105 學年度 (本部) 期中考的第 1(d) 題解答

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1. (d) 請判斷級數 $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$ 的收斂姓.

Sol: If $n \in \mathbb{N}$ and $n > e^{e^2} \equiv n_0$, then we see that

$$\ln n \geq \ln(e^{e^2}) = e^2 > 1.$$

Then we immediately have $(\ln n)^{\ln n} \geq (e^2)^{\ln n} = e^{2\ln n} = n^2$ for all $n > n_0$. Thus, it follows from the Direct Comparison Test and

$$a_n \equiv \frac{1}{(\ln n)^{\ln n}} \leq \frac{1}{n^2} \equiv b_n \quad \forall\, n >> n_0$$

that the series $\sum a_n$ converges, since $\sum b_n$ is a convergent p-series.