

最大公因數與最小公倍數

Greatest Common Factor and Least Common Multiple

Class: _____ Name: _____

1. 公因數與最大公因數

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Which numbers are factors of both 24 and 36? Circle them!

A common factor(公因數) is a whole number that is a factor of two or more numbers.

List all common factors of 24 and 36.

The greatest common factor(GCF) is the largest factor that is a factor of two or more numbers.

What is the GCF of 24 and 36?

The GCF of 24 and 36 can be written as (24,36).

2. 利用短除法求最大公因數

Example. Find the GCF of 120 and 165.	Exercise. Find the GCF of 24 and 39.
Example. Find the GCF of 84, 140, and 168.	Example. Find the GCF of 25, 50, and 85.
Example. Find the GCF of $2^4 \times 3^2 \times 5$ and $2^2 \times 3^5 \times 7^3$.	Example. Find the GCF of $5^3 \times 11^3$ and $3 \times 5^4 \times 7$.

3. 互質(coprime)

Two numbers are coprime if their GCF is 1.

(1) Are 25 and 80 coprime?

(2) Are 8 and 27 coprime?

(3) Are 3 and 57 coprime?

4. 公倍數與最小公倍數

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, ...

Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, ...

Which numbers are factors of 6 and 9? Circle them!

A common multiple(公倍數) is a whole number that is a multiple of two or more numbers.

List at least 10 common multiples of 6 and 9.

The least common multiple(LCM) is the smallest number that is a multiple of two or more numbers.

What is the LCM of 6 and 9?

The LCM of 6 and 9 can be written as [6,9].

5. 利用短除法求最小公倍數

Example. Find the LCM of 40 and 88.	Exercise. Find the LCM of 33 and 77.
Example. Find the LCM of 315, 675, and 900.	Example. Find the LCM of 42, 77, and 154.
Example. Find the LCM of $2^3 \times 11^4$ and $2 \times 7 \times 11^5$.	Example. Find the LCM of $2 \times 7 \times 11^2$ and $5^2 \times 13^4$.

6. 最大公因數與最小公倍數的關聯

(1) Find the GCF of 12 and 20.

(2) Find the LCM of 12 and 20.

(3) Calculate 12×20 .

(4) Find the relationship between the answer in (1), (2), and (3).

★小結論： $(a,b) \times [a,b] = a \times b$

一、設計理念：

1. 學生國小已學習過利用短除法求最大公因數及最小公倍數，國中部分僅增加求數字以標準分解式呈現的最大公因數及最小公倍數。在前一個小節已學過因數與倍數。
2. 互質的英文本文使用 coprime，實際上亦可使用 relatively prime。
3. 因數的英文可使用 factor 或 divisor，本文中的因數均使用 factor，故公因數使用 common factor，最大公因數則使用 greatest common factor。另外，最大公因數亦有 highest common factor(HCF)的用法。
4. greatest common factor 簡寫為 GCF，least common divisor 簡寫為 LCM，中間不需要縮寫點(period)，發音則將字母逐字唸出即可。
5. 國外教最大公因數時通常會連著最大公因式(greatest common factor of monomial)一起教，最小公倍數連著最小公倍式(least common multiple of monomial)一起教。

二、英文詞彙：

中文	英文
因數	factor
公因數	common factor
最大公因數	greatest common factor(GCF)
倍數	multiple
公倍數	common multiple
最小公倍數	least common multiple(LCM)
互質	coprime
短除法	short division
質因數分解	prime factorization
標準分解式	prime factorization in exponent form
乘積	product

三、數學英文用法：

數學表示法	英文
6 是 3 的倍數	Six is a multiple of three.
3 是 6 的因數	Three is a factor of three.
6 被 3 整除	Six is divisible by three.
2 是 4 和 12 的公因數	Two is the common factor of four and twelve.
8 和 12 的最大公因數是 4	The GCF of eight and twelve equals four.
8 是 2 和 4 的公倍數	Eight is the common multiple of two and four.
2 和 3 的最小公倍數是 6	The LCM of two and three equals six.
8 和 9 互質	8 and 9 are coprime.

四、教學參考範例：

1	<p>我們前面學過因數的概念，先讓我們複習一下因數。What are the factors of 24? The factors of 24 are numbers that can be divisible by 24. For example, 24 can be divisible by 3, so 3 is a factor of 24. 24 is not divisible by 5, so 5 is not a factor of 24. Then, let's list all factors of 24. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. What are the factors of 36? The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.</p>
2	<p>接下來，我們要介紹公因數及最大公因數。Some numbers are both the factors of 24 and 36. Which numbers are factors of 24 and 36? 1, 2, 3, 4, 6, and 12. We say these numbers are the common factors of 24 and 36. “Common factor” 就是公因數。 Then, the largest of common factors of 24 and 36 is called the greatest common factor of 24 and 36, or we can say the GCF of 24 and 36. “GCF”就是最大公因數“greatest common factor”每個字的第一個字母縮寫，習慣上可以直接寫 GCF。 Also, the GCF of 24 and 36 can be written as (24,36), so we can write (24,36)=12.</p>
3	<p>Next, we want to find the GCF of 120 and 165. We can list all factors of 120 and 165, list all factors of 120 and 165, and find the largest number of common factors. But finding all factors of a large number is difficult. We can also use short division to find GCF of 120 and 165. 我們曾經學的短除法除了可以用在將一個數質因數分解外，現在也可以用來幫我們找最大公因數。</p>
4	<p>We can easily find that 3 is the common factor of 120 and 165, so we write 3 on the left. Then, 120 divided by 3 equals 40, so we write 40 below 120. 165 divided by 3 equals 55, so we write 55 below 165. We can keep finding the common factor of 40 and 55. These two numbers are divisible by 5, so we write 5 on the left. Dividing 40 and 55 by 5, we get 8 and 11, we write down 8 and 11 below 40 and 55. Now, 8 and 11 are coprime, so we stop right here, and the GCF of 120 and 165 is the product of all the numbers on the left. The GCF of 120 and 165 is 3 times 5, which equals 15. 用短除法求最大公因數時，將兩個數一直除到互質之後，最大公因數就是把最左邊的那排相乘。</p>

$$\begin{array}{r|l}
 3 & 120 \quad 165 \\
 5 & 40 \quad 55 \\
 & 8 \quad 11
 \end{array}$$

五、引入語言使用建議：

第 1 段	先用中文提示現在要先複習因數，並以英文讓學生練習找出 24 和 36 的所有因數，同時幫學生複習因數的數學概念及熟悉英文。
第 2 段	用中文概述要介紹公因數及最大公因數，因為數學概念上並不難，所以先用英文介紹這兩個概念，再用中文介紹詞彙對應的中文名稱。
第 3 段	由於此部分仿照前面的做法，故整段皆使用英文，結束後在用中文補充說明。
第 4 段	計算過程因涉及英文不難，且會同時在黑板上演示，故直接利用英文操作，結束後再以中文總結。

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