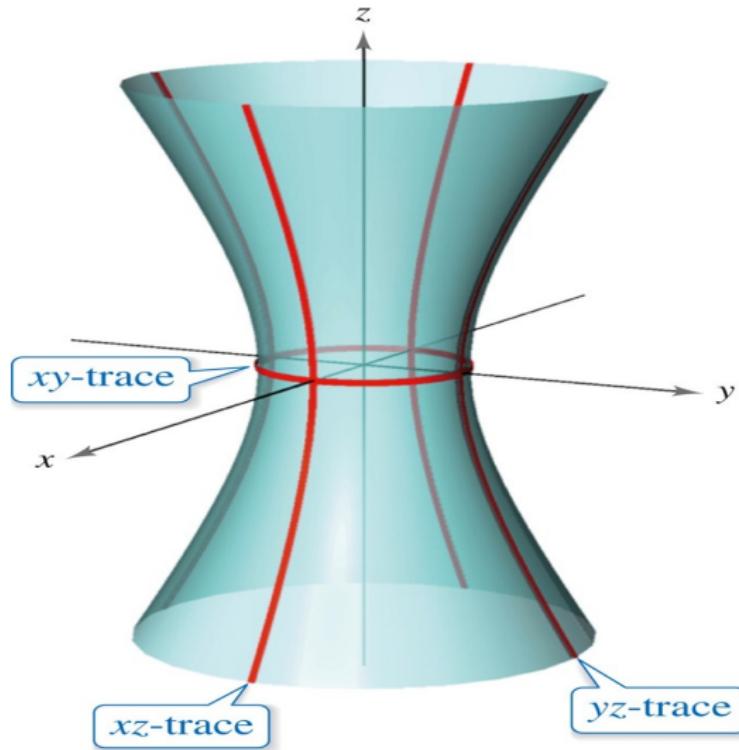
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \text{with } a, b, c > 0.$$

Then the xy -trace, xz -trace and yz -trace of the surface are

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \frac{x^2}{a^2} - \frac{z^2}{c^2} = 1 \quad \text{and} \quad \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$$



Hyperboloid of One Sheet 的示意圖



Type II: Quadratic Surfaces

(3) Hyperboloid of Two Sheets: (雙片雙曲面)

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{with } a, b, c > 0.$$

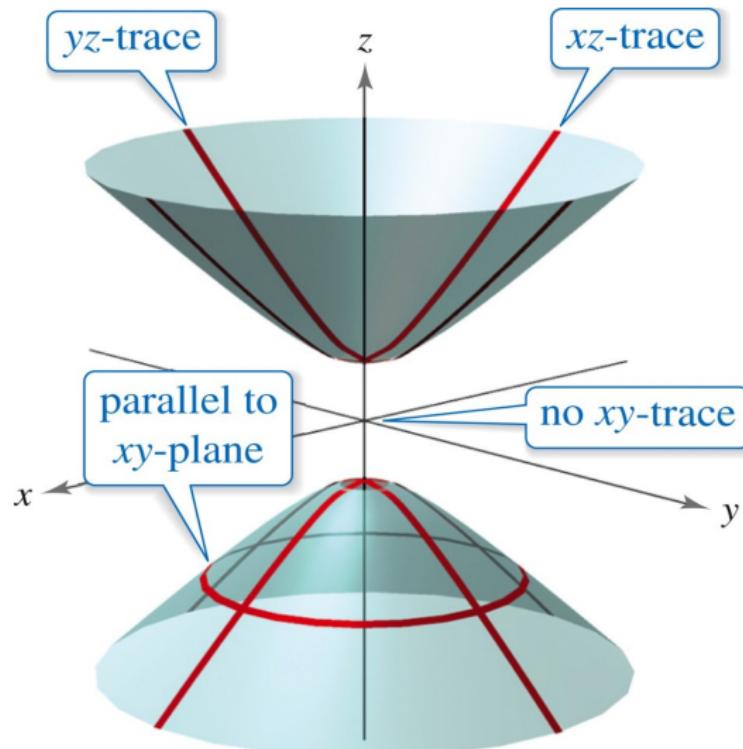
Then the xz -trace and yz -trace of the surface are

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} = 1 \quad \text{and} \quad \frac{z^2}{c^2} - \frac{y^2}{b^2} = 1,$$

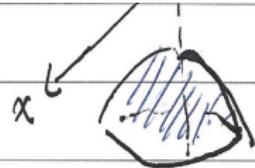
but no xy -trace!



Hyperboloid of Two Sheets 的示意圖



Example 2: Sketch the surface



$$4x^2 - 3y^2 + 12z^2 + 12 = 0$$

Sof: $4x^2 - 3y^2 + 12z^2 + 12 = 0$

$$\Rightarrow 3y^2 - 4x^2 - 12z^2 = 12$$

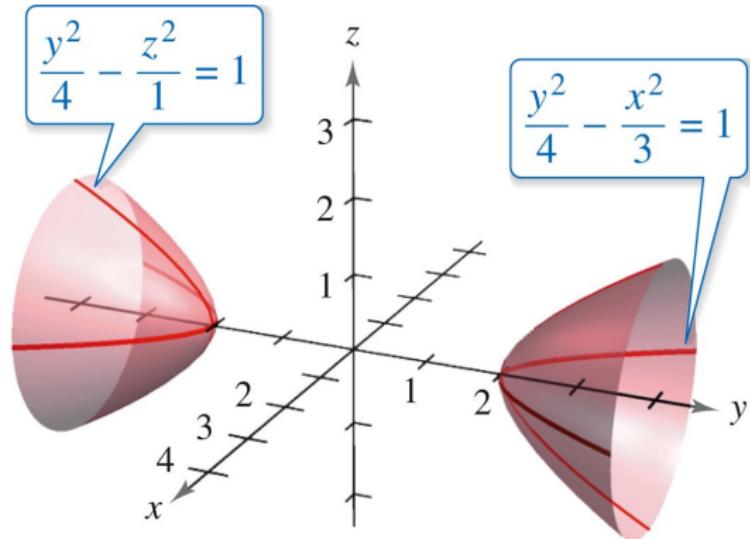
$$\Rightarrow \frac{y^2}{4} - \frac{x^2}{3} - \frac{z^2}{1} = 1$$

is the surface of a hyperboloid
of with two sheets.

See Figure 9.59.



Example 2 的示意圖



Hyperboloid of two sheets:

$$\frac{y^2}{4} - \frac{x^2}{3} - z^2 = 1$$



Type II: Quadratic Surfaces

(4) Elliptic Cone: (橢圓錐面)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \quad \text{with } a, b, c > 0.$$

Then the xy -trace, xz -trace and yz -trace of the surface are

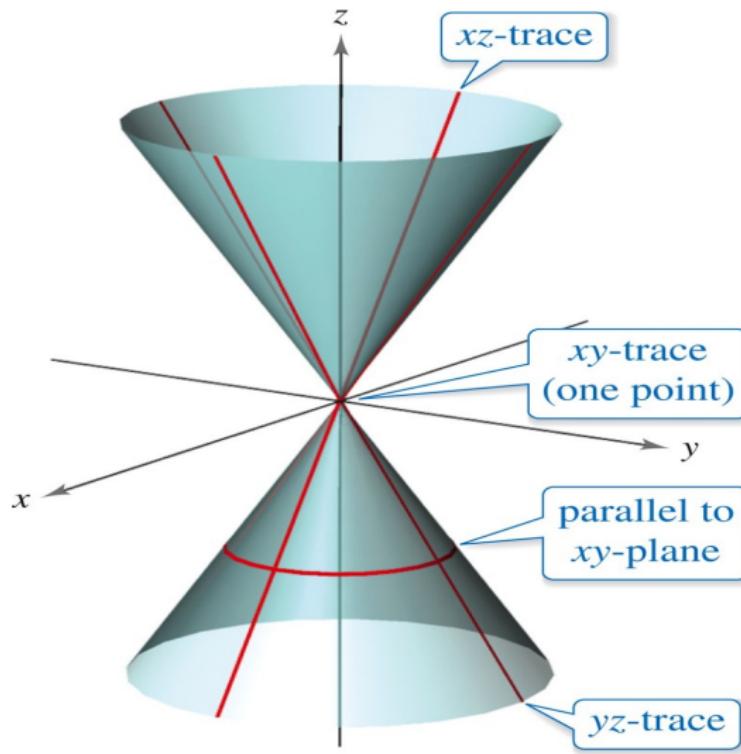
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = k \quad \text{for some } k \geq 0,$$

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 0 \implies z = \pm \frac{c}{a}x \quad \text{and}$$

$$\frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \implies z = \pm \frac{c}{b}y.$$



Elliptic Cone 的示意圖



Type II: Quadratic Surfaces

(5) Elliptic Paraboloid: (橢圓拋物面)

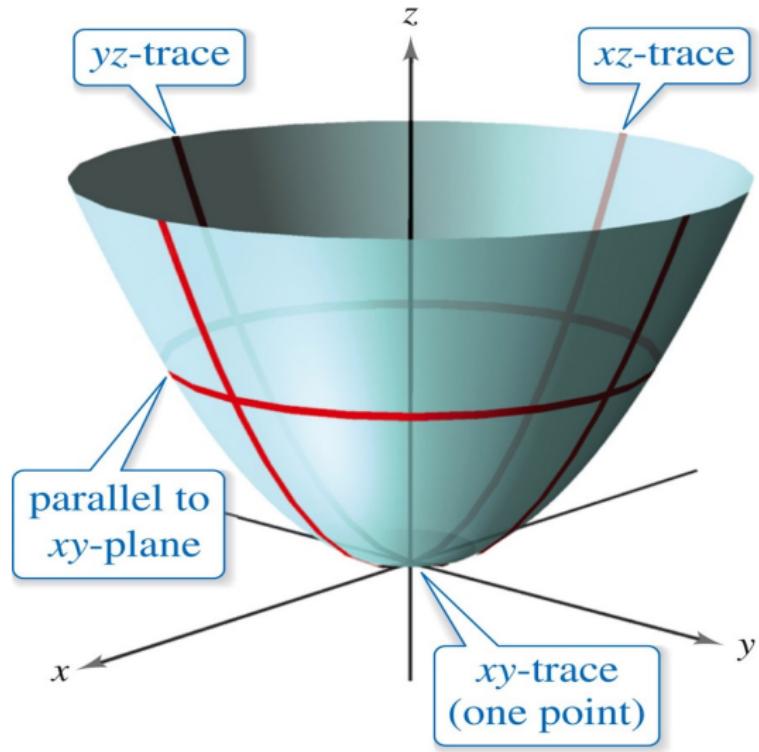
$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2} \quad \text{with } a, b > 0.$$

Then the xy -trace, xz -trace and yz -trace of the surface are

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = k \quad \text{for some } k \geq 0, \quad z = \frac{x^2}{a^2} \quad \text{and} \quad z = \frac{y^2}{b^2}.$$



Elliptic Paraboloid 的示意圖



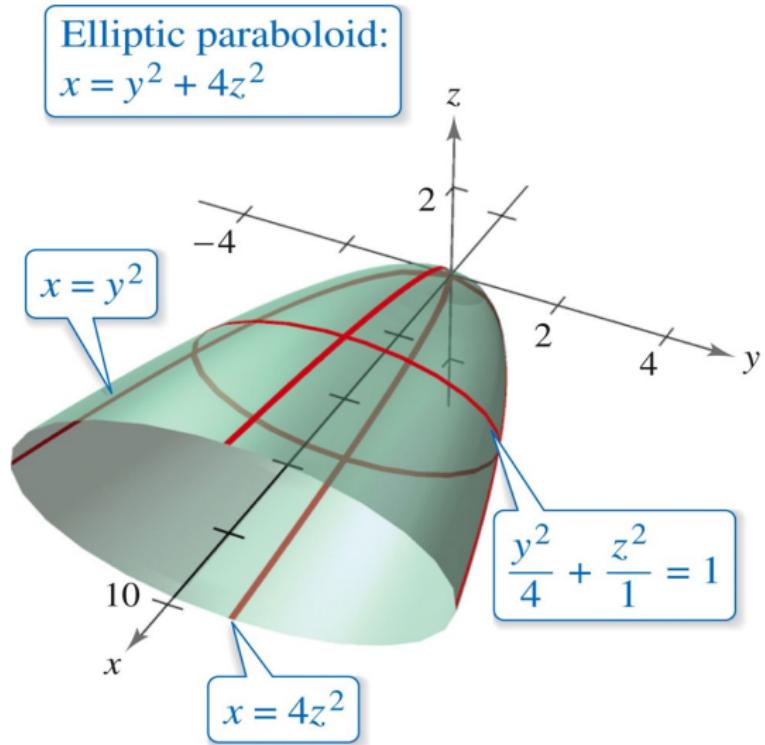
Example 3: Sketch the surface $x^2 - y^2 - 4z^2 = 0$.

Sol: xy-trace: $x = y^2$ and xz-trace: $x = 4z^2$.

For $x=4$, $\frac{y^2}{4} + \frac{z^2}{1} = 1$ is an ellipse. See Figure 9.60.



Example 3 的示意圖



Type II: Quadratic Surfaces

(6) Hyperbolic Paraboloid: (雙曲拋物面)

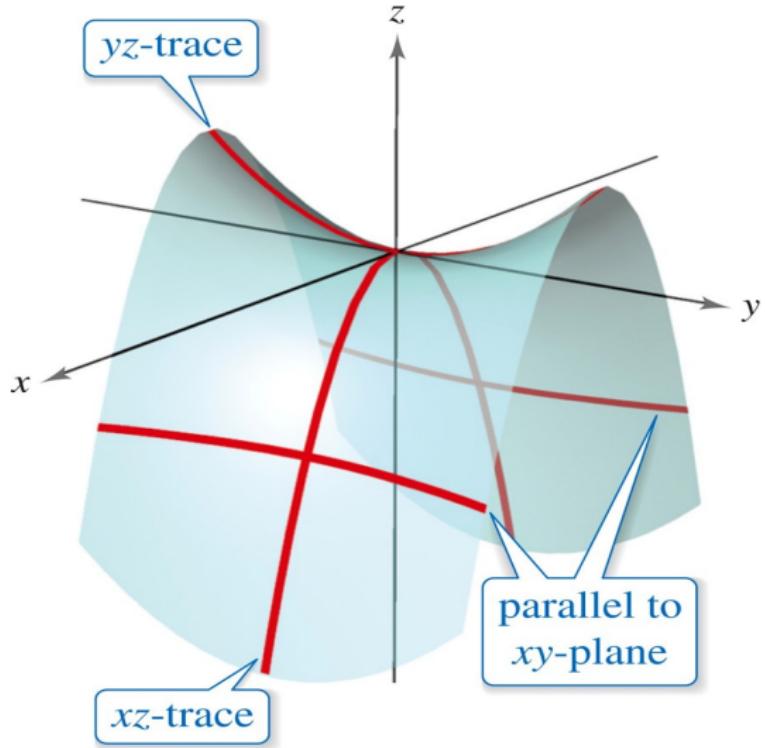
$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2} \quad \text{with } a, b > 0.$$

Then the xy -trace, xz -trace and yz -trace of the surface are

$$y = \pm \frac{b}{a}x, \quad z = -\frac{x^2}{a^2} \quad \text{and} \quad z = \frac{y^2}{b^2}.$$



Hyperbolic Paraboloid 的示意圖



Section 9.7

Cylindrical and Spherical Coordinates (柱面坐標與球面坐標)



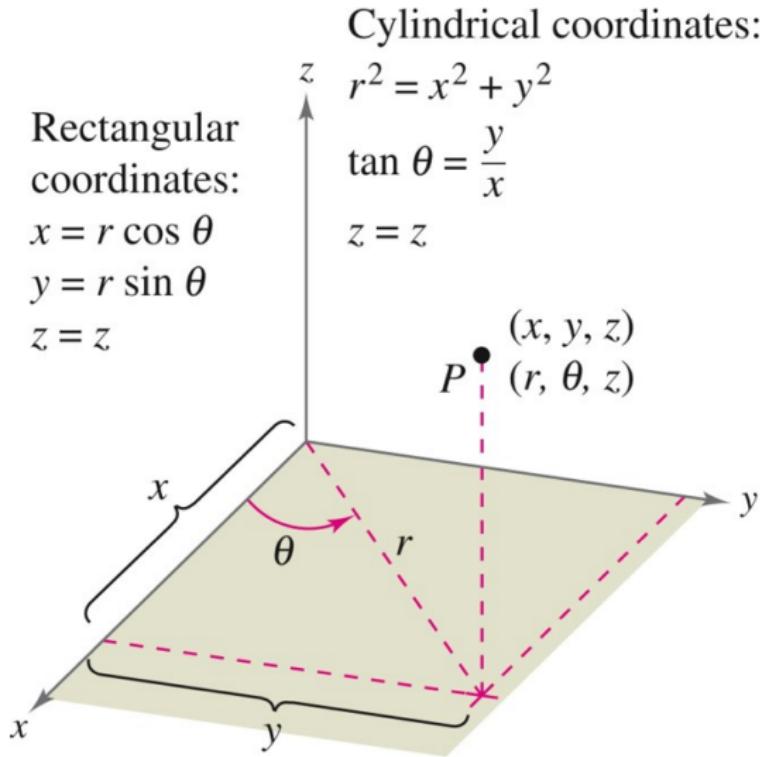
Def. (柱面坐標系統)

In the cylindrical coordinate system, a point $P(x, y, z) \in \mathbb{R}^3$ is represented by an ordered triple (r, θ, z) with

- (r, θ) is the polar coordinates of the (orthogonal) projection $P_0(x, y, 0)$ of P in the xy -plane.
- z is the directed distance from P_0 to P .



柱面坐標的示意圖 (承上頁)



* Cylindrical Coord. v.s. Rectangular Coord.:

① Cylindrical to Rectangular :

$$(r, \theta, z) \mapsto (x, y, z) \in \mathbb{R}^3$$

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z.$$

② Rectangular to Cylindrical : $(x, y, z) \in \mathbb{R}^3 \mapsto (r, \theta, z).$

$$r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}, \quad z = z.$$



Example 3: Find an equation in Cylindrical coordinates for each rectangular equation.

$$(a) x^2 + y^2 = 4z^2.$$

Sol: Let $x = r \cos \theta$ and $y = r \sin \theta \Rightarrow r^2 = x^2 + y^2$

From $x^2 + y^2 = 4z^2$, $r^2 = 4z^2$ we obtain $r^2 = 4z^2$

$$(b) y^2 = x \Rightarrow (\sin \theta)^2 = r \cos \theta \text{ if we let } x = r \cos \theta \text{ and } y = r \sin \theta.$$

$$\Rightarrow r^2 \sin^2 \theta = r \cos \theta \Rightarrow r^2 \sin^2 \theta = r \cos \theta.$$

$$\Rightarrow r(r \sin^2 \theta - \cos \theta) = 0 \Rightarrow r \sin^2 \theta = \cos \theta.$$

$$\Rightarrow r = \frac{\cos \theta}{\sin^2 \theta} = \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta} = \csc \theta \cot \theta.$$



Example 4: Find an equation in rectangular coordinates for the cylindrical equation $r^2 \cos 2\theta + z^2 + 1 = 0$.

Sol: Let $x = r \cos \theta$ and $y = r \sin \theta$. Then since

$$\cos 2\theta = \frac{1 + \cos 2\theta}{2} - \frac{1 - \cos 2\theta}{2} = \cos^2 \theta - \sin^2 \theta, \text{ we see that}$$

$$r^2 \cos 2\theta + z^2 + 1 = 0 \Rightarrow r^2 (\cos^2 \theta - \sin^2 \theta) + z^2 + 1 = 0.$$

$$\Rightarrow (x \cos \theta)^2 - (y \sin \theta)^2 + z^2 + 1 = 0.$$

$$\Rightarrow x^2 - y^2 + z^2 + 1 = 0 \Rightarrow y^2 - x^2 - z^2 = 1 \text{ is the graph}$$

of a hyperboloid of two sheets. See Figure 9.73.



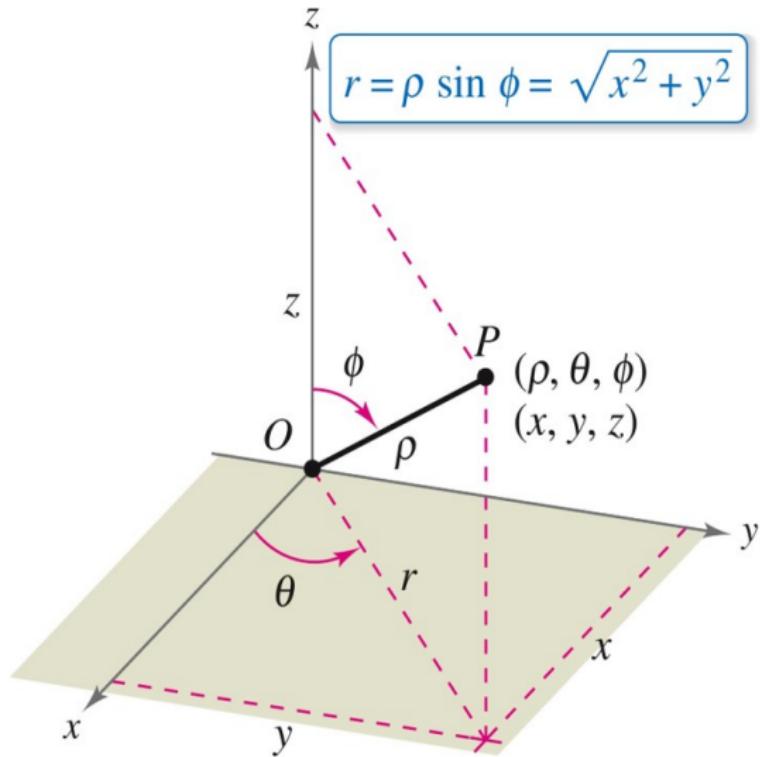
Def. (球面坐標系統)

In the spherical coordinate system, a point $P(x, y, z) \in \mathbb{R}^3$ is represented by an ordered triple (ρ, θ, ϕ) with

- $\rho = |\overline{OP}| = \sqrt{x^2 + y^2 + z^2} \geq 0.$
- θ is the directed angle from the positive x -axis to $\overline{OP_0}$, where $P_0(x, y, 0)$ is the (orthogonal) projection of P in the xy -plane.
- ϕ is the directed angle from the positive z -axis to \overline{OP} .
 $(0 \leq \phi \leq \pi)$



球面坐標的示意圖 (承上頁)



* Spherical Coord. vs. Rectangular Coord.:

① $(\rho, \theta, \phi) \mapsto (x, y, z) \in \mathbb{R}^3$

$$x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$$

② $(x, y, z) \mapsto (\rho, \theta, \phi)$:

$$\rho = \sqrt{x^2 + y^2 + z^2}, \theta = \tan^{-1}\left(\frac{y}{x}\right), \phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$



Example 5: Rectangular-to-Spherical Conversion.

(a) Cone: $x^2 + y^2 = z^2 \Rightarrow \phi = \frac{\pi}{4}$ or $\phi = \frac{3\pi}{4}$.

(b) Sphere: $x^2 + y^2 + z^2 - 4z = 0 \Rightarrow \rho = 4 \cos \phi$. For $0 \leq \phi \leq \frac{\pi}{2}$.



Solutions of Example 5

(a) Since $\rho^2 = x^2 + y^2 + z^2$ and $z = \rho \cos \phi$ for $0 \leq \phi \leq \pi$, it follows that

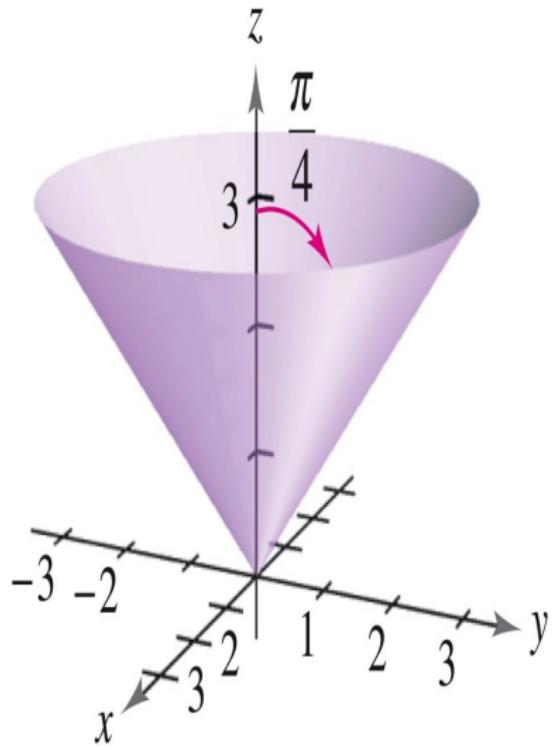
$$\begin{aligned}x^2 + y^2 = z^2 &\implies (x^2 + y^2 + z^2) - 2z^2 = 0 \\&\implies \rho^2 - 2\rho^2 \cos^2 \phi = 0 \implies 1 - 2\cos^2 \phi = 0 \\\implies \cos \phi &= \pm \frac{1}{\sqrt{2}} \implies \phi = \frac{\pi}{4} \quad \text{or} \quad \phi = \frac{3\pi}{4}.\end{aligned}$$

(b) Since $\rho^2 = x^2 + y^2 + z^2$ and $z = \rho \cos \phi$, it follows that

$$\begin{aligned}x^2 + y^2 + z^2 - 4z &= 0 \implies \rho^2 - 4\rho \cos \phi = 0 \\\implies \rho(\rho - 4 \cos \phi) &= 0 \\\implies \rho &= 4 \cos \phi \quad \text{for } 0 \leq \phi \leq \frac{\pi}{2}.\end{aligned}$$

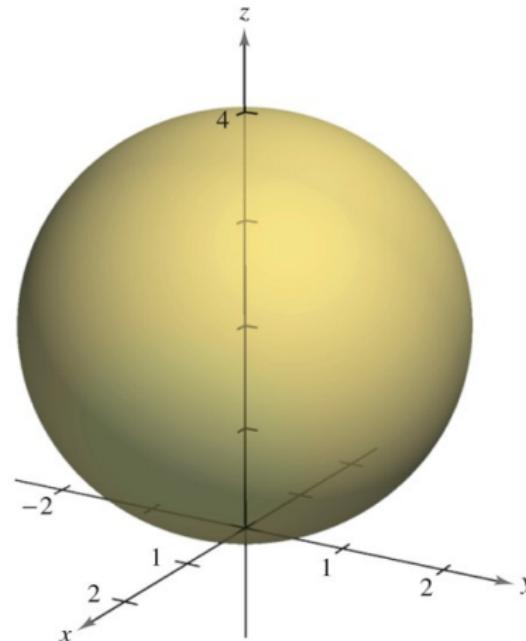


Example 5 的示意圖



Rectangular:
 $x^2 + y^2 + z^2 - 4z = 0$

Spherical:
 $\rho = 4 \cos \phi$



Thank you for your attention!

