教材 10 比例式 proportion

Ratio and Proportion （比與比例式）

A ratio is a comparison of two quantities by division, it can be represented in three different forms, such as $a : b$; $\frac{a}{b}$; or a to b.

Therefore, the ratio defines the relationship between two quantities such as a: b, where b is not equal to 0. Example: The ratio of 2 to 4 is represented as 2:4 = 1:2. And the statement is said to be in proportion here.

This type of ratio has no units. If different quantities are compared, this special type of ratio is called a rate and it has units.

The quantities in a ratio are sometimes called terms.
In a ratio, the first term is also called antecedent, and the second term is also called consequent.

Proportion is an equation that defines that the two given ratios are equivalent to each other. For example, the time taken by train to cover 100 km per hour is equal to the time taken by it to cover the distance of 500 km for 5 hours. Such as 100km/hr = 500km/5hrs.

Sometimes, students get confused with the concept of ratio and proportion.

For example, $\frac{4}{5}$ is a ratio and the proportion statement is $\frac{20}{25} = \frac{4}{5}$.

課本內容：

1 比與比值

　　我們常利用比與比值的概念來表示兩數量之間的關係，例如 12 包餅乾與 3 包餅乾的比為 12：3，其中 12 稱為前項，3 稱為後項，前項除以後項所得的數稱為比值，因此 12：3 的比值為 12÷3＝4，即 12 包餅乾是 3 包餅乾的 4 倍。

> 翻譯示例：
>
> We often use the concept of ratio to represent the relation between two quantities.
> For example, the ratio of 12 packs of biscuits to 3 packs of biscuits is 12:3,
> where 12 is called the antecedent and 3 is called the consequent.
>
> It can be represented in three different forms, such as 12：3; $\frac{12}{3}$; or 12 to 3.

練習:

　　假設 7 年 1 班有 10 位男生和 12 位女生，則在 7 年 1 班中男生人數：全班人數的比為 10：(10+12)＝10：22。

其比值為 $\frac{10}{22}$。

　　也就是說，男生人數是全班人數的 $\frac{10}{22}$ 倍。

> 翻譯示例：
>
> Suppose there are 10 boys and 12 girls in Class 701.
> The ratio of the number of boys and all students in Class 701 is 10：(10+12)＝10：22.
>
> The value of ratio is $\frac{10}{22}$.
>
> In other words, the number of boys is $\frac{10}{22}$ times the number of all students.

課本內容：

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【比與比值】

a、b 兩數的比記為 $a：b$，則

(1) a 稱為比的前項，b 稱為比的後項。

(2) $a：b$ 的比值為 $\dfrac{a}{b}$，即 $a÷b$ 的值（$b≠0$）。

(3) 若 $a÷b＝k$，表示 a 是 b 的 k 倍。

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國小時曾學過正數的比值，事實上，我們也可以求負數的比值。

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翻譯示例：

【ratio】

Given a ratio of *a* and *b* (represented as *a* : *b*), then:

(1) *a* is called the antecedent of the ratio, and *b* is called the consequent of the ratio.

(2) The value of ratio of *a* : *b* is $\dfrac{a}{b}$, that is, the value of a ÷ b (b ≠ 0). 問比值

(3) If a ÷ b = k, it means that *a* is *k* times *b*.

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練習：

Find the value of the following ratio.

(1)（-20）：24

(2) $10：\dfrac{1}{9}$

課本內容：

比值相等的兩個比，稱為相等的比。，「8：20」與「12：30」的比值相等，就稱這兩個比相等，可記成 8：20＝12：30。

由於分數經過約分或擴分後，其值不變。

因此一個比的前項與後項同除以或同乘以一個不為 0 的數，其比相等。

> 翻譯示例：
> Equivalent ratios are those that can be simplified or reduced to the same value.
> Two or more ratios can be compared with each other to check whether they are equivalent or not.
> For example, 8:20 and 12:30 are equivalent ratios.
> It can be written as 8：20＝12：30
> Whether you're expanding or reducing a fraction, you are not changing the amount that the fraction represents.
> Therefore, if the antecedent and the consequent of a ratio are divided or multiplied by a number that is not 0, the ratios are equivalent.

【相等的比】

已知 $m \neq 0$，則

(1) $a：b＝(a÷m)：(b÷m)$。

(2) $a：b＝(a×m)：(b×m)$。

課本內容：

　　一個比如果前項、後項都是整數，且此兩個整數的絕對值互質，這個比就稱為**最簡整數比**。例如 $2：5$ 與（-7）$：6$ 是最簡整數比；$\frac{1}{3}：2$ 與 $8：18$ 不是最簡整數比。

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翻譯示例：

A simple whole number ratio is when both the numerator and denominator are integers of any size, but are coprime (i.e. they share no common factors, hence the ratio is simple).

A ratio that has integers for both quantities and that cannot be reduced any further (using integers) is said to be in simplest form or lowest terms.

For example:

2: 5 and (-7): 6 are in simplest form;

$\frac{1}{3}$: 2 and 8: 18 are not in simplest form.

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練習：　最簡整數比

Simplify ratios and reduce to simplest form.

(1) $50：35$　　　　　　　　　(2) $\frac{1}{12}：\frac{1}{3}$

練習： 應用問題

1.Jack bought 5 gallons of grape juice and 7 gallons of apple juice for a party. What is the ratio of gallons of apple juice to gallons of grape juice he bought?

2.A survey asks 40 people if they prefer grape juice or apple juice. The data show that 12 people prefer grape juice.

①Write a ratio, in three ways, to describe the relationship between the people who prefer grape juice and all the people surveyed.

②Write a ratio, in three ways, to describe the relationship between the people who prefer apple juice and all the people surveyed.

③Write a ratio, in three ways, to describe the relationship between the people who prefer grape juice and apple juice.

3.There are 5 green pens and 20 blue pens in a bag.

①What is the ratio of green pens to total number of pens?

②Find three equivalent ratios that represent this relationship.

練習： 應用問題

參考資料來源

1. 110 國中數學 1 下翰林版課本

2. Into Math - Advanced 1

☆老師們可以自己從中選擇以做出適合自己學生程度的學習單或是在課堂中適時補充這些英文。

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