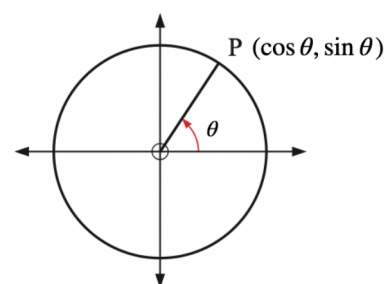


Topic: The Sine Function and Radian Measurement of Angles

1. Review: Suppose the moving point P moves around the unit circle (the radius equals 1), so the angle \overline{OP} made with the positive horizontal axis is θ . We've learned that the coordinate of P is $(\cos \theta, \sin \theta)$. Use the trigonometric ratios of general angles to finish the following table.



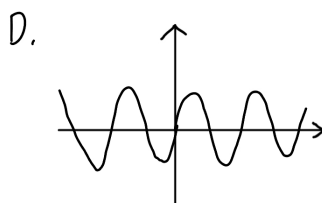
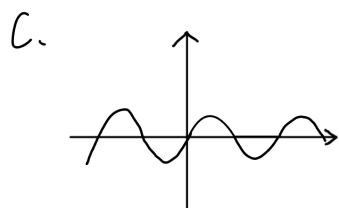
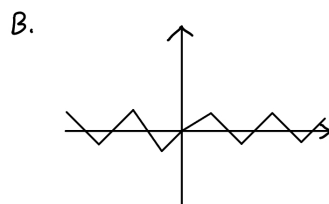
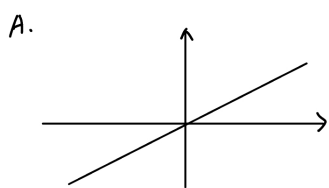
θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin \theta$									
θ	210°	225°	240°	270°	300°	315°	330°	360°	390°
$\sin \theta$									

使用建議

教學活動安排	<ol style="list-style-type: none"> 1. 複習高一學過廣義三角比定義 2. 讓學生填表格的用意是為了讓學生看到對應關係好連接後續sin函數的定義 3. 填表的過程中讓學生多回憶可以利用圖形的對稱(或角度換算) 加速找$\sin \theta$的值 4. 這裡選擇從複習開始，而不是從角度的弧度量開始，是為了後續還會鋪陳利用畫圖有弧度量的需求才去做角度的弧度量
英文提問 / 開場	<p>Today we're going to learn the sine function and radian measure of angles. We will start with the sine function and then move on to measurement of angles.</p> <p>So, what is the sine function? Have we learned about the term "sine" before? YES! It's one of the trigonometric ratios.</p> <p>Let's complete the review to jog our memory.</p>

2. The investigation of the sine function and its graph

From the table you completed above, we found the relationship " $\theta \rightarrow \sin \theta$ " (from θ to $\sin \theta$) is a function $y = \sin \theta$. We call it the sine function. Now we use the above table to sketch the graph of $y = \sin \theta$. Which of the following options could be the graph of $y = \sin \theta$? State your reason



使用建議

教學
活動
安排

在這裡讓學生猜測 $y = \sin \theta$ 函數圖形的長相，希望學生可以從表格中觀察到圖形有週期性，故 A 選項不可能。而 C、D 選項比起 B 選項更有可能是因為函數值 $\sin \theta$ 是單位圓上的 y 坐標，也許圖形偏向圓弧。至於 C、D 的差異教師可引導學生 x 軸上 θ 是角度，那角度 30 度就等於實數 30 嗎？這樣比例不對！所以我們來介紹角度的另一種測量方式弧度量。

英文
提問
/
開場

Based on the table we just completed, we found that the relationship from θ to $\sin \theta$ is actually a function. We denote it as $y = \sin \theta$. This particular function is known as the sine function.

Now, carefully observe the table we just completed to determine which of the following options could potentially represent the graph.

Is the sine function a linear function? Why?

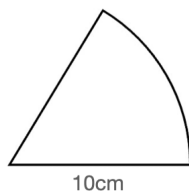
What are the differences between options B and C? In your opinion, which of these two is more likely to be the graph of $y = \sin \theta$?

What are the differences between options C and D? In your opinion, which of these two is more likely to be the graph of $y = \sin \theta$?

The units on the x -axis are in degrees. Does an angle measurement of 30 degrees represent the numerical value of 30? This doesn't make sense! So, let's introduce another way to measure angles called radians.

3. The investigation of radian measurement of angles

There are two fans, A and B, with fan rib lengths (radii) of 10 centimeters and 20 centimeters, respectively. Now, you hold the two ends of the fan ribs and slowly open them. You will see that the edge of the fan (arc length) gradually increases.



a. Fill in the table below.

Fan A (radius=10cm)

angle	45°	90°	135°	180°	360°
arc length					
$\frac{\text{arc length}}{\text{radius}}$					

Fan B (radius=20cm)

angle	45°	90°	135°	180°	360°
arc length					
$\frac{\text{arc length}}{\text{radius}}$					

b. Will the ratio of the arc length to the radius be the same for a radius of 10 centimeters and 20 centimeters when the angles are equal?

c. When the angle is given, will the arc length to the radius remain constant for different radii? Why or why not?

d. If the radius is r , what is the arc length at which the ratio of the arc length to the radius is 1?

e. Sketch an angle where the ratio of the arc length to the radius is 2. Compare the angle you sketch with your classmates. Are the sizes of the two angles the same?

使用建議

教學活動安排

讓學生透過兩個半徑不同的扇形裡，只要給定角度（圓心角）弧長跟半徑比值就能唯一決定。最後一個問題e則是讓學生感受到此比值可以決定角度大小,因此比值可以作為測量單位。

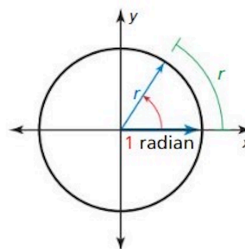
英文提問 / 開場

Degree measurement divides a circle into 360 equal parts, with each part being 1 degree. Is there another way to measure or describe the size of an angle? Let's investigate another way we call it radian measurement. We'll check it in five minutes.

4. Radian measurement of angles

Using Radian Measure

Angles can also be measured in *radians*. To define a radian, consider a circle with radius r centered at the origin, as shown. One **radian** is the measure of an angle in standard position whose terminal side intercepts an arc of length r .



We indicate degrees with a small $^\circ$.
To indicate radians we use a small c or else use no symbol at all.



使用建議

教學活動安排

介紹弧度量

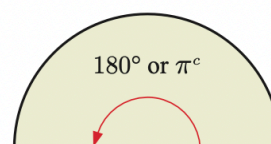
使用建議

英文提問 / 開場	In question e, you were asked to sketch an angle where the arc length is twice the length of the radius. This means the size of that angle is 2 radians. Now, using the same concept, you can also sketch angles of 3 radians and 0.5 radians. So, we have angles measured in radians. Let's take a look at the definition on the worksheet.
參考資料	1. https://www.haesemathematics.com/books/mathematics-hl-core-3rd-edition 2. https://www.mathsisfun.com/geometry/radians.html

5. Degree–Radian Conversions

If the radius of a circle is r , then an arc of length πr , or half the circumference, will subtend an angle of π radians.

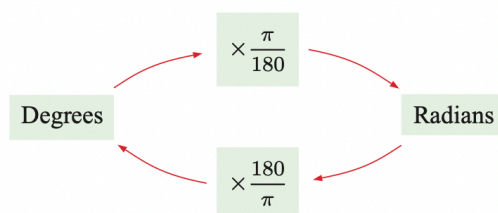
Therefore, π radians \equiv 180° .



So, $1^\circ = \left(\frac{180}{\pi}\right)^\circ \approx 57.3^\circ$ and $1^\circ = \left(\frac{\pi}{180}\right)^\circ \approx 0.0175^\circ$.

To convert from degrees to radians, we multiply by $\frac{\pi}{180}$.

To convert from radians to degrees, we multiply by $\frac{180}{\pi}$.



使用建議

教學活動安排	介紹度度量與弧度量換算
--------	-------------

使用建議

英文提問 / 開場	<p>How do we convert an angle measured in radians to degrees? And, how do we convert an angle measured in degrees to radians?</p> <p>Let's do it by sketching a half circumference.</p> <p>What is the ratio between the length of the semicircular arc and the radius?</p> <p>Yes, it is π (pi). This means that an angle of 180 degrees is equal to π radians.</p> <p>So, how many degrees are equal to 1 radian?</p> <p>How many radians are equal to 1 degree?</p> <p>Good! I think all of you are grasping the process of converting between radians and degrees.</p> <p>Awesome! Now, let's put our skills into action and practice with the worksheet.</p>
參考資料	<p>https://www.haesemathematics.com/books/mathematics-hl-core-3rd-edition</p>

6. Practice makes perfect

- a. Convert 45° to radians in terms of π . (Note: Angles in radians may be expressed either in terms of π or as decimals.)
- b. Convert $\frac{\pi}{5}$ to degrees.
- c. Convert the following degrees to radians in terms of π

Degrees	0	30	60	90	120	150	180	210	240	270	300	330	360
Radians													

使用建議

教學活動安排	練習
--------	----

使用建議

英文
提問
/
開場

We'll check it in five minutes.
Anyone wants to share?

7. The graph of $y = \sin x$

a. Find the value of $\sin \frac{\pi}{6}$

b. Complete the following table

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin x$									

c. Use the table to sketch the graph of $y = \sin x$ ($0 \leq x \leq 2\pi$)

d. Scan the QR code to see the graph of $y = \sin x$



使用建議

教學
活動
安排

畫出 $y = \sin x$ 的圖形

先讓學生求出當自變數是弧度量的角度時的 $\sin x$ 的值

接著讓學生完成表格並畫出圖形，表格中的角度都是以45度為基準的理由是考量學生會比較沒有耐心完成太多。且讓學生嘗試出大概的樣子即可。

接著教師可以讓學生猜測那如果我們角度不限制在360度以內，甚至是負的角度時，圖形會長什麼樣子呢？猜完後讓我們掃QR code 來看用科技軟體畫出來的圖長什麼樣子

使用建議

英文提問 / 開場	<p>Now, we are ready to sketch the actual the graph of sine function.</p> <p>Follow the instruction of a,b, and c on the worksheet.</p> <p>We'll check it in five minutes.</p> <p>(當完成a,b,c後)</p> <p>Can you make a prediction about the graph of the sine function when the domain is not limited to the interval from zero to two pi?</p> <p>Well done.</p> <p>Let's scan the QR code to see if our prediction is correct or not. Exciting times ahead!</p>
-----------	---

製作者：台北市立育成高中 數學科教師 林玉惇