我的英文不好，數學雙語教學真的行嗎？
中學數學雙語教學實務初探

陳界山
國立臺灣師範大學數學系特聘教授兼理學院院長
Berri Hsiao
Math Faculty and Learning Center Co-coordinator
Mathematics Department
Clackamas Community College, Oregon, USA

一、前言

行政院於 2018 年 12 月通過國家展委員會《2030 雙語國家政策發展藍圖》，期能達成「提升國家競爭力」與「厚植國人英語力」兩大目標。在此政策的指引下，教育部推動中小學部分領域或學科採英語授課。基本上，以英語作為各領域或學科的學習語言，其共同原因為增加自然情境下使用英語的機會。至於將數學列入雙語教學之學科，不僅因為數學與科學是決定國家競爭力的要項，也因為數學與科學知識的傳播主要以英語為媒介，如同 Tan and Saw Lan（2011）指出，以英語作為數學與科學的教學語言，其主要目標在於培養出能夠流利使用英語取得科技知識的新一代，使他們畢業後進入職場能持續以英語取得最新科技知識，為國家經濟與發展作出貢獻。在數學雙語領域方面，臺北市於 109 學年度首度招考中學數學雙語教師，國立臺灣師範大學則領先全國於 110 學年度開設數學雙語教程，完成培育條件者可於教師證加註雙語次專長。然而，除了職前端的雙語數學師資培育以外，不論從雙語師資需求的急迫性，或是從教師專業發展作為推動教育政策的永續基礎而言，在職教師的雙語教學增能培訓實刻不容緩。

二、現場數學教師的聲音

筆者自 2021 年 8 月開始舉辦中學數學雙語教學工作坊、數學雙語教學演示觀摩工作坊（陳界山，2021-2023），蒐集了許多現場數學教師的聲音與剛開始的憂慮，大致上可歸納成下列幾個面向：

1. 我的英文不好，害怕講很長的英文，沒有信心嘗試數學雙語教學。

2. 我不知道我的英文是否可以應付數學雙語教學，也不知道如何著手。

3. 儘管可以嘗試數學雙語教學，我無法確定學生是否理解學科內容。

除了教師端以外，數學學科本身就是較難理解的學科，以雙語教學為之是否合適，更是家長與學生關心的面向。這仍是一個具有正反意見的議題，筆者撰寫此文的目的不在於鼓吹數學雙語教學，而是站在幫助有意了解並嘗試數學雙語教

學的數學教師的角度上，了解他們困難點與需要幫忙的地方，提供實務上的專業鋪陳作法。呂妍慧、袁媛（2020）曾提出數學 CLIL（Content and Language Integrated Learning）教學案例，該文獻主要以國小數學教學為主要對象，本文則著重於中學數學雙語教學的層面。同時，筆者也希望藉由此文特別提醒一些雙語培訓中心的做法，避免讓數學雙語教師變成英文老師。

上述工作坊的主要講師為臺灣出生長大，具有英文與數學雙專長，目前任職於美國波特蘭 Clackamas Community College 的蕭弘玫老師（Berri Hsiao），蕭老師兩年多來實際接觸參與工作坊的第一線數學教師，彼此建立良好的互動與密切關係。從現場老師的回饋與主持工作坊的實際經驗，提出了一些關於數學雙語教學的看法。

The question "Why do I need to teach bilingual math? " frequently arises from my interactions over the last three years while conducting bilingual math education workshops for both current and preservice math teachers in Taiwan. The question itself might initially seem like a resistance to change, even though there are numerous compelling reasons for adopting bilingual math education, benefitting both students and Taiwan's global competitiveness. Enhancing English proficiency is crucial to bolster the international mobility of Taiwanese, aligning with the Bilingual 2030 initiative outlined by the National Development Planning (Bilingual 2030). Although the importance of this goal itself and the methods for achieving true bilingual education are topics of debate, my engagements with math educators underscore that this 'why' question can be an opening to other important discussions. It often represents an expression of the sentiment, 'This is challenging, and I need guidance.' Many teachers find teaching students to appreciate and engage with math in their native language difficult in itself, adding another layer of English exacerbates this task. Teachers also are concerned that their own English is not good enough to command a classroom. Acknowledging these valid concerns in my opinion is vital for successful implementation of an ambitious educational reform like Bilingual 2030 where math educators are major stakeholders. Establishing a platform for dialogue and valuing teachers' input is the first step in this arduous journey. Yes, teaching bilingual math is hard. Yes, it is challenging to add another layer of complexity to an already formidable subject. Yes, it is a burden to the teachers to improve their English skills with everything else they have in their lives. Yes, it is time and energy consuming. My objective is not to champion the belief in this national plan upon teachers, but rather to support those interested in implementing bilingual math education by providing concrete and practical knowledge and skills. Drawing from my three decades of experiences as both a math student and a math teacher in the United States, alongside my prior English

teaching experiences in Taiwan, I strive to share the essential components of a successful bilingual math education in this article. The views presented here reflect both my teaching experiences and the insights from feedback gathered during workshops conducted on this topic.

Everyday Classroom English

The easiest way that a teacher can start a bilingual classroom is to have some classroom specific English in their toolbox. This could range from simple greetings like 'Good morning class!' or 'Hello math students!' or 'I need everyone's attention.' to more complex instructions like 'That's it for today. See you next class!' or 'Please turn your textbook to page 10'. Encouraging students to respond in English helps create a relaxed English-speaking environment. Other classroom-specific English phrases such as those used for managing group work and calling students' attention can be introduced at leisure. These easy-to-use English phrases are valuable and easy to implement, as they can be used in every class. In a world where students are exposed to English across various platforms, using these expressions is not overtly daunting. (Berri Hsiao, 2021 - 2023)

Adapting to anything new takes time to get used to. I recommend teachers initiate a conversation with students outlining the intended changes in your plan for implementing Everyday Classroom English. Ask the students what phrases they know and like and incorporate these suggestions when possible or appropriate. It's valuable to acknowledge everyone's initial unease and awkwardness. It also may come as a surprise to discover the value in sharing your own apprehension as a teacher with your students. For this purpose, I created a document in my workshop that serves as a 'template' that provides ready-to-use phrases to use in different phases of a math classroom. Teachers can easily swap one word for a different word, depending on your personal style and the situation. Authenticity is key to driving substantial change.

Basic Math Vocabulary

Regardless of the math level, mastering basic math vocabulary is paramount. While numbers are understood in English by most people, math extends far and beyond them. For instance, most teachers recognize 'addition, subtraction, multiplication, and division' as the four basic math operations in English. However, teachers might not know how to say an equation or an expression in English. For example, '2x+3–4y=5' is read as 'two x plus three minus four y equals five'. In addition, the way we say the

symbol '=' can be equals or is equal to. How do we say simple things such as: $\frac{2}{3}$ or x^2 or $\sqrt{9}$? Understanding the nuances and identifying the most common and accessible expressions in math across subject levels is crucial for bilingual math education. In my workshops, I illustrate diverse ways of conveying the same mathematical idea, focusing on the simplest or most prevalent usage. Workshop participants generally find this approach interesting and engaging, as bridging the gap between their own mathematical comprehension and English articulation is more straightforward. Proficiency in basic math vocabulary is transferable across all levels of math, even in a Calculus class. These math words play a vital role.

Subject-specific Math English

Math's inherent complexity and abstraction pose challenges for students to master and for educators to teach. Math, as a system, relies heavily on symbols and notations for concise communication of ideas and logic. This is why even for native speakers, math is a language that necessitates explicit teaching in its vocabulary, pronunciation, and grammar structure. I assure Taiwanese math educators that even in my own classroom, American students require guidance in how to read or state a math expression or an equation. My average American students don't know how to say $\sum_{i=1}^{10} 2i$ or $\lim_{x \to 2^-} \frac{x^2-4}{x-2}$ or even $f(x) = x^2 - 1$ without being specially taught. I hope this is a comforting thought for Taiwanese teachers in their journey to become bilingual math educators. Simultaneously, enlisting and training people with expertise in both math and English is vital for the success of a bilingual math education reform. Merely treating bilingual math as an English-focused class is both inadequate and inappropriate. Proficiency in English doesn't guarantee mastery of math concepts or the correct application of English within mathematical contexts.

In order to implement bilingual math education, teachers and preservice teachers need tailored training for specific subjects. A geometry teacher's needs are different from an Algebra teacher or a Calculus teacher. Once the basic foundational math vocabulary is built, ongoing and accessible training and ready to use resources must be in place, considering that becoming a proficient bilingual math teacher is a gradual process that takes time and energy investment. Just as Rome was not built in one day, linguistic acquisition and mastery requires patience and continuous learning. There

needs to be a comprehensive plan of professional development opportunities for teachers of all levels of English abilities. I firmly believe a good first workshop experience can serve as a catalyst for individuals who understandably harbor reservations and apprehension. It can encourage them to open their minds and cultivate the willingness to embark on the journey of bilingual teaching/learning and teaching. My hope for the workshops I conduct is to inspire the notation of 'trying', fostering the belief that anyone can embrace bilingual math teaching, even if not immediately perfect. Proficiency comes through a combination of trial and error, dedication, time, and quality resources.

Drawing from over two decades of teaching a variety of math courses to a diverse range of student population in the United States, I have realized that embodying the role of a learner best enhances my teaching prowess. Occupying the seat of a student helps me empathize with the students' experience and underscores the profound impact of the teacher's attitude and approach. Relating to student struggles and comprehending their learning journey enhances my effectiveness as an educator. These compelling reasons alone highlight the values of attempting bilingual math teaching. I applaud all the math teachers and aspiring teachers who voluntary attended bilingual math education training and those who are considering taking the giant first step in trying, as I believe embracing change and continuous learning are keys to teaching excellence. Bilingual math education is indeed possible, achievable through willingness to try, learn, and evolve.

三、數學雙語教學的因應策略

筆者認為推動數學領域雙語教學可思考底下三大面向：

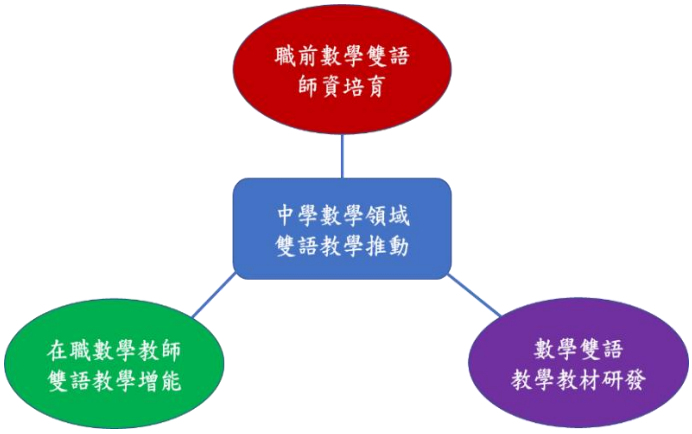

圖 1 推動數學領域雙語教學的三大面向

首先，職前端數學雙語師資培育，目前許多師培大學已持續進行培育。在職數學教師雙語教學增能，建議透過計畫中心或培訓中心提供增能工作坊幫助數學

教師學習營造雙語教學環境、數學雙語教學實務技巧。然而，筆者對於目前數學學科的雙語培訓以語文角度切入，過度強調語文學習目標，有一些擔憂。若是雙語政策持續進行，未來接軌國際，雙語教材的研發、開放中學數學教學使用原文書或是制定符合數學雙語教學的教材，是未來必須面對的議題，可交由國家教育研究院研議其可行性。事實上，筆者的團隊也針對數學雙語教材部分設計了一些相關教材（陳界山，2023），可供現場的數學教師參考使用。整體而言，建議推動數學雙語教學的過程中，朝向三個互相關聯的正循環目標，來幫助數學教師了解數學雙語教學的樣貌，去除疑慮，進而面對它，如下圖所示：

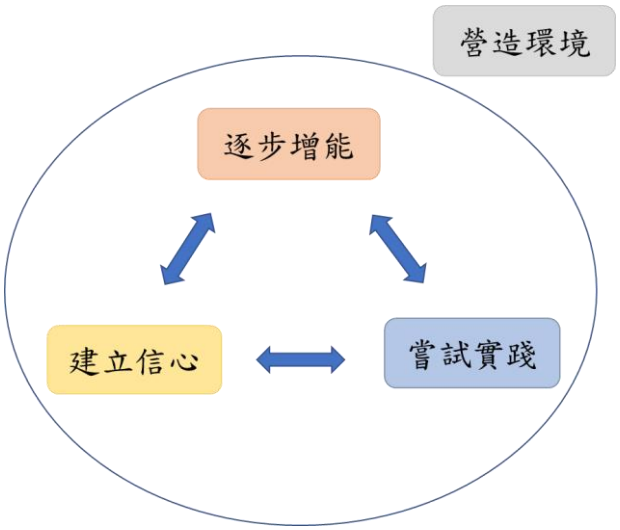

圖 2 營造環境的正循環

(一) 逐步增能

經由分階段辦理數學雙語教學工作坊，逐步提升中學數學教師的數學專業英文用語及一般課堂英文之運用能力。

(二) 建立信心

設計進階工作坊提供中學數學教師設計雙語教學微課程之實作機會，教師可選擇在自己熟悉的課堂進行教學，並將教學錄影於工作坊中發表，聽取工作坊講師與同儕教師之回饋意見，建立雙語教學的信心。

(三) 嘗試實踐

基於數學教師本身對雙語教學的熟悉度與考量學生的接受度，逐步增加數學雙語教學的份量，不必執著於多少比例（林子斌，2020）是以雙語進行。藉由初階工作坊巡迴全國之推廣，以及進階工作坊表現傑出教師於成果展示之分享，逐步提升中學數學教師嘗試英文雙語教學之意願。

另外，筆者從參與工作坊的數學教師的回饋中，發現中學數學教師希望能藉由參加數學雙語教學工作坊，學習較常用到的英文詞彙和句子，亦希望能營造團體學習氣氛，像是組成讀書會來練習數學雙語教學，能一起合作學習，彼此互相鼓勵與成長。數學雙語教學的推動應由數學專業背景的人士來推動較為適宜，若全由外文專長的人士來推動，可能產生數學雙語教學等同於全英語教學的疑慮。另外也可能有一些後遺症，例如：現場數學教師對數學雙語教學的背景與目標缺乏正確認知，或者因為對個人的英語程度缺乏自信，或對數學雙語教學的樣貌缺乏理解，因而退卻、排斥或產生過多壓力。總而言之，主要目標是幫助他們建立信心與勇氣，並讓他們理解這是漫長的路（林子斌、黃家凱，2020）。

四、以數學專業來鋪陳數學雙語教學

筆者再次強調數學雙語教學要以數學專業來鋪陳數學雙語教學，不能以語文的角度切入去訓練或要求數學教師。雙語教學培訓中心的做法，應該思考如何避免讓數學雙語教師變成英文老師的擔憂。如何以數學專業來鋪陳數學雙語教學？主要是依據下列三個方向去做鋪陳：Math Vocabulary、Subject-specific Math English、Classroom English。筆者舉一些簡單的例子來闡述實際的做法。

(一) Math Vocabulary

1. Operations: plus, minus, times, divided by, add, subtract, multiply, divide.
2. $-3+2\times(12-5)$→ negative three plus two times the quantity of twelve minus five (negative 3 plus 2 times 12 minus 5 in parenthesis).
3. fractions: numerator, denominator.
4. $\frac{1}{2}$ → one half (one over two).
5. $\frac{1}{3}$ → one third (one over three).
6. $\frac{1}{4}$ → one quarter (one over four). (proper fraction).
7. $\frac{9}{5}$ → nine over five (nine fifths). (improper fraction).
8. $4\frac{2}{7}$ → four and two over seven. (mixed number).
9. x^2 → x squared.
10. y^3 → y cubed.
11. 4^5 → 4 to the 5th power (or 4 to the 5th).
12. y^{-4} → y to the negative four.

13. $(-3)^7$ → the quantity of negative three to the 7th.
14. $-3^2 \neq (-3)^2$→ negative three squared is not equal to the quantity of negative three squared.
15. 23.4 → twenty three point four.
16. $3.0\overline{45}$ → three point zero four five with four five repeating.
17. $\sqrt{5}$ → square root of five.
18. $\sqrt[3]{y}$ → cube root of y.
19. $\pm\sqrt[3]{k}$ → plus or minus the cube root of k.
20. $\sqrt[4]{x+2}$ → fourth root of x plus 2.
21. $log_2 8$→ log of eight with base two (log base 2 of 8).
22. 4^5 → 4 to the 5th power (or 4 to the 5th).
23. 5^{4+3} → 5 to the power of the quantity of four plus three (or 5 to the 4 plus 3).
24. $\frac{x^3}{y^{-1}}$ → x cubed over y to the negative one.
25. $\frac{1-x}{\sqrt{x}+2}$ → one minus x over square root of x plus 2.

(二) Subject-specific Math English

1. Find the sum of 3+4.
2. Compute/Calculate -5^3.
3. Perform the given operations: $(-3) + (-8) \times (-4)$.
4. Evaluate the expression $x^2 - 4xy + y^3$ for $x = -2$ and $y = 4$.
5. Simplify the expression $-2x + 4y + 6x - y$.
6. Reduce the fraction $\frac{45}{12}$ to simplest form.
7. Substitute: When we are asked to "Determine if $x = 3$ is a solution to the equation: $2x - 5 = 1$", we substitute 3 for x (plug in $x = 3$) and check if both sides of the equation are the same quantity.
8. Solve the equation: $x^2 - 2x = -x^2 + 5x + 4$.
9. Simplify the following expression by combining the like terms: $2x - x^2 + xy - y + 4x^2 - 7x - 5 + 4y$.
10. Solve the following equation by completing the square: $x^2 - 8x + 10 = 0$.
11. Identify the leading coefficient and the degree of the polynomial $-x^3 + 3x + 5$.

12. Make a table of values of at least 5 points for the function $f(x) = 2^x - 1$.
13. Graph the system of linear equations $\begin{cases} x - 2y = 8 \\ 2x + y = 1 \end{cases}$
14. Factor the polynomial completely $-3x^2 + 9x + 12$.
15. Using “Rationalize the numerator” to evaluate the limit: $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$.
16. Round 2.34561 to 3 decimal places (6 is great than 5, so we round up 6 to get an approximation 2.346).
17. Round 2.34531 to 4 decimal places (1 is less than 5, so we get rid of 1 to get an approximation 2.3453).
18. Describe the position of a point.
19. We refer to the ordered pair [r, θ] as the polar coordinates of P.（稱作）
20. Express 42880000 in scientific notation.
21. Convert nanometer into centimeter by times.
22. Determine whether the following series forms a geometric sequence.
23. Expand the equation $(x-1)^2 + (y-2)^2 = 4$.
24. Sketch/Draw the graph of the ellipse.

(三) Classroom English

這部分臺灣師大雙語教學研究中心已出版課室英文手冊(有聲版)，內容涵蓋學校場所、設施、課前準備、講解、教學活動、教室管理、分派作業、考試、點名等相關的課室英文，內容豐富完整足夠提供給各學科的課室英文運用，不再贅述。

簡單地說，數學教師實踐雙語教學的過程中，所使用的英文大多數和數學專業有關，數學教師不用擔心要講很長的英文句子，課室英文是輔助的角色不是數學雙語教學的重點。數學教師的英文能力並不需要達到流利的程度，不必執著於多少比例（林子斌，2020）是以英語進行。同時，數學雙語教學不等同於全英文教學，不能以全英文教學強迫數學教師為之，數學雙語教學應以數學專業為鋪陳，而非以語文的角度切入，數學學科知識的傳授仍是重點。在實踐或嘗試數學雙語教學的過程，有幾個技巧可以加以運用：

1. 數學名詞或專有術語用英文介紹，數學課堂板書可用英文書寫，重要的數學概念還是以母語進行，數學考卷可以用英文呈現。
2. 準備 Teaching worksheet，將該課堂可能出現的英文專有名詞呈現出來。

3. 在黑板寫完解答、概念、圖形時，是練習雙語教學的好時機。例題或活動設計可以用 PPT 呈現，以免雙語教學時，一面要講解專業知識，一面思索英文詞彙，容易停頓中斷。

五、結語

數學雙語教學有其正反意見，也有利弊得失，在推動的進程中應該讓現場的數學教師有充分的管道了解數學雙語教學的樣貌，提供專業發展的增能工作坊，讓數學教師逐步增能並願意定期嘗試，希冀經由雙語教學活化並整合各類創意教學。另一方面，設法讓數學教師去除疑慮，避免產生負面的作用，有幾個大原則必須掌握。

1. 數學雙語教學不等同於全英文教學，不能以全英文教學強迫數學教師為之。
2. 數學雙語教學應以數學專業為鋪陳，而非以語文的角度切入，數學學科知識的傳授仍是重點。
3. 考量學生的學習成效，數學學習目標為主，語文學習目標為輔。
4. 認識數學雙語教學的樣貌，逐步增能並定期嘗試。
5. 數學雙語教學的共同語言是數學，而非英語。

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