雙語教材 1　數列與級數 Sequences and Series

數列 **Sequences**

1. 認識數列 Understand sequences
2. 等差數列 Arithmetic sequences
3. 等比數列 Geometric sequences

等差級數 **Arithmetic series**

1. 等差級數的和 Sum of a finite arithmetic series
2. 等差級數和的公式 Formula for the sum of a finite arithmetic series

核心單字	英文	參考音標
數列	Sequence	/ˈ**siː.kwəns/**
級數	Series	/ˈ**sɪr.iːz/**
數列的項	Term of a squence	
等差數列	Arithmetic sequence	/**əˈrɪθ.mə.tɪk/**
等差級數	Arithmetic series	
公差	Common difference	/ˈ**kɑː.mən/** /ˈ**dɪf.ɚ.əns/**
等比數列	Geometric sequence	/ˌ**dʒiː.əˈmet.rɪk/**
等比級數	Geometric series	
公比	Common ratio	

數列 Sequence

1 認識數列 Understand sequences

數列英文版定義：

Definition: A number sequence is a pattern of numbers arranged in a particular order according to a rule.
Each individual number, or element, of a sequence is called a term.

課本內容：

右表是新北市免費公車明志線 F216 例假日的班次時刻表，其中臺北車站北三門的發車時間都是整點，可以簡記為：11，12，13，14，15，16，21，22 。像這樣將數排成一列，並以逗點分開，稱為**數列**。

班次	臺北車站北三門發車時間
1	11：00
2	12：00
3	13：00
4	14：00
5	15：00
6	16：00
7	21：00
8	22：00

雙語使用參考範例：

- ✧ The right table(右表) is the schedule of Bus F216. The departure time of the North third gate of Taipei station is 11, 12, 13, 14, 15, 16, 21 and 22 o'clock.
- ✧ Numbers arranged in a row like this and separated by commas(逗號) are called sequences.
- ✧ Here are some number sequences.
  2, 4 , 6, 8, …
  100, 400, 1600, 6400,…
  1 , 4, 9, 16, 25,…
  3, 7, 10, 12, 13, …

課本內容：

在上面的數列中：

11 稱為這個數列的**第 1 項或首項 (First term)**，通常記為 a_1；

12 稱為這個數列的第 2 項**(Second term)**，記為 a_2；

13 稱為這個數列的第 3 項**(Third term)**，記為 a_3；

這個數列的第 n 項，記為 a_n；**而數列中的最後一項稱為末項 (Last term)**。

翻譯示例：

In the above sequence: 11 is called the first term of this sequence, which is usually recorded as a_1.

雙語使用參考範例：

- ✧ You can also use the notation a_n to denote the nth term of a sequence, where n is a positive integer.
- ✧ 如何念a_n　T: How to say a_n : a subscript n or a sub n
- ✧ Remember that n, the term number, will always be a whole number.

  We could not have a 0.25th term, or a $\frac{1}{4}$ th term.

備註：

基數	序數	唸法	寫法
1～3	不規則變化	first、second、third	1st、2nd、3rd
4～19	字尾加 th * ve 結尾改成 th * t 結尾直接加 h * e 結尾去 e 加 th	fourth,fifth,sixth, seventh,eighth,nonth...	數字後加上 th。 （例：4th, 20th, 100th）
20, 30, 40...	字尾去 y 加 ieth	twentieth thirtieth fortieth	
100, 1000...	字尾加 th	hundreth, thousandth, millionth	
21～99 （0 結尾以外）	個位數改為序數，中間加連字號（-）	twenty-first, thirty-fourth, seventy-second	數字後加上 th， 但個位數為 1～3 者除外， 1～3 分別加上 st, nd, rd。 （例：21st、32nd、43rd）

雙語使用參考範例：

在數列2, 4, 6, 8, 10, 12, 14中

✧ 哪一個是首項? T: Which one is the first term?

✧ 6是第幾項? T: Which term is 6? & 6 is the _____ term? S: Third

✧ 這個數列總共有幾項? T: How many terms are there in this sequence?

✧ 請找出/寫下前三項 T: Please find/write down the first three terms in this sequence.

練習

數列 1, 4, 7, 10, 13, 16 中，

第 1 項 a_1＝ 1 ，第 2 項 a_2＝ 4 ，第 3 項 a_3＝ 7 ，

第 4 項 a_4＝ 10 ，第 5 項 a_5＝ 13 ，第 6 項 a_6＝ 16 。

翻譯示例：

In the sequence 1, 4, 7, 10, 13, and 16,

The first term a_1 = 1, the second term a_2 = 4, the third term a_3 = 7,

the fourth term a_4 = 10, the fifth term a_5 = 13,and the sixth term a_6 = 16.

課本內容:

數列可能具有某種規律。例如：

翻譯示例：

The sequence may have some mathematical patterns. For example,

可以這麼說：

✧ 我們可以使用規律來...規律可以幫助我們...

We can use patterns to… / Patterns can help us…

make predictions(預測)about the future and calculate how long it will take for an investment to double in value.

觀察規律：

數列如下：

135, 137, 139, 141, ……，

可以發現這個數列的規律是：相鄰兩項中，後一項等於前一項加 2。

135 ， 137 ， 139 ， 141 ， ……

＋2 ＋2 ＋2 ＋2

翻譯示例：

The address numbers of the shops on the street are as follows:

135, 137, 139, 141, ……，

It can be found that the rule of this sequence is: among the two adjacent terms, the latter term is equal to the previous term plus 2.

*鄰近的 adjacent /əˈdʒeɪ.sənt/

練習

已知下列各數列分別隱含某種規律，依其規律在空格中填入適當的數。

It is known that each of the following series implies a certain pattern.

Please fill in the appropriate number in the space according to its pattern.

(1) 10, 18, 26, ____, 42, 50 (2) 2, 4, 8,____, 32, 64

翻譯示例：

It is known that each of the following series implies a certain pattern.

Please fill in the appropriate number in the space according to its pattern.

*適當的 appropriate /əˈproʊ.pri.ət/

課本內容:

有些具有規律的數列，其第 n 項 a_n 可用含有未知數 n 的式子表示，例如：

翻譯示例：

For some regular sequences, the nth term a_n can be expressed by the formula containing the unknown n. For example:

(1) 從 1 開始的連續正整數的倒數所形成的數列如下：

$\frac{1}{1},\frac{1}{2},\frac{1}{3},\frac{1}{4},\cdots\cdots$

第 1 項	第 2 項	第 3 項	……	第 n 項
a_1	a_2	a_3	……	a_n
$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$		$\frac{1}{n}$

翻譯示例：

The sequence formed by the reciprocal of consecutive positive integers starting from 1 is as follows: $\frac{1}{1},\frac{1}{2},\frac{1}{3},\frac{1}{4},\cdots\cdots$

*倒數 reciprocal /rɪˈsɪp.rə.kəl/

*連續的 consecutive /kənˈsek.jə.ʈɣv/

*正整數 positive integers

(2) 從 2 開始的連續正偶數所形成的數列如下：

2, 4, 6, 8,……

第 1 項	第 2 項	第 3 項	……	第 n 項
a_1	a_2	a_3	……	a_n
2	4	6		$2n$
2×1	2×2	2×3		$2\times n$

翻譯示例：

The sequence formed by consecutive positive even numbers starting from 2 is as follows: 2, 4, 6, 8,……

*偶數 even numbers

*奇數 odd numbers

像這類具有規律的數列，如果能寫出第 n 項的表示式，就可以找到該數列的任意一項。上述例 (2) 中，第 n 項 $a_n=2n$ 稱為此數列的**一般項**。

英文版定義：

Definition: If we can write the expression of the nth term in a regular sequence like this, we can find any term of the sequence. In the above example (2), the nth term $a_n=2n$ is called the general term of the sequence.

可以這麼說：

✧ 如何找出一般項 T: How to find the general term?

✧ 一般項為何? T: What is the general term?

✧ 請寫下這個數列的一般項 T: Write a general term of the sequence.

✧ 你觀察/注意到甚麼? T: What did you observe/notice?

✧ 觀察一般項：2, 4, 6, 8,……

T: You can notice that the value of each term is two times n.

✧ 觀察一般項：8, 11, 14, 17, 20, 23, 26,……

T: You can notice that the value of each term is three bigger than the value of the previous term.

若要求第 100 項 a_{100}，只要在 $a_n=2n$ 的式子中，n 以 100 代入即可求得：$A_{100}=2\times100=200$。

翻譯示例：

If we are asked to find a_{100}, as long as n is substituted with 100 in the formula $a_n=2n$.

$A_{100}=2\times100=200$

*用…代替 substitute /ˈsʌb.stə.tuːt/

例題 求第 *n* 項

已知某數列的第 n 項 $a_n=n(n+1)(n+2)$，

(1) 求此數列的前 3 項。

(2) 求此數列的第 10 項。

(3) 若此數列的第 k 項為 336，求 k。

翻譯示例：

Find nth term

Given the nth term of a sequence, $a_n=n(n+1)(n+2)$.

(1) Find the first three terms of this sequence.

(2) Find the 10^{th} term of this sequence.

(3) If the kth term of this sequence is 336, find k.

參考資料來源

1. 110 國中數學 2 下翰林版課本
2. IB Maths SL Book Oxford

   Chapter 6 Patterns, sequences andseries
3. Holt McDougal Larson Algebra 2

   Chapter 7 Sequences and Series

☆老師們可以自己從中選擇以做出適合自己學生程度的學習單或是在課堂中適時補充這些英文。

製作者：臺北市立誠正國民中學　　陳怡伶