雙語教學主題(國中九年級上學期教材):　三角形的重心

Topic: The centroid of a triangle

The teaching materials for introducing circles in our textbooks have been changed quite a bit due to the **108 syllabus.** Therefore, the content here is based on the official textbooks-NANI, KANG HSUAN and HANLIN.

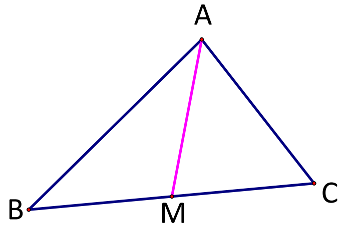
由於 108 新綱教材大改，所以這個單元參考 108 新課綱及南一、康()¯軒及翰林版國中數學課本第五冊

Vocabulary

centroid, median, vertex, vertices, congruent, equidistant,

Before we start introducing this new lesson, we want to introduce a new word median.

As shown in the figure on the right, in ΔABC, point M is the midpoint of segment AB. Then $\overline{AM}$ is a median.



We can easily prove that

The area of ΔABM=the area of ΔACM=$\frac{1}{2}$the area of ΔABC

For teachers' reference (students can try to prove it themselves)

Pf:

In ΔABC, $\overline{AH}\perp\overline{BC}$ and $\overline{AH}$ intersects $\overline{BC}$ at point H, as shown in figure 1.

The area of ΔABC=$\frac{1}{2}\overline{BC}$x$\overline{AH}$

The area of ΔABM=$\frac{1}{2}\overline{BM}$x$\overline{AH}$

The area of ΔACM=$\frac{1}{2}\overline{CM}$x$\overline{AH}$

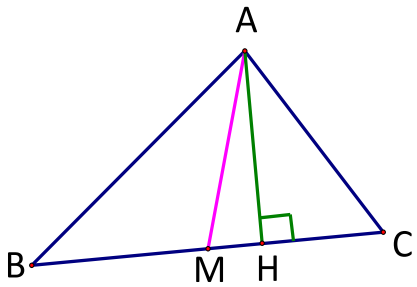


Figure 1

Since point M is a midpoint of segment BC

$$\overline{BM}=\overline{CM}=\frac{1}{2}\overline{BC}$$

We get

The area of △ABM= $\frac{1}{2}\overline{BM}$x$\overline{AH}$= $\frac{1}{4}\overline{BC}$x$\overline{AH}$=$\frac{1}{2}$the area of △ABC

The area of △ACM= $\frac{1}{2}\overline{CM}$x$\overline{AH}$= $\frac{1}{4}\overline{BC}$x$\overline{AH}$=$\frac{1}{2}$the area of △ABC

That is:

The area of △ABM=the area of △ACM=$\frac{1}{2}$the area of △ABC

So the statement is true.#

Now let's learn what the centroid of a triangle is.

Centroid:

Figure 1 shows that $\overline{BE}$ and $\overline{CD}$ are two medians in △ABC, and they intersect each other at point G.

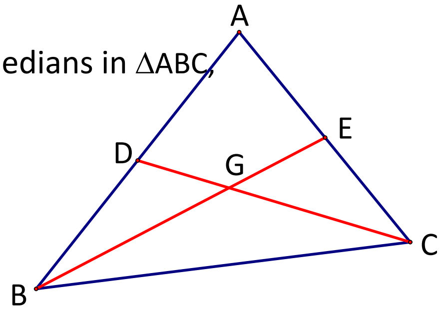


Figure 1

As we mention in the beginning, $\overline{BE}$ is a median of △ABC, then
the area of △ABE =the area of △CBE., this means the center of gravity (center of mass)of this triangle must lie on $\overline{BE}$. $\overline{CD}$ is also a median of △ABC. Therefore, we know that the intersection point G is the center of gravity of △ABC.

We have learned that in a triangle, three perpendicular bisectors of three sides will intersect at one point, and three angle bisectors of three interior angles will intersect at one point. If we construct the third median in ΔABC, will these three medians also intersect at one point?

Connect $\overline{DE}$ as shown In figure 2.

By midsegment theorem, $\overline{DE}//\overline{BC}$ and $\overline{DE}=\frac{1}{2}\overline{BC}$

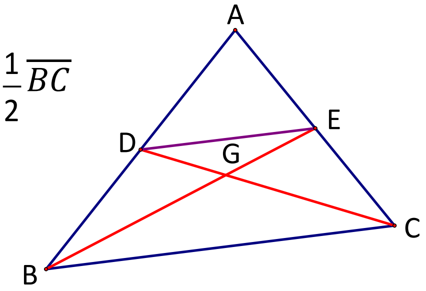


Figure 2

In ΔDEG and ΔCBG,

∠DEG=∠CBG ($\overline{DE}//\overline{BC}$, the alternate interior angles are equal)

∠DGE=∠CGB(the vertical angles are equal)

(of course you can use the same reason as the first one with ∠EDG=∠BCG)

We get ΔDEG~ ΔCBG (AA)

Then $\overline{DG}:\overline{CG}=\overline{EG}:\overline{BG}=\overline{DE}:\overline{BC}$=1:2 ($\overline{DE}=\frac{1}{2}\overline{BC}$)

In figure 3, connect $\overline{AF}$ where point F is the midpoint of $\overline{BC}$. And assume $\overline{AF}$ intersects $\overline{BE}$ at a point G'.

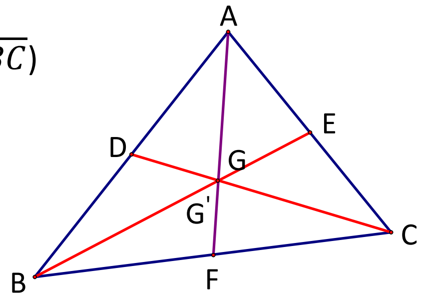


Figure 3

Similarly, $\overline{EG'}:\overline{BG'}$=1:2, but $\overline{EG}:\overline{BG}$=1:2

Point G and point G' are both on segment BE

Therefore, Point G and point G' must be the same point

That means the third median $\overline{AF}$ also passes through the centroid

We get the conclusion that

Three medians of a triangle will intersect at one point, centroid.

We can see in figure 4 that no matter what kind of triangle it is, the centroid of the triangle always stays inside the triangle.

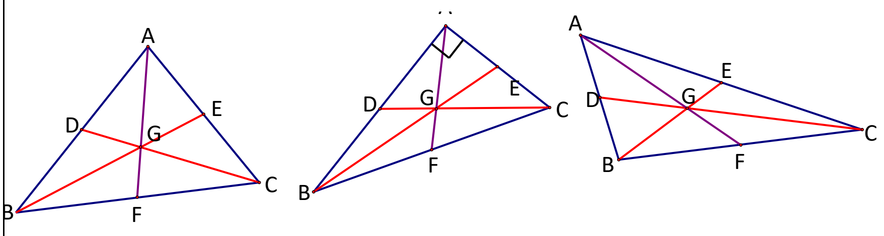


Figure 4

From the discussion above, we get a very important result that

$\overline{AG}=2\overline{FG}$, $\overline{BG}=2\overline{EG}$, and $\overline{CG}=2\overline{DG}$:

Now let's talk about some important properties concerning centroids of triangles

In figure 1 on the right side, point G is the centroid of ΔABC, connect $\overline{AG}$, $\overline{BG}$, and $\overline{CG}$.

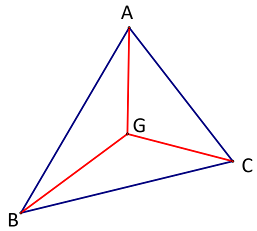
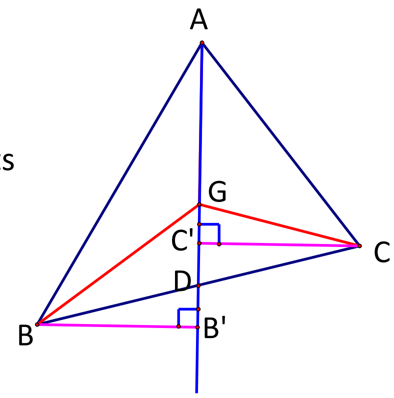


Figure 1

Then the area of ΔAGB= the area of ΔBGC = the area of ΔCGA

$=\frac{1}{3}$ (the area of ΔABC)



Pf: in figure 2, construct $\overrightarrow{AG}$, and $\overrightarrow{AG}$ intersects $\overline{BC}$ at point D. And construct $\overline{BB'}\perp\overrightarrow{AG}$ and intersects $\overrightarrow{AG}$ at point B',

construct $\overline{CC'}$ $\perp\overrightarrow{AG}$ and intersects $\overrightarrow{AG}$ at point C'.

In △BB’D and △CC’D,

∠BB’D=∠CC’D=90° ($\overline{BB'}\perp\overrightarrow{AG}$,$\overline{CC'}\perp\overrightarrow{AG}$) Figure 2

∠ BDB’=∠CDC’ (vertical angles are equal)

$\overline{BD}=\overline{CD}$ (point G is the centroid, so point D is the midpoint of segment BC.)

Then △BB’D≅△CC’D (AAS)

$\overline{BB'}=\overline{CC'}$ (corresponding segments are congruent)

In △ABG, the area of △ABG=$\frac{1}{2}\cdot\overline{AG}\cdot\overline{BB'}$

and in △ACG, the area of △ACG=$\frac{1}{2}\cdot\overline{AG}\cdot$

the area of △ABG= the area of △ACG (same base and congruent height)

Similarly, we can get the conclusion that

the area of △ABG= the area of △ACG= the area of △BCG

∵the area of △ABC

=the area of △ABG+the area of △ACG+the area of △BCG

We get the area of △ABG= the area of △ACG= the area of △BCG

=$\frac{1}{3}$ (the area of △ABC) #

You can have lots of methods to get the result above, please try your best to think and enjoy the process.

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If we construct three medians of a triangle, we can get the result as follows:

In figure 3, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are medians of ∆ABC.

Let the area of ∆AFG=a, ∆BFG=b, ∆BDG=c, ∆CDG=d, ∆CEG=e, and ∆AEG=f,
then a=b=c=d=e=f

Pf:

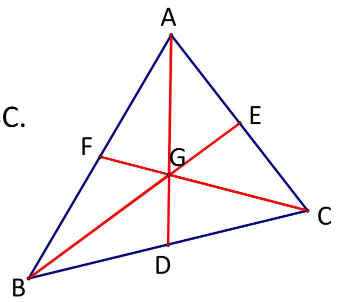


Figure 3

In figure 4, construct $\overline{GH}\perp\overline{BC}$ and intersects $\overline{BC}$ at point H.

Then c=$\frac{1}{2}\cdot\overline{BD}\cdot\overline{GH}$=$\frac{1}{2}\cdot\overline{CD}\cdot\overline{GH}$=d   ($\overline{BD}$=$\overline{CD}$)

With the similar process, we get

a=b=c=d=e=f#

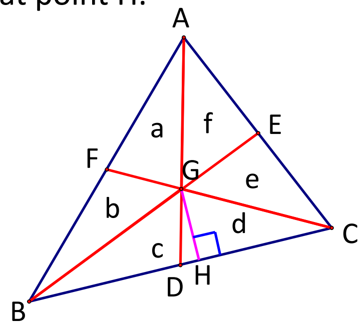
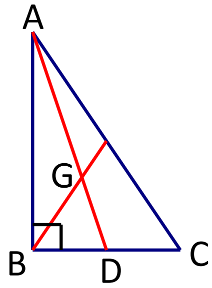


Figure 4

Let's do some examples here.

Ex 1:

Point G is the centroid of right triangle ABC. Point D is the midpoint of segment BC, $\overline{AC}$=10, $\overline{BC}$=6.



Please find out:

(1)the length of $\overline{AG}$

(2)the area of ΔABG

Sol:

(1) $\overline{AG}=\frac{2}{3}\overline{AD}=\frac{2}{3}\sqrt{\overline{AB}^2+\overline{BD}^2}$

By Pythagorean theorem

$\overline{AB}^2=\overline{AC}^2-\overline{BC}^2=10^2-6^2=64$

$\overline{BD}=\frac{1}{2}\overline{BC}=\frac{1}{2}\cdot 6=3$

$\overline{AG}=\frac{2}{3}\overline{AD}=\frac{2}{3}\sqrt{\overline{AB}^2+\overline{BD}^2}=\frac{2}{3}\sqrt{64+9}=\frac{2}{3}\sqrt{73}$

(2) the area of ΔABG$=\frac{1}{3}$the area of ΔABC

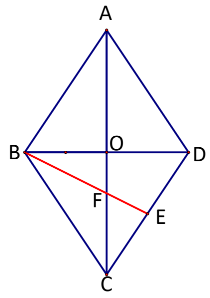
$=\frac{1}{3}\cdot\frac{1}{2}\cdot 8\cdot 6$

$=8$#

Ex 2:

Two diagonals in rhombus ABCD $\overline{AC}$ and $\overline{BD}$

intersect each other at point O. Point E is the midpoint of segment CD.



$\overline{EF}$=2, $\overline{CF}$=4. Please find out:

(1)the length of segment BF
(2) the length of segment OF
(3) the length of segment BO
(4) the area of △CEF
(5) the area of rhombus ABCD

Sol:

The two diagonals of a rhombus are perpendicular and bisect each other.

Point O is the midpoint of $\overline{BD}$ and point E is the midpoint of $\overline{CD}$.

(1) $\overline{BF}=2\overline{EF}=2\cdot 2=4$ (property of centroid)

(2) $\overline{OF}=\frac{1}{2}\overline{CF}=\frac{1}{2}\cdot 4=2$

(3) apply the Pythagorean theorem,

$$\overline{OB}^2=\overline{BF}^2-\overline{OF}^2=4^2-2^2=12$$

$$\overline{OB}=2\sqrt{3}$$

(4) the area of △CEF= the area of △OBF

$$=\frac{1}{2}\cdot\overline{OB}\cdot\overline{OF}$$

$$=\frac{1}{2}\cdot 2\sqrt{3}\cdot 2$$

$$=2\sqrt{3}$$

(5) the area of rhombus ABCD=12·the area of △OBF

$$=12\cdot 2\sqrt{3}=24\sqrt{3}_{\#}$$

Let's wrap up the key information as follows:

TYPE / FEATURES	CIRCUMCENTER	INCENTER	CENTROID
INTERSECTION POINT OF	PERPENDICULAR BISECTORS OF THREE SIDES	ANGLE BISECTORS OF THREE INTERIOR ANGLES	THREE MEDIANS
POSITION	ACUTE ∆ INTERIOR OF ∆ RIGHT ∆ ON THE MIDPOINT OF HYPOTENUSE OBTUSE ∆ EXTERIOR OF ∆	ALWAYS INTERIOR OF ∆	ALWAYS INTERIOR OF ∆
EQUIDISTANT TO	THREE VERTICES	THREE SIDES	X
CENTER OF	CIRCUMCIRCLE	INCIRCLE	X

There is so much to learn in this lesson. Please review it thoroughly. And do more practice is a must.

Watch and enjoy: Center of gravity
https://youtu.be/R8wKV0UQtlo

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