

Topic: Logarithmic Functions and Their Graphs

1. Review:

Find the value for each of the following expressions:

a. $\log_2 2$ b. $\log_2 4$ c. $\log_2 8$ d. $\log_2 16$ e. $\log_2 1024$

f. complete the table

x	$\frac{1}{3}$	1	3	9	27
$\log_3 x$					

g. The logarithm ($\log_a b$) is defined for what values of a and b ?

2. From the table you completed above in part f, we found that the relationship

“ $x \rightarrow \log_3 x$ ” (from x to log base of three x) is a function: $y = \log_3 x$.

(給定任意實數 $x > 0$, 對數 $\log_3 x$ 的值都隨之唯一確定)

Definition of Logarithmic Function with Base a

For $x > 0$, $a > 0$, and $a \neq 1$, the function

$$y = \log_a x \quad (\text{Read as “log base } a \text{ of } x)$$

is the logarithmic function with base a .

The domain of $y = \log_a x$ is $\{x \mid x > 0\}$ (the set of all positive real numbers).

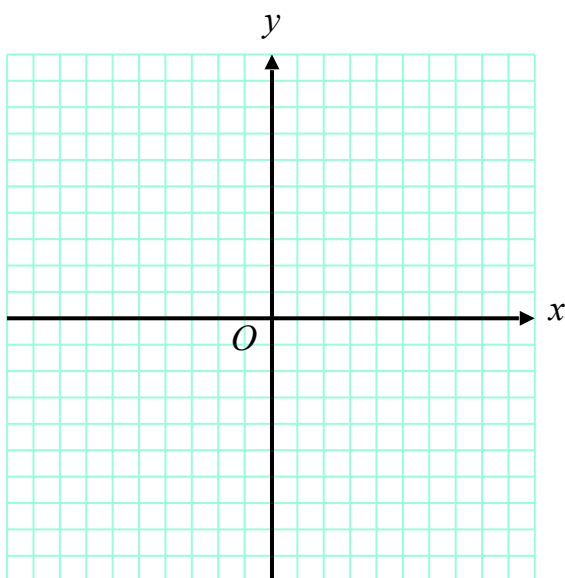
The range of $y = \log_a x$ is $\{y \mid y \in \mathbb{R}\}$ (the set of all real numbers)

3. Graphs of logarithmic functions

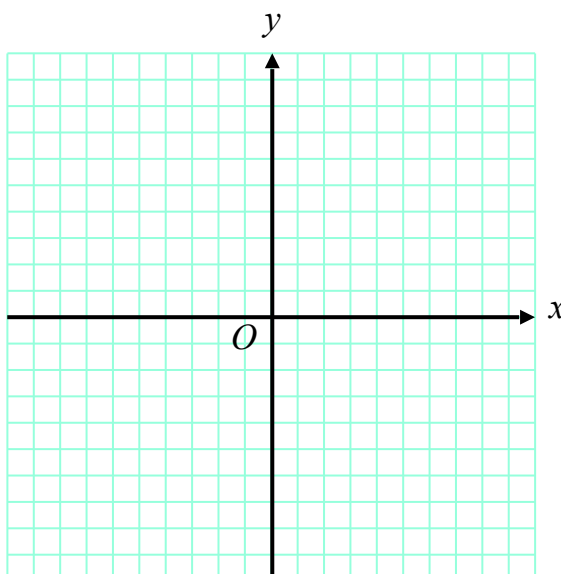
a. $y = \log_3 x$

complete the table, plot the points from the table, and connect them with a smooth curve then sketch the function in the coordinate plane.

x	$\frac{1}{9}$	$\frac{1}{3}$			
y			0	1	2



b. Consider that $y = \log_{\frac{1}{3}} x = -\log_3 x$ (why?), use the concept of symmetry to sketch the graph $y = \log_{\frac{1}{3}} x$.



4. 在 Desmos 中輸入函數 $f(x) = \log_a x$ 及選擇新增滑桿 a (亦可選擇其他軟體操作) 拉動滑桿觀察圖形變化並回答下列問題:

a. How do you describe the shape of these graphs?

(Read this article <https://www.mathsisfun.com/calculus/concave-up-down-convex.html>.)

b. Under what conditions are the graphs increasing or decreasing??

c. Do the graphs have any asymptotes?

d. Do the graphs intersect x-axis or y-axis?

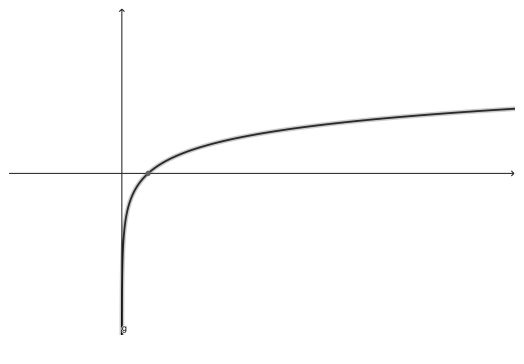
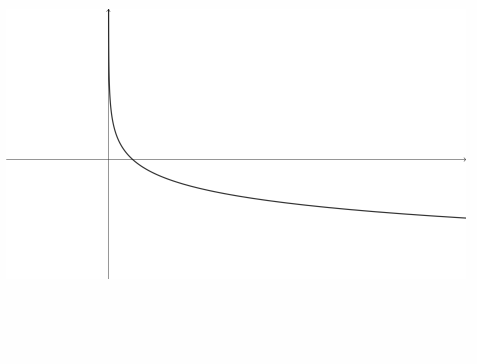
e. How many zeros do these functions have and why?

f. Describe the main features of the graphs of all logarithmic functions of the form

$$f(x) = \log_a x, a > 0, a \neq 1$$

5. Conclusion

The characteristics in the graph of $y = \log_a x$

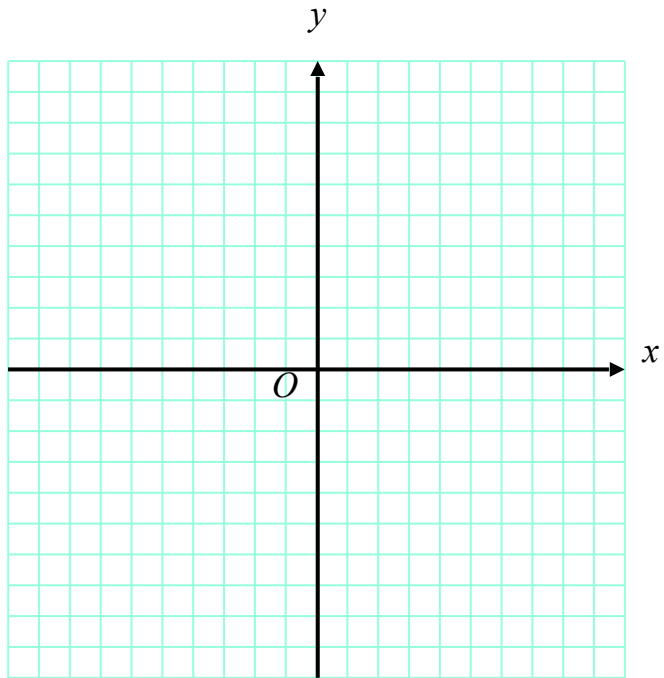
	$a > 1$	$0 < a < 1$
Graph		
Characteristics	<ul style="list-style-type: none"> • x-intercept: (1,0) • vertical asymptote: $x = 0$ • increasing (when the y-value increases as the x-value increases) • concave downward 	<ul style="list-style-type: none"> • x-intercept: (1,0) • vertical asymptote: $x = 0$ • decreasing (when the y-value decreases as the x-value increases) • concave upward

6. Graphs of logarithmic functions and exponential functions

Construct a table of values for the following functions. Plot the points from the table and connect them with a smooth curve then sketch each function in the same coordinate plane.

a. $f(x) = 2^x$

b. $g(x) = \log_2 x$



c. Are the graphs $y = 2^x$ and $y = \log_2 x$ symmetric to the line $y = x$? State your reason.

d. Are the graphs $y = a^x$ and $y = \log_a x$ symmetric to the line $y = x$? State your reason.

Topic: Logarithmic Functions and Their Graphs 及使用建議

內容	<p>1. Review</p> <p>....以下省略</p>												
使用建議	<p>[教學活動安排]</p> <p>複習對數定義; 填表格是為引出對數函數定義做鋪陳。</p> <p>[可參考的英文問句/提問/開場]</p> <p>Today we're going to learn the logarithmic functions and their graphs. Before that, let's do the review, we will check it in two minutes. OO(點一個同學) Would you like to share your answer?</p> <p>[參考答案]</p> <p>1. Review:</p> <p>Find the value for each of the following expressions:</p> <p>a. $\log_2 2 = 1$ b. $\log_2 4 = 2$ c. $\log_2 8 = 3$ d. $\log_2 16 = 4$ e. $\log_2 1024 = 10$</p> <p>f. complete the table</p> <table border="1" data-bbox="504 1263 1243 1417"> <tr> <td>x</td> <td>$\frac{1}{3}$</td> <td>1</td> <td>3</td> <td>9</td> <td>27</td> </tr> <tr> <td>$\log_3 x$</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> </table> <p>g. The logarithm ($\log_a b$) is defined for what values of a and b? $a > 0$ and $a \neq 1$, $b > 0$</p>	x	$\frac{1}{3}$	1	3	9	27	$\log_3 x$	-1	0	1	2	3
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內容	2. From the table...以下省略
使用建議	<p>[教學活動安排]</p> <p>介紹對數函數的定義</p> <p>[可參考的英文問句/提問/開場]</p> <p>What is the logarithmic function? From the table you complete of the review, we found that there “$x \rightarrow \log_3 x$” (x to the log base three of x). 因為從表格中，我們看到給定任意實數$x > 0$,對數$\log_3 x$的值都隨之唯一確定。所以x跟$\log_3 x$ 是函數關係，寫成 $y = \log_3 x$. This is the logarithmic function with base three.</p> <p>So, the definition of logarithmic function with base a is $y = \log_a x$ (y equals to log base a of x). And we have to notice that the restriction of the variable x should be positive and the base a should be positive but not equal to one.</p>
內容	3. Graphs of logarithmic functions...以下省略
使用建議	<p>[教學活動安排]</p> <p>接續前面的舉例讓學生畫$y = \log_3 x$的圖形並利用對稱的概念畫$y = \log_{\frac{1}{3}} x$的圖形</p> <p>[可參考的英文問句/提問/開場]</p> <p>What does the graph of the logarithmic function look like? Let's take $y = \log_3 x$ for example. Follow the direction on the worksheet to sketch the graph $y = \log_3 x$. We will check it in two minutes.</p> <p>OO(點一個同學) Would you like to share your answer? Let's use the concepts we've learned before to sketch the graph of $y = \log_{\frac{1}{3}} x$.</p> <p>Any volunteers would like to explain why $y = \log_{\frac{1}{3}} x = -\log_3 x$? 那我們可以怎麼利用對稱畫出$y = \log_{\frac{1}{3}} x$ 的圖形呢? 還是想不到的同學就先畫表格找 $y = \log_{\frac{1}{3}} x$ 的點。 從這個例子，你觀察到什麼嗎? (對於同樣的x代入,函數$y = \log_{\frac{1}{3}} x$ 跟$y = \log_3 x$ 的y值差一個負號。所以他們圖形對稱y軸。)</p>

內容	4.在 Desmos 中...以下省略
使用建議	<p>[教學活動安排]</p> <p>透過Desmos讓學生探索不同底數的對數函數圖形的特徵。有關圖形凹向性在此僅補充讓學生有個概念,至於如何驗證就留待微積分再詳細說明。</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's use Desmos to explore more graphs of logarithmic functions with different bases.</p> <p>Follow the directions on the worksheet and use desmos to answer the questions. We will discuss in five minutes.</p>
內容	5. Conclusion....以下省略。
使用建議	<p>[教學活動安排]</p> <p>對數函數圖形特徵整理。</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's tidy up what we discussed about the characteristics of logarithmic functions graphs.</p> <p>Let's go through the conclusion on the worksheet.</p>
內容	6. Graphs of logarithmic functions and exponential functions...以下省略

使用建議

[教學活動安排]

用底數2當例子,將指數函數與對數函數圖形畫在同一個座標平面去觀察對稱。

[可參考的英文問句/提問/開場]

We know that the logarithm is a different way to look at the exponent. So, do their graphs have relationships? Let's take base 2 for example. Follow the directions on the worksheet and answer the questions. We will discuss in five minutes.

We observed that the graph of $y = \log_2 x$ is the reflection of the graph of $y = 2^x$ in the line $y = x$. Is it right for any bases of logarithmic functions and exponential functions? Let's take the base 2 for example. The explanation:

$P(\alpha, \beta)$ is any point on the graph $y = 2^x$, therefore we get $\beta = 2^\alpha$, which can also write in the form of $\alpha = \log_2 \beta$. Hence, the point (β, α) is on the graph $y = \log_2 x$, vice versa. So, the graph of $y = \log_2 x$ is the reflection of the graph of $y = 2^x$ in the line $y = x$.

[參考答案]

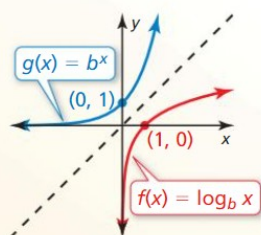
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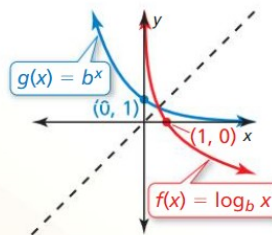
Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$. Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$



Graph of $f(x) = \log_b x$ for $0 < b < 1$



Note that the y -axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of $f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

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