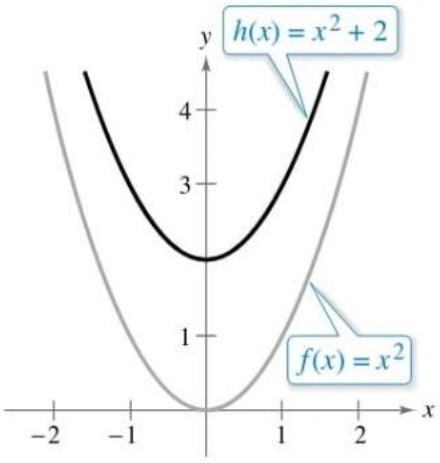
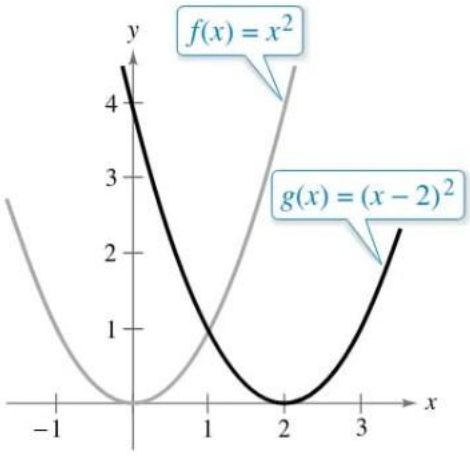
Topic: The transformations of exponential functions and logarithmic functions

1. **Review**: Shifting graphs. Take $f(x) = x^2$ for example.
Look at the graphs below and describe how the graph of g is obtained by shifting of the graph of f and write the formula to make such shifts happen.
(如圖，請回答各圖發生了什麼平移？函數式間有什麼關係？)

a.



b.



We can obtain the graph of $h(x) = x^2 + 2$ by shifting the graph of $f(x) = x^2$ up ___units. In function notation, $f$and h are related as $h(x) = x^2 + 2 = f(x) + \_\_\_\_.$	We can obtain the graph of $g(x) = (x-2)^2$ by shifting the graph of $f(x) = x^2$ to the right ____units. In function notation, $f$and g are related as $g(x) = (x-2)^2 = f(x - \_\_\_).$

The list below summarizes this discussion about horizontal and vertical shifts.

Vertical and Horizontal Shifts

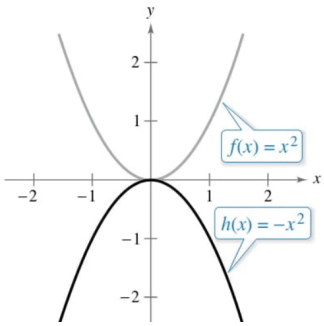
Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *up:* $h(x) = f(x) + c$
2. Vertical shift c units *down:* $h(x) = f(x) - c$
3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

2. Review: Reflecting Graphs

Take $f(x) = x^2$ for example.

The graph $h(x) = -x^2$ is the reflection of $f(x) = x^2$ about the x–axis.



Reflecting about the Coordinate Axes

Reflections about the coordinate axes of the graph of $y = f(x)$ are represented as follows.

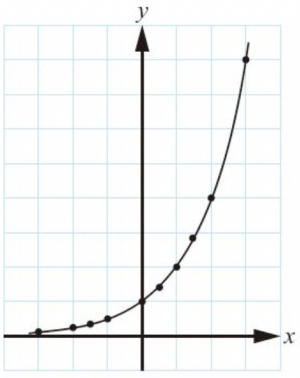
1.Reflection about the x–axis: $h(x) = -f(x)$

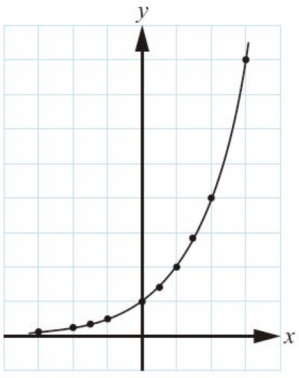
2.Reflection about the y–axis: $h(x) = f(-x)$

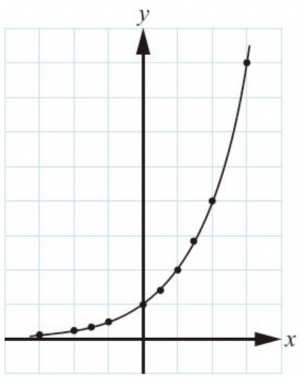
3. The transformation of exponential functions and logarithmic functions

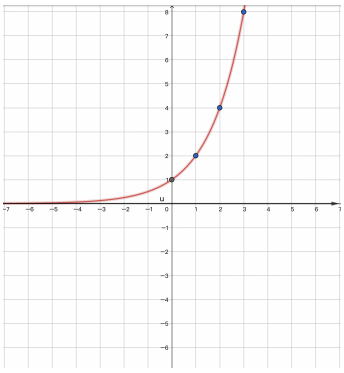
(1) Use the graph of $y = 2^x$ to sketch the graph of each function and its asymptote.

a. $y = 2^{x-1}$ b. $y = 2^{x-1} + 1$ c. $y = 2^{-x} = (\frac{1}{2})^x$ d. $y = -2^x$





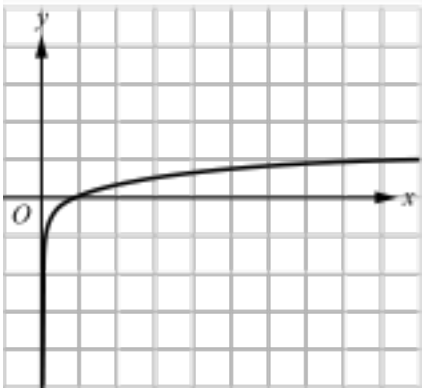




(2) Use the concept of transformations to win this challenge of Desmos.https://teacher.desmos.com/activitybuilder/custom/566b317b4e38e1e21a10aafb?lang=zh-TW&collections=featured-collections%2C5da6462c8b305273be677729

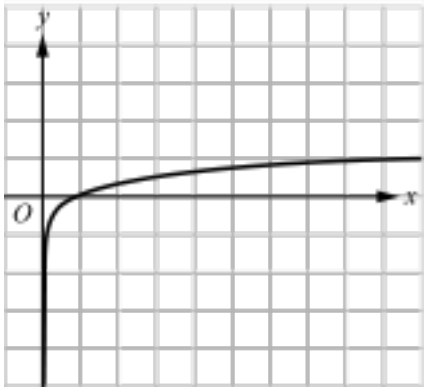
(3) Use the graph of $y = logx$ to sketch the graph of each function and its asymptote.

a. $y = log(x-1)$

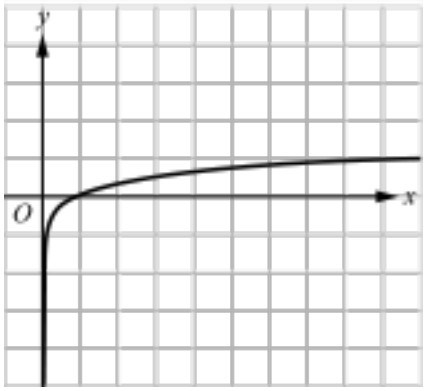


$y = -logx = log_{\frac{1}{10}}x$

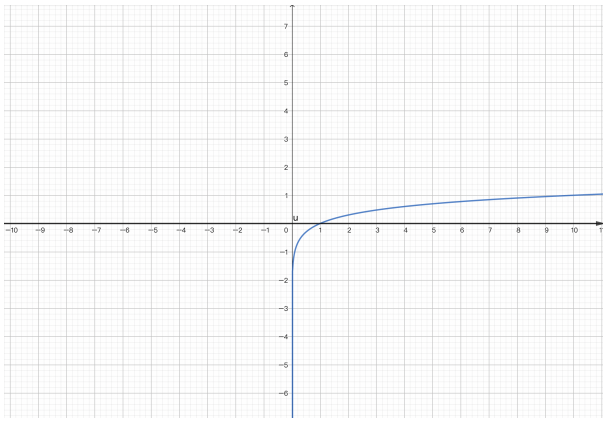
b. $y = log(x-1)+2$



c.



d. $y = log(-x)$



4. Review: Nonrigid Transformations

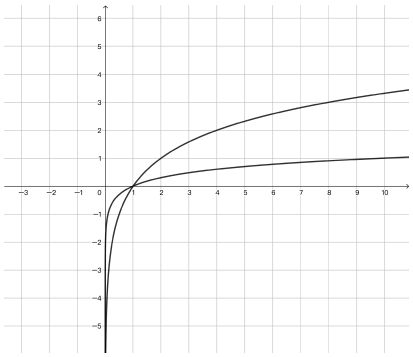
There are four common types of nonrigid transformations:

	The graph of $y = f(x)$ is represented by $g(x) = cf(x)$
Vertical stretch	$c > 1$
Vertical shrink	$0 < c < 1$
	The graph of $y = f(x)$ is represented by $h(x) = f(cx)$
Horizontal stretch	$0 < c < 1$
Horizontal shrink	$c > 1$

Take $f(x) = x^2$ for example. Use Desmos to compare the graph of each function below with the graph of $f(x) = x^2$ and state which type of nonrigid transformation is.

a. $g_1(x) = 2x^2$ b. $g_2(x) = \frac{1}{4}x^2$ c. $h_1(x) = f(\frac{1}{3}x) = (\frac{1}{3}x)^2$ d. $h_2(x) = f(3x) = (3x)^2$

5. (Challenge) We plot the $f(x) = logx$ and $g(x) = log_2 x$ on the same coordinate plane. Use the change base formula to explain that $g(x) = log_2 x$ is a vertical stretch of $f(x) = logx$



6.參考資料：https://www.mathsisfun.com/sets/function-transformations.html

Topic: The transformation of exponential functions and logarithmic functions 及使用建議

內容	1. Review:Shifting graphs.....以下省略
使用建議	[教學活動安排] 複習高一學過的函數圖形的平移與函數式之間的關係。 [可參考的英文問句/提問/開場] Today we're going to learn the transformations of exponential functions and logarithmic functions. We have learned the transformations of functions before. Let's jog our memories. We will check it in two minutes. Would anyone like to share the answer?
內容	2. Review:Reflecting Graphs....以下省略
使用建議	[教學活動安排] 複習高一學過的函數圖形的對x軸對稱、對y軸對稱與函數式之間的關係 [可參考的英文問句/提問/開場] Another type of transformation is reflecting graphs. It is an easy one. But be careful, what should we notice here? Would anyone like to share? $h(x) = -f(x)$ which the negative sign is on the outside of f of x. This would result in a reflection along the x–axis. $h(x) = f(-x)$which the negative sign is (found) within the brackets This negative sign applied to the x would result in a reflection along the y–axis 參考資料：https://youtu.be/OpH7ocRalRs
內容	3.The transformations of exponential functions and logarithmic functions....以下省略

使用建議	[教學活動安排] 3. (1) 是利用複習過的概念 ,應用在已給的$y = 2^x$的圖形去畫給定的函數的圖形 3.(2)是 Desmos的活動，讓學生可以觀察圖形拉動滑桿 (平移) 改變函數式去贏得挑戰。是動態的練習中讓學生熟悉概念。 3.(3) 是利用複習過的概念 ,應用在已給的$y = logx$的圖形去畫給定的函數的圖形 [可參考的英文問句/提問/開場] Let's use the concept of transformations to sketch the graph of each exponential function. We take $y = 2^x$ for example. See the worksheet 3(1). Let's do a.$y = 2^{x-1}$ together. The exponent of base 2 is x minus 1 . So,which transformation is it ? 點學生回答。學生回答後回饋 Well done. Then we can very quickly sketch the graph. We don't have to find the points and then connect them.Now it's your turn. Complete the rest of this example. We'll check it in five minutes. Once you get more familiar with how the functions are related to the transformations of graphs, you quickly get what functions graph look like. So let's do this desmos challenge to help us to get this skill.Bonus points for the top three.(此項挑戰的前三名可以加分！) Apply the same skill to sketch the graphs of logarithmic functions.We'll check it in five minutes.
內容	4. and 5.....以下省略

使用建議	[教學活動安排] 簡單的複習四種伸縮變換，並利用單項二次函數作為例子複習，再讓學生應用在對數函數圖形上。 [可參考的英文問句/提問/開場] Horizontal shifts, vertical shifts, and reflections are rigid transformations because the basic shape of the graph is unchanged. These transformations change only the position of the graph in the coordinate plane. Nonrigid transformations are those that cause a distortion–a change in the shape of the original graph. Look at the sheet then do the example of $y = x^2$. We'll check it in five minutes. Would anyone like to share? Let's see the challenge. Looking at the graph, we can see the graph of $g(x) = log_2 x$ is a vertical stretch of $f(x) = log x$. Why? How do we know? Try to use the change base formula to explain it.

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