Topic: The cubic functions and their graphs

1. Let's read the news together and then answer the following questions

a. Why might this Greenland's ice rapidly melt in warm weather matter to me, to people around me(family, friends, city, nation), and to the world?

b. Why might this Greenland's ice rapidly melt in warm weather matter to people living in Greenland?
(https://vip.udn.com/newmedia/2022/greenland/climate/)

c. Although there are many factors that affect ice melts, we simplify and consider the function of the change in volume of a 1 cubic centimeter ice cube over time in seconds at a temperature of 16 degrees Celsius to be $V(t) = (-\frac{1}{285}t + 1)^3$

What is the highest degree of this function?
How long does it take for this ice to melt?
Based on the equation of this function, what can you say about its graph?

使用建議	[教學活動安排] 利用生活實例及氣候變遷的議題來引起學生學習三次函數及其圖形的興趣及對特定議題的關懷。 [參考資料] 1.https://numeracylab.com/archives/475 2.https://vip.udn.com/newmedia/2022/greenland/climate/ 3.問題c.d.e.f 改編自https://pz.harvard.edu/sites/default/files/The%203%20Ys.pdf 用於引發學生對全球議題的思考 4.qrcode的連結 https://edition.cnn.com/2022/07/20/world/greenland-heat-wave-ice-melting-climate/index.html [可使用的英文提問/開場] We've learned the quadratic functions and their graphs. We've also learned that some real-life contexts could be modeled by a quadratic function. Now let's move on to cubic functions. What are cubic functions? What is the highest degree of this function? Let's start with the example "ice-melting". What is the highest degree of this function? When I read the news about climate change, I found this interesting example. Read the example on the worksheet and answer the questions. We'll check it in five minutes. Anyone wants to share your thought on the news? Anyone wants to share your work on the "ice-melting" function?

2. The Definition of a Cubic Function

Let a, b, c and d be real numbers with $a \neq 0$. The function

$$f(x) = ax^3 + bx^2 + cx + d$$

is a cubic function.

The basic cubic function is $f(x) = x^3$. We also call it the Cube Function.

使用建議	[教學活動安排] 介紹三次函數的定義 [參考資料] 1.https://www.cuemath.com/calculus/cubic-function/ 2.https://www.mathsisfun.com/sets/function-cube.html [可使用的英文提問/開場] In the ice-melt example, the highest degree is three. So the key feature of cubic functions is the highest degree of variable x is three. A cubic function is a polynomial function of degree three and is in the form $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are real numbers with $a \neq 0$. The basic cubic function is $f(x) = x^3$. We also call it the cube function. Then what do their graphs look like? Let's start with the cube function.

3. Investigate the cubic function $f(x) = x^3$ and its graph

a. Plot the points from the table and connect them with a smooth curve.

x	-2	-1.7	-1.5	-1	-0.8	-0.6	-0.5	-0.4	-0.2	0
$f(x) = x^3$	-8	-4.91	-3.38	-1	-0.51	-0.22	-0.13	-0.06	-0.01	0
x	2	1.7	1.5	1	0.8	0.6	0.5	0.4	0.2	X
$f(x) = x^3$	8	4.91	3.38	1	0.51	0.22	0.13	0.06	0.01	X

b. How do you describe the shape of the graph?

c. Is it increasing or decreasing?

d. Is it symmetrical about the $x-$axis or $y-$axis ?

e. Does it have a rotational symmetry about the origin ?

f. Observe the coordinates of the points we used to plot the graph and evaluate $f(-x)$ and $-f(x)$. What does the result relate to rotational symmetry about the origin?

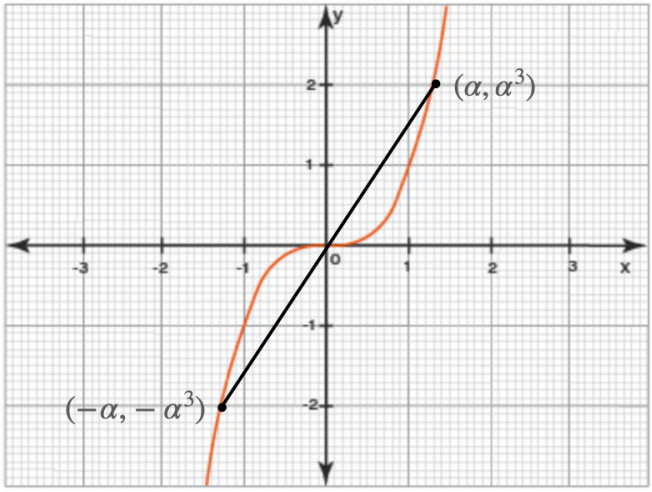
使用建議	[教學活動安排] 1.帶點讓學生去畫出$f(x) = x^3$圖形。 因考量每位教師讓學生操作計算機的習慣不同，故這裡為節省時間，直接給學生點，讓學生只操作將點標在坐標平面上並畫出圖形。 2.透過提問引導學生觀察圖形性質及函數對原點對稱的性質 3.點對稱圖形的定義國中沒教，故這裡用影片補充。 https://youtu.be/s4tS-ZmpJfw 4. 補充函數圖形對稱性質 https://www.mathsisfun.com/algebra/functions-odd-even.html [可使用的英文提問/開場] Let's investigate the graph of the cubic function. Follow the instructions on the worksheet. Let's do question a, b ,c and d. We will check it in five minutes. Anyone wants to share your works? Excellent! Good job. Now let's complete the questions e and f. It will take us more time to discuss. Be patient! Let's watch the clip together. (After watching the clip) What is the main point of this video? What is a rotational symmetry? Share with your partner in three minutes, I will pick one of you to share! Does the graph has a rotational symmetry about the origin ? How do you confirm that? (Students might answer, we can just rotate the graph on the worksheet to confirm.) Are there some ways to confirm that a function has a rotational symmetry? Let's do question f. We'll check it in three minutes. What did you notice from question f ? For a function that satisfies $f(-x) = -f(x)$, the graph of the function has a rotational symmetry about the origin. We also call the function is an "odd"function.

4. The graph of the cubic function $f(x) = x^3$

a. It is an increasing function.

b. It flattens out at $(0,0)$.

c. Since any point (α, α^3) is on the graph of $f(x) = x^3$, and its rotational symmetry about the origin $(-\alpha, -\alpha^3)$ is also on the graph of $f(x) = x^3$. It has a rotational symmetry about the origin.



使用建議	[教學活動安排] 統整 $f(x) = x^3$ 的函數圖形性質 [可使用的英文提問/開場] Let's sum up the properties of the graph of cube function. Let's read the properties on the worksheet. Any questions?

5. Investigate the cubic functions $f(x) = ax^3$ and their graphs

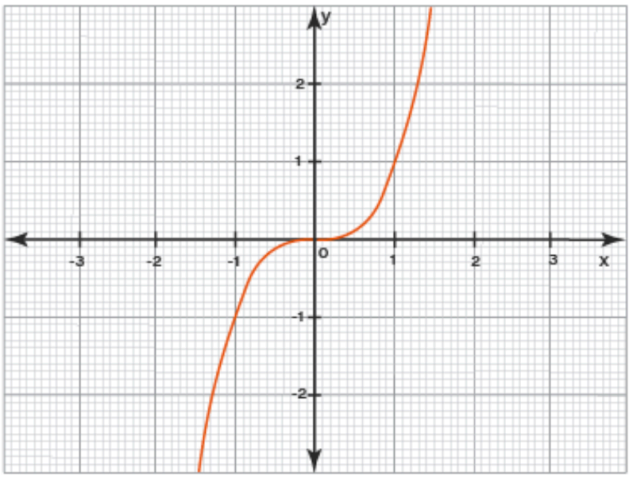
Use the concept of transformations to sketch the following functions and state what transformations from $f(x) = x^3$

a. $F(x) = -x^3$

b. $f_1(x) = 2x^3$

c. $f_2(x) = \frac{1}{2}x^3$

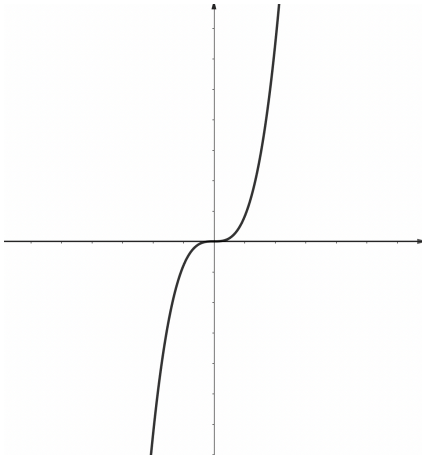
d. $f_4(x) = -2x^3$

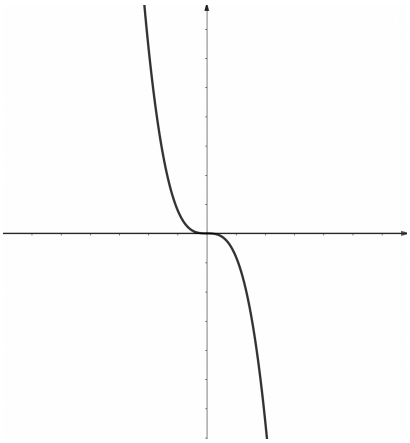


使用建議	[教學活動安排] 讓學生使用之前學過的伸縮變換來探索首項係數對三次單項函數圖形的作用。 教師可以視學生狀況看是否需要複習首項係數與伸縮變換的關係 There are four common types of nonrigid transformations: [可使用的英文提問/開場] We've learned how the leading coefficients are related to the graphs in the quadratic functions. Let's use the concept of transformations to sketch the following functions. We will check it in five minutes. Anyone wants to share your work?

	The graph of $y = f(x)$ is represented by $g(x) = cf(x)$
Vertical stretch	$c > 1$
Vertical shrink	$0 < c < 1$
	The graph of $y = f(x)$ is represented by $h(x) = f(cx)$
Horizontal stretch	$0 < c < 1$
Horizontal shrink	$c > 1$

6. The cubic functions $f(x) = ax^3$ $(a \neq 0)$and their graphs





a. When the leading coefficient is positive $(a > 0)$, the graph is increasing.

b. When the leading coefficient is negative $(a < 0)$, the graph is decreasing.

c. The graphs of cubic functions $f(x) = ax^3$ $(a \neq 0)$ have rotational symmetry about the origin.

使用建議	[教學活動安排] 統整$f(x) = ax^3$圖形的性質 [可使用的英文提問/開場] Let's sum up the properties of the graph of $f(x) = ax^3$ Let's read the properties on the worksheet.

7. Investigate the cubic functions $f(x) = ax^3 + px$ and their graphs

a. $f(x) = x^3 + 2x$

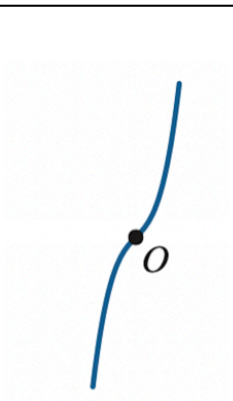
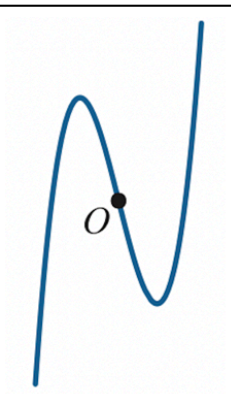
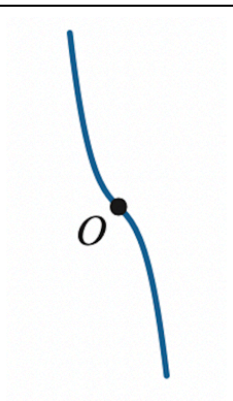
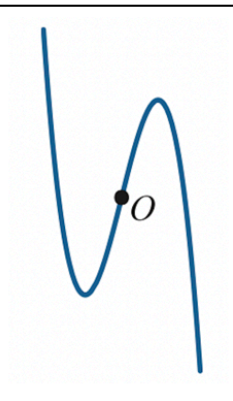
b. $f(x) = x^3 - x$

c. Use the Desmos to investigate what conditions of a and p make the graphs of $f(x) = ax^3 + px$ in different types.

The conditions	The graph
$a > 0, p > 0$	
$a > 0, p < 0$	
$a < 0, p < 0$	
$a < 0, p > 0$	

使用建議	[教學活動安排] 1.讓學生探索$f(x) = ax^3 + px$的圖形 2.教師可依自己使用Desmos或GGB的習慣,考量學生程度來決定要先讓學生帶點畫圖或是直接使用科技軟體去探索圖形 [可使用的英文提問/開場] Based on what we have done before, you might think that there are only two types of graphs for cubic functions. Actually, there are two standard forms of cubic functions. The first form is the one we have already explored, and the second form is the one we will investigating next. Let's do question 7 a. $f(x) = x^3 + 2x$ and b. $f(x) = x^3 - x$ Create a table by calculator. Plot the ponits and connect them with a smooth curve. Anyone wants share your work? What do you notice? Actually, there are four types of graphs for this standard form($f(x) = ax^3 + px$). Now let's use desmos to explore more and fill in the table on the worksheet. Let's check what you have been investigate. Let's see sum-up in 8.

8. The cubic functions $f(x) = ax^3 + px$ and their graphs

The conditions	$a > 0, p > 0$	$a > 0, p < 0$	$a < 0, p < 0$	$a < 0, p > 0$
The graph				

Note that :

a. All the graphs of the cubic functions $f(x) = ax^3 + px$ have rotational symmetry about the origin.

b. When the leading coefficient is positive ($a > 0$), as x tends to infinity y tends to infinity. (圖形最右方都是上升的)

c. When the leading coefficient is negative ($a < 0$), as x tends to infinity y tends to negative infinity. (圖形最右方都是下降的)

使用建議	[教學活動安排] 統整$f(x) = ax^3 + px$圖形的性質 [可使用的英文提問/開場] Let's read the notes on the worksheet. a. All the graphs of the cubic functions $f(x) = ax^3 + px$ have a rotational symmetry about the origin. b. When the leading coefficient is positive ($a > 0$), as x tends to infinity y tends to infinity. (圖形最右方都是上升的) c. When the leading coefficient is negative ($a < 0$), as x tends to infinity y tends to negative infinity. (圖形最右方都是下降的)

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