分數的四則運算 2
Operations with Fractions 2

Class:_____ Name: _____

1. Multiplying fractions

To multiply fractions, multiply the numerators, and multiply the denominators.

For example, $\frac{3}{4}\times\frac{5}{7}=\frac{3\times5}{4\times7}=\frac{15}{28}$.

Example: $\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}=?$ 教師可參考後方教學範例，先做直接乘完最後再約分，再教於過程中約分，左方例題有教師操作，右方則讓學生練習。	Exercise: $(-\frac{7}{2})\times(-\frac{8}{3})\times\frac{9}{14}=?$

2. Reciprocal(倒數)

當兩個數的乘積等於 1 時，我們稱這兩個數互為倒數。

Any number multiplied by its reciprocal is 1.

For example, the reciprocal of 4 is $\frac{1}{4}$, because 4 multiplied by $\frac{1}{4}$ equals 1.

Let's find the reciprocal of a mixed fraction by definition.

What is the reciprocal of $2\frac{3}{5}$? □$2\frac{5}{3}$ □$3\frac{2}{3}$ □$\frac{5}{13}$

此部分先由教師帶同學透過檢查定義的方式確認何者為倒數

In the following example, we will learn how to find the reciprocal of a mixed fraction faster.

Example: Find the reciprocal of $7\frac{1}{2}$ 藉由上面的例題，請同學在過程中看出帶分數的倒數需先換成假分數後才能計算，並由教師示範完成例題。	Exercise: Find the reciprocal of $-4\frac{3}{7}$

Let's find the reciprocal of a negative number by definition.

What is the reciprocal of $-\frac{2}{3}$? □$\frac{3}{2}$ □$\frac{2}{3}$ □$-\frac{3}{2}$

此部分先由教師帶同學透過檢查定義的方式確認何者為倒數

In the following example, we will learn how to find the reciprocal of a mixed fraction faster.

Example: Find the reciprocal of -4 藉由上面的例題，請同學在過程中看出富庶的倒數仍為負數，故性質符號無須改變，並由教師示範完成例題。	Exercise: Find the reciprocal of 1

3. Dividing fractions

Dividing a fraction by a number is the same as multiplying it by the reciprocal of that number.

For example, $\frac{5}{6}\div\frac{10}{13}=\frac{5}{6}\times\frac{13}{10}=\frac{13}{12}$.

Example:	Exercise:
$\frac{7}{2}\div(-\frac{5}{4})\div(-\frac{5}{3})=?$	$\frac{1}{3}\div\frac{2}{5}\times(-\frac{6}{11})=?$

4. Operations of fractions(分數的四則運算)

Before we see example 1, think about the following question:

When we calculate $\frac{3}{7}-\frac{2}{5}\div(-\frac{4}{15})$, which operation should we do first?

這個問題希望在做 Example 1 前先提醒學生先乘除、後加減的概念，故題目與該題相同。

□ $\frac{3}{7}-\frac{2}{5}$　　□ $\frac{2}{5}\div(-\frac{4}{15})$

Example 1:	Exercise 1:
$\frac{3}{7}-\frac{2}{5}\div(-\frac{4}{15})=?$	$\frac{5}{4}\times2\frac{1}{3}+\frac{1}{2}=?$
Example 2:	Exercise 2:
$-3+\frac{7}{2}\div0.7-\frac{2}{3}=?$ 本題為混合分數與小數的四則運算	$\frac{3}{4}\div(-1\frac{5}{7})-\frac{8}{5}\times0.25=?$

一、設計理念：

1. 學生在國小時曾經學過正分數的概念及四則運算，在國中時僅增加負分數的部分。
2. 前一節課已複習過分數的加法與減法，本節課針對乘法、除法及四則運算。
3. 語言部分考量學生國小可能未受過雙語數學的教育，故分數的英文用法皆從頭教學。
4. 帶分數的英文可使用 mixed fraction 或 mixed number，為求與真分數、假分數的一致性此處採用 mixed fraction。
5. 分數的唸法有兩種，以 $\frac{2}{3}$ 為例，可使用 two-thirds 或 two over three，但由於七年級學生尚未學習序數的英文，故本文統一使用 a over b 的用法。但無論何種唸法，英文習慣上分數由分子唸到分母，與中文從分母唸到分子的習慣顛倒。
6. 由於分數的加減法需通分，有學生會誤以為乘除法亦須通分，教師須特別留意。

二、英文詞彙：

中文	英文
最簡分數	fraction in simplest form
假分數	improper fraction
帶分數	mixed fraction
分子	numerator
分母	denominator
倒數	reciprocal

三、數學英文用法：

數學表示法	英文
$\frac{1}{5}$	one over five
$-\frac{7}{5}$	negative seven over five
$2\frac{3}{4}$	two and three over four
$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$	one over two plus one over three equals five over six
$\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$	two over three minus one over two equals one over six
$\frac{2}{9}\times\frac{1}{2}=\frac{1}{9}$	two over nine times one over two equals one over nine
$\frac{3}{4}\div\frac{3}{8}=2$	three over four divided by three over eight equals two
3 的倒數是 $\frac{1}{3}$	The reciprocal of three is one over three

四、教學參考範例：

<table>
<tr>
<td rowspan="4">1
【分數的乘法】</td>
<td colspan="2">講義內容
1. Multiplying fractions
To multiply fractions, multiply the numerators, and multiply the denominators.
For example, $\frac{3}{4}\times\frac{5}{7}=\frac{3\times5}{4\times7}=\frac{15}{28}$.</td>
</tr>
<tr>
<td>計算過程

$\frac{3}{4}\times\frac{5}{7}$
$=\frac{3\times5}{4\times7}$
$=\frac{15}{28}$</td>
<td>課堂用語
We just learned how to add or subtract fractions. Next, let's see how to multiply fractions. To multiply fractions, we just need to multiply the numerators and denominators.
For example, when we calculate $\frac{3}{4}\times\frac{5}{7}$ (three over four times five over seven), the numerator in our product is 3 times 5, which is equal to 15, and the denominator in our product is 4 times 7, which is equal to 28. Therefore, the product is $\frac{15}{28}$.</td>
</tr>
<tr>
<td colspan="2">講義內容
Example: $\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}=?$

計算過程
$\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}$
$=-\frac{8}{3}\times\frac{6}{5}\times\frac{5}{4}$
$=-\frac{8\times6\times5}{3\times5\times4}$
$=-\frac{240}{60}$
$=-4$</td>
</tr>
<tr>
<td>計算過程

$\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}$

+*-=-
-*+=-
=>+*-*+=-

$\frac{8}{3}\times\frac{6}{5}\times\frac{5}{4}$</td>
<td>課堂用語
Let's take a look at a more difficult example. Calculate $\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}$ (eight over three, times negative six over five and times five over four).
As we multiply integers, we can determine whether the result is positive or negative first. What is the product of positive times negative times positive? Positive times negative equals negative, negative times positive equals positive, so the result is negative.
Then, we calculate $\frac{8}{3}\times\frac{6}{5}\times\frac{5}{4}$. Multiplying the numerators, we get 8 times 6 times 5, which equals</td>
</tr>
</table>

	$=\frac{8\times6\times5}{3\times5\times4}$ $=\frac{240}{60}$ $\frac{240}{60}$ $=4$ $\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}=-4$	240. Multiplying the denominators, we get 3 times 5 times 4, which equals 60. The product of these three fractions is $-\frac{240}{60}$. However, this is not our final answer. Don't forget to simplify the fraction. Both the numerator and the denominator are divisible by 60, so we can divide them by 60 together. 240 divided by 60 is 4, and 60 divided by 60 is 1. As a result, the final answer is -4.
1 【分數的乘法】	講義內容 Example: $\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}=?$	計算過程 $\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}$ $=-\frac{8}{3}\times\frac{6}{5}\times\frac{5}{4}$ $=-\frac{2}{1}\times\frac{2}{1}\times\frac{1}{1}$ $=-\frac{2\times2\times1}{1\times1\times1}$ $=-\frac{4}{1}$ $=-4$
	計算過程 $\frac{8}{3}\times\frac{6}{5}\times\frac{5}{4}$ $=\frac{2}{1}\times\frac{2}{1}\times\frac{1}{1}$ $=\frac{2\times2\times1}{1\times1\times1}$ $=\frac{4}{1}$ $\frac{8}{3}\times(-\frac{6}{5})\times\frac{5}{4}=-4$	課堂用語 In fact, we can simplify the fraction before we multiply the the numerators and the denominators. To reduce the fraction faster, we can reduce it in the beginning. Because 3 and 6 are both divisible by 3, so we divided both of them by 3. 3 divided by 3 equals 1, and 6 divided by 3 equals 2. Similarly, divided 8 and 4 by 4 and we get 2 and 1. Then we divided 5 by 5 and we get 1. Therefore, the numerator of the product is 2 times 2 times 1, which equals 4, and the denominator of the product is 1 times 1 times 1, which equals 1. Don't forget the result is negative as we discuss in the beginning. The final answer is -4.

<table>
<tr><td rowspan="4">2
【倒數的介紹】</td><td colspan="2">講義內容
2. Reciprocal(倒數)
當兩個數的乘積等於 1 時，我們稱這兩個數互為倒數。
Any number multiplied by its reciprocal is 1.
For example, the reciprocal of 4 is $\frac{1}{4}$, because 4 multiplied by $\frac{1}{4}$ equals 1.</td></tr>
<tr><td colspan="2">課堂用語
The reciprocal is the number which when multiplied by the original number, you get 1. For example, the reciprocal of 4 is $\frac{1}{4}$, because 4 multiplied by $\frac{1}{4}$ equals 1.</td></tr>
<tr><td colspan="2">講義內容
Let's find the reciprocal of a negative number by definition.
What is the reciprocal of $-\frac{2}{3}$? □$\frac{3}{2}$ □$\frac{2}{3}$ □$-\frac{3}{2}$</td></tr>
<tr><td>計算過程
$-\frac{2}{3}\times\frac{3}{2}=-1$
$-\frac{2}{3}\times\frac{2}{3}=-\frac{4}{9}$
$(-\frac{2}{3})\times(-\frac{3}{2})=1$</td><td>課堂用語
We are going to find the reciprocal of $-\frac{2}{3}$.
Let's check the following numbers. $-\frac{2}{3}$ times $\frac{3}{2}$ equals -1, so $\frac{3}{2}$ is NOT the reciprocal of $-\frac{2}{3}$.
Please check the other two numbers by yourself and we'll check it later. Time's up. Is $\frac{2}{3}$ the reciprocal of $-\frac{2}{3}$? No. $-\frac{2}{3}$ times $\frac{2}{3}$ equals $-\frac{4}{9}$. Then, is $-\frac{3}{2}$ the reciprocal of $-\frac{2}{3}$? Yes, $-\frac{2}{3}$ times $-\frac{3}{2}$ equals 1. It is what we are trying to find. The reciprocal of $-\frac{2}{3}$ is $-\frac{3}{2}$. We can find that the reciprocal of a negative number is always a negative number because negative times negative equals positive.</td></tr>
</table>

<table>
<tr><td rowspan="2">2
【倒數的介紹】</td><td colspan="2">講義內容
Let’s find the reciprocal of a mixed fraction by definition.
What is the reciprocal of $2\frac{3}{5}$? □$2\frac{5}{3}$ □$3\frac{2}{3}$ □$\frac{5}{13}$</td></tr>
<tr><td>計算過程
$2\frac{3}{5}\times 2\frac{5}{3}$
$=\frac{13}{5}\times\frac{11}{3}$
$=\frac{143}{15}$</td><td>課堂用語
Next, find the reciprocal of $2\frac{3}{5}$? Check $2\frac{3}{5}$ multiplied by which number equals 1. Does $2\frac{3}{5}$ times $2\frac{5}{3}$ equals 1? When multiplying the mixed fraction, we have to change it into the improper fraction first. $2\frac{3}{5}$ times $2\frac{5}{3}$ equals $\frac{13}{5}$ times $\frac{11}{3}$, which equals $\frac{143}{15}$, not 1. Therefore, the reciprocal of $2\frac{3}{5}$ is NOT $2\frac{5}{3}$. Try to do another two numbers by yourself.</td></tr>
<tr><td rowspan="2">3
【分數的除法】</td><td colspan="2">講義內容
3. Dividing fractions
Dividing a fraction by a number is the same as multiplying it by the reciprocal of that number.
For example, $\frac{5}{6}\div\frac{10}{13}=\frac{5}{6}\times\frac{13}{10}=\frac{13}{12}$.</td></tr>
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$\frac{5}{6}\div\frac{10}{13}$
$=\frac{5}{6}\times\frac{13}{10}$
$=\frac{1}{6}\times\frac{13}{2}$
$=\frac{13}{12}$</td><td>課堂用語
How can we divide fractions? Dividing a fraction by a number is the same as multiplying by its reciprocal. For example, $\frac{5}{6}$ divided by $\frac{10}{13}$. It is the same as $\frac{5}{6}$ multiplied by the reciprocal of $\frac{10}{13}$. The reciprocal of $\frac{10}{13}$ is $\frac{13}{10}$. Then, we already know how to multiply fractions. Divide both 5 and 10 by 5, we get 1 and 2. Therefore, $\frac{5}{6}$ divided by $\frac{10}{13}$ is $\frac{13}{12}$. Remember, when dividing a fraction, always change it into multiplying by its reciprocal.</td></tr>
</table>

補充：去括號法則

<table>
<tr><td rowspan="2">4
【去括號法則】</td><td>計算過程
$\frac{7}{13}-(\frac{8}{23}-\frac{6}{13})$
$=\frac{7}{13}-\frac{8}{23}+\frac{6}{13}$
$=\frac{7}{13}+\frac{6}{13}-\frac{8}{23}$
$=1-\frac{8}{23}$
$=\frac{15}{23}$</td><td></td></tr>
<tr><td>計算過程
$\frac{7}{13}-(\frac{8}{23}-\frac{6}{13})$
$-(\frac{8}{23}-\frac{6}{13})$
$=-\frac{8}{23}+\frac{6}{13}$
$\frac{7}{13}-\frac{8}{23}+\frac{6}{13}$
$=\frac{7}{13}+\frac{6}{13}-\frac{8}{23}$
$=1-\frac{8}{23}$
$=\frac{15}{23}$</td><td>課堂用語
Let’s solve this problem. We know that we should do the operation in the parathesis first, so we should calculate $\frac{8}{23}-\frac{6}{13}$ in the beginning. However, when dividing two fractions, we should change the denominators of two fractions into the same denominator. Then, the denominator will become a large number, and the process will be too complex.
Observing the problem, we can find $\frac{7}{13}$ and $\frac{6}{13}$ have the same denominator. If we can calculate these two numbers first, it might be easier. Therefore, we should start with removing the parathesis first. There is a negative sign before the parathesis, so by the distributive property, the positive in the parenthesis becomes negative, and the negative in the parenthesis becomes positive. Then, the negative parathesis of $\frac{8}{23}$ minus $\frac{6}{13}$ equals negative $\frac{8}{23}$ plus $\frac{6}{13}$.
Next, we can change the order of operation. Let’s add $\frac{7}{13}$ and $\frac{6}{13}$ first, and the sum is 1. Then, 1 minus $\frac{8}{23}$ equals $\frac{15}{23}$. Therefore, the final answer is $\frac{15}{23}$.</td></tr>
</table>

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