雙語教學主題(國中九年級教材): 圓-1
Topic: introducing circles-1

The teaching materials for introducing circles in our textbooks have been changed quite a bit due to the **108 syllabus.** Therefore, the content of introducing circles here is based on one of the official textbooks- NANI, KANG HSUAN, and HANLIN.
由於 108 新綱教材大改，所以這個單元參考 108 新課綱及南一、康軒及翰林版國中數學課本第五冊

這個單元常用到的一些用語
The vocabulary we will use in this topic
Center of a circle, circumference, radius, diameter, arc, major /minor arc 優/劣弧, chord, sector, segment in a circle, semicircle, tangent, secant, central angle, inscribed angle, area, perimeter, bisector, perpendicular, perpendicular bisector 中垂線, right angle 直角, segment of a circle 弓形, intersect 相交, external, internal, point of tangency 切點,

**Definition of a circle**: It's a closed 2D figure in which the set of all the points in a plane has the same distance from a given point called the **center.**
Introducing terms of a circle:

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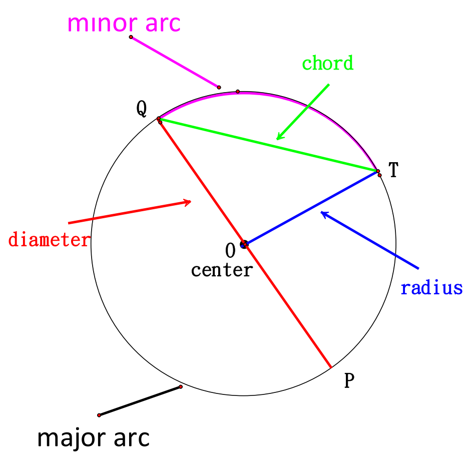
| <br>The diameter is two times the radius<br>$\overline{PQ}=2\overline{OT}=2r$<br><br>The circumference of the circle is $2\pi r$<br>The area of the circle is $\pi r^2$ | This is a circle O, and O is the center of the circle.<br>Radius: the fixed distance from the center of a circle to any point on the circle itself., usually written as r, $\overline{OT}$=r<br><br>Chord: a line segment that connects two points on a circle, $\overline{QT}$ is a chord<br><br>Diameter: the chord that passes through the center of a circle, $\overline{PQ}$ is the diameter of the circle, $\overline{PQ}$=2r<br>(the length of the segment PQ is 2r)<br>We always say: diameter is the longest chord in a circle.<br>Circumference: the distance around the circle, like the perimeter of a circle |

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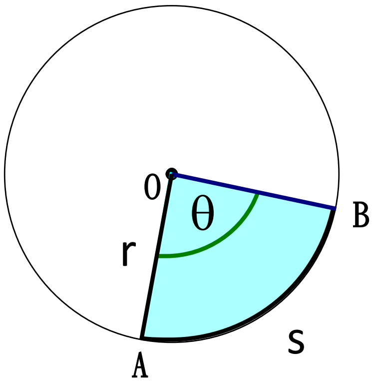
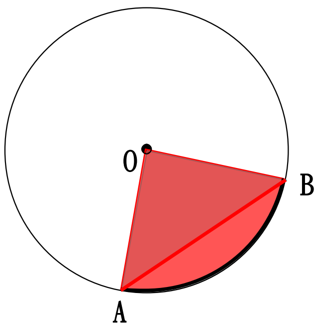
| | Arc: a section of the circumference<br><br>arc QT denotes as $\overset{\frown}{QT}$<br>The chord of a circle intercepts the circumference into two parts<br>$\overset{\frown}{QPT}$ (arc QPT) is a **major arc** and $\overset{\frown}{QT}$ is a minor arc<br>Whenever an arc is denoted with its two endpoints, it always refers to a minor arc. |
| <br><br><br>The area of the sector OAB is the fraction of the area of the circle<br>$=(\frac{\theta}{360})\pi r^2$<br><br><br> | Sector: a section of a circle, sector OAB<br><br>Let $\angle$AOB=θ, radius=r, $\overset{\frown}{AB}$=s<br>(arc AB is s)<br><br>θ over 360 times π times r squared<br><br>A segment of a circle: the region that is bounded by an arc and a chord of the circle, shown as the red area in the left figure.<br>We can also calculate the area and the "perimeter" of the segment of a circle. We will introduce them soon. You can do your thinking first. |
| | |

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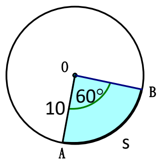
<td>
Ex1: The radius of the circle is 10, ∠AOB=60°<br>
Please find out the following of the circle:<br>

<br>
(1) the diameter of the circle<br>
(2) the circumference of the circle<br>
(3) the area of the circle<br>
(4) the area of the sector OAB
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Answers:<br>
(1) the diameter of the circle<br>
$=2r=2\cdot 10=20$<br>
(2) the circumference of the circle<br>
$=2\pi\cdot 10=20\pi\cdot$<br>
(3) the area of the circle$=\pi\cdot 10^2=100\pi$<br>
(4) the area of the sector OAB<br>
$=(\frac{60}{360})100\pi=\frac{50}{3}\pi$<br>
(60 over 360 times 100π equals 50 over 3 times π)
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Most of the information mentioned above is like a review of what you already learned about circles in elementary school.

Now we are going to introduce some new knowledge about circles here. We will discuss the relationship between points and circles, and also lines and circles.

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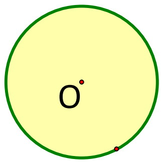
Points and circles<br>
On the coordinate plane, a circle divides the plane into 3 regions, interior of the circle, on the circle, and exterior of the circle.<br>
We compare the distance from point P to the center of circle O to decide the location of point P.<br>


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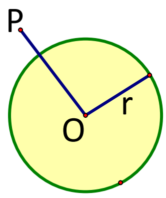
<td>
r is the radius of circle O.<br>

<br>
$\overline{OP}>r$<br>
P is external to the circle O<br>
P is outside the circle
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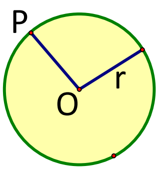
<br>
$\overline{OP}=r$<br>
P is on the circle
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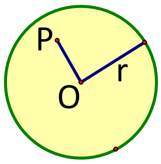
<br>
$\overline{OP}<r$<br>
P is internal to the circle O<br>
P is inside the circle
</td>
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</table>

Let's do this example together

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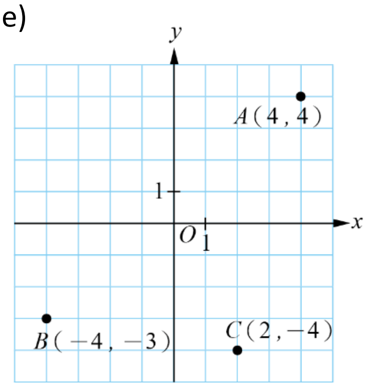
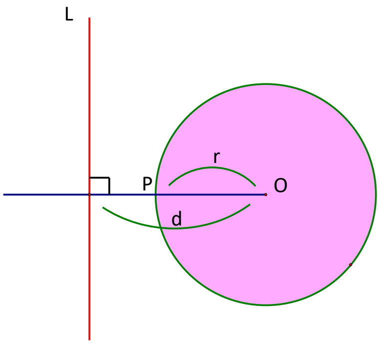
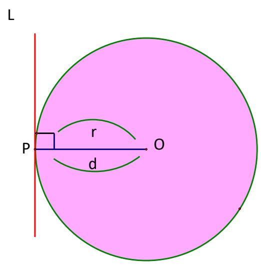
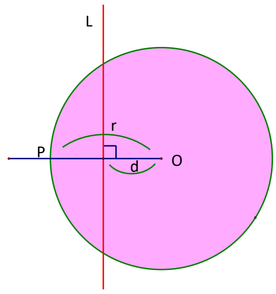
| Ex: On the coordinate plane, there are three points A(4,4), B(-4, -3), and C(2, -4). If the center of the circle O is the origin, and its radius is 5.<br>Please decide the location for each of the three points. (inside, outside, or on the circle)<br><br><br><br>Sol: | Sol:<br>Remember the distance formula between two points on the plane?<br>The distance between two points P(a,b) and Q(x,y) is<br>$\overline{PQ}=\sqrt{(a-x)^2+(b-y)^2}$<br>(line segment PQ equals the square root of parentheses a minus x squared plus parentheses b minus y squared)<br>Here we have<br>$\overline{OA}=\sqrt{4^2+4^2}=4\sqrt{2}=5.657...$<br>(line segment OA equals the square root of 4 squared plus 4 squared equals 4 times the square root of 2 and approximately equals 5.657 by using a calculator)<br>$\overline{OA}>r$<br>$\overline{OB}=\sqrt{(-4)^2+(-3)^2}=5$<br>(line segment OB equals the square root of parentheses negative 4 squared plus parentheses negative 3 squared equals 5)<br>$\overline{OB}=r$<br>$\overline{OC}=\sqrt{2^2+(-4)^2}=2\sqrt{5}=4.472...$<br>(line segment OC equals the square root of the quantity of 2 squared plus parentheses negative 4 squared equals 2 times the square root of 5 and approximately equals 4.472 by using a calculator)<br>$\overline{OC}<r$<br>∴point A is external to the circle O<br>point B is on the circle O<br>point C is interhal to the circle O |

Next, we want to introduce the relationship between lines and circles on the coordinate plane. We will discuss the relationship in terms of the distance from a line to the center of the circler. There are some important lines introduced here, please learn with your heart and review them thoroughly.
Attention: whenever we mention the distance from a point to a line, we always refer to the perpendicular distance.

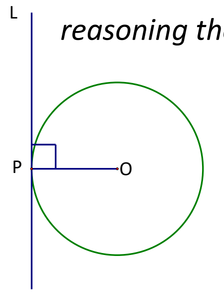
| | | |
|---|---|---|
| On the coordinate plane, we have a line L and a circle O, the radius of the circle is r. P is a point on the circumference of the circle. $\overline{OP}$=r, $\overleftrightarrow{OP}\perp$L(line OP is perpendicular to line L),<br>d is the distance from point O to line L. | | |
|   |   |   |
| d>r<br>line L is external to the circle O<br><br>Number of the intersection point: 0 | d=r<br>We move line L towards P and keep $\overleftrightarrow{OP}\perp$L. When L touches the circle at one single point P, L is **tangent** to the circle. L is the tangent line and P is the point of tangency.<br>Number of the intersection point: 1 | d<r<br>We move line L towards the center O, line L intersects the circle at two points. Line L is secant to the circle. L is the secant line of the circle.<br><br>Number of the intersection point: 2 |

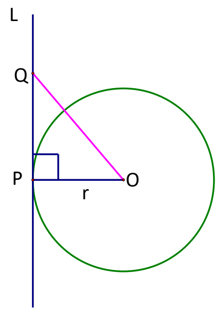
- A tangent line always has only one intersection point on the circle.
- The line segment between the intersection point and the center of the circle is always perpendicular to the tangent line.

  $\overleftrightarrow{OP}\perp$L

  But why? Why is it true that when $\overleftrightarrow{OP}\perp$L, line L can touch the circle at only one point P? is it possible that line L intersects the circle at two or more different points?

  *Hint: get a distinct point Q (from P) on the line L, reasoning the distance between Q and the center O to decide if Q is on the circle.*




For teachers:

$\overleftrightarrow{OP}\perp$L and point P is on the circle, $\overline{OP}$=r, $\Delta$OPQ is a right triangle.

By Pythagorean Theorem

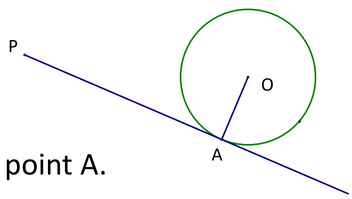
In $\Delta$OPQ, $\overline{OQ}$>$\overline{OP}$=r means point Q is external to circle O.

So point Q is not on the circumference of the circle.

There is a lot to talk about in terms of tangent lines. Let's understand what the length of a tangent line means. We know that the length of a line is infinite, but when we say the length of a tangent, it means the length of $\overline{PA}$.

P is a point outside the circle, and $\overleftrightarrow{PA}$ is tangent to circle O while point A is on the circle.

Then the length of a tangent means the distance between point P and point A.

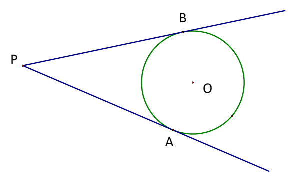



A question for you(to students) :

Point P is outside the circle $\overline{PA}$ and $\overline{PB}$ are two tangent lines of circle O. Is it true that $\overline{PA}$=$\overline{PB}$?

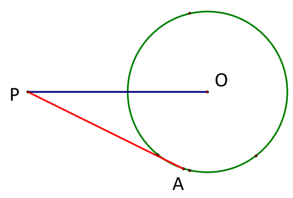
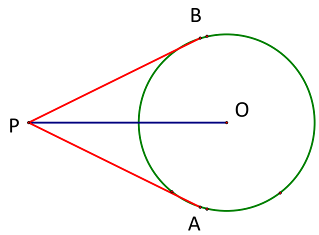
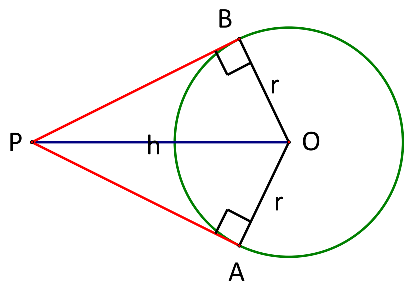
If not, give me a reason.

If yes, prove it!



**For teachers:**

Give students hints. Let them explain ~~it~~ with the symmetry of a circle, prove it with the congruence of triangles, or prove it with the Pythagorean Theorem…

| | |
|---|---|
| Symmetry<br>A circle is a line symmetric figure. Any line through the center of the circle is an axis of symmetry for the circle.<br><br>$\overline{PA}$ is tangent to circle O, point A is the point of tangency. $\overleftrightarrow{PO}$ is an axis of symmetry for the circle. $\overline{PB}$ is the symmetric segment by reflecting $\overline{PA}$ across $\overleftrightarrow{PO}$. Therefore, $\overline{PB}$ is also tangent to circle O and point B is the point of tangency. And $\overline{PA}=\overline{PB}$ | <br> |
| Pythagorean Theorem<br>$\overline{PA}$ and $\overline{PB}$are tangent to circle O.<br>Connect $\overline{OA}$ and $\overline{OB}$, then<br><br>$\overline{OA} \perp \overleftrightarrow{OP}$, $\overline{OB} \perp \overleftrightarrow{OP}$<br>Let $\overline{OP}$ be h and $\overline{OA}=\overline{OB}$=r.<br>$\Delta$POA and $\Delta$POB are right triangles.<br>According to the Pythagorean Theorem<br>$\overline{PA}^2=\overline{OP}^2-\overline{OA}^2=h^2-r^2$ ...... (1)<br><br>$\overline{PB}^2=\overline{OP}^2-\overline{OB}^2=h^2-r^2$ ...... (2)<br>($\overline{OA}$ and $\overline{OB}$ are radii)<br>From equations (1) and (2), conclude<br>$\overline{PA}^2=\overline{PB}^2 \Rightarrow \overline{PA}=\overline{PB}$ | <br>(line segment OA is perpendicular to line segment OP)<br><br>(line segment PA squared is equal to line segment OP squared minus line segment OA squared) |
| | |

Triangle congruence

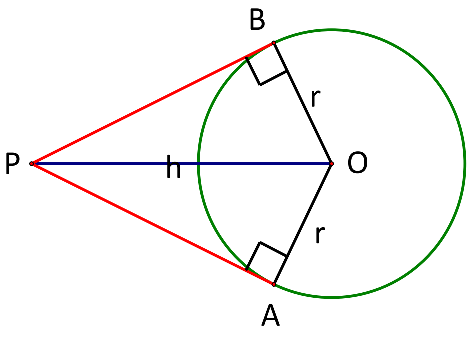
In ΔPOA and ΔPOB,

$$\begin{cases} \angle PAO = \angle PBO = 90^\circ \\ \overline{OA} = \overline{OB} = r \\ \overline{OP} = \overline{OP} \quad (common\ \ side) \end{cases}$$

∴ ΔPOA≅ΔPOB (RHS)

Then

$\overline{PA}$=$\overline{PB}$

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太多資料斯要討論，留待下次繼續。請同學們先好好複習圓的性質，往下學習才能得心應手。

There’s a lot of information for us to learn in this section. Please review it again and again, make sure you can distinguish which is what.

Listen and relax, enjoy!

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