雙語教學主題(國中八年級教材):介紹以因式分解解二元一次方程式
Topic: solving quadratic equations by factoring

這個單元常用到的一些用語
The vocabulary we will use in this topic
squared, square root , ± plus or minus 正負 , complete the square , solution or root , polynomial, coefficient, leading coefficient, binomial, trinomial, identity 恆等式, *ZERO PRODUCT PROPERTY*, GCF=greatest common factor, factoring polynomial , factoring out , grouping , repeated roots 重根,

各位老師，用因式分解法來解二元一次方程式這個單元，在內容上主要是因式分解的方法，國內外稍有不同。在這裡我主要介紹我們常用的因式分解方法來解二次方程。
We first want to introduce *ZERO PRODUCT PROPERTY*

If a·b=0, then a=0 or b=0

It means if the product of a times b equals 0, then either a=0 or b=0. No two numbers other than 0 can be multiplied together and get 0.
Actually, we used the *ZERO PRODUCT PROPERTY* when we solved linear equations with one variable before. For instance,

3x=0

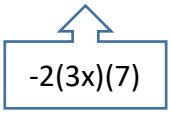
3 times an unknown number x equals 0
We know that x has to be 0, So the solution to this equation is x=0

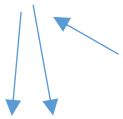
Let's start solving quadratic equations by using different factoring methods. I highly recommend that we ~~have to~~ review all we learned about factoring polynomials in another class before we get into this lesson.
We usually set the quadratic equations in the *general standard form* $\mathbf{ax^2+bx+c=0}$

Ex1: solving the quadratic equations by using factoring the GCF method.	
1. $x^2+2x=0$	x squared plus 2x equals 0
$x^2+2x=0$	Both terms have a common variable x
$x(x+2)=0$	Factor out their GCF x
$\Rightarrow$ $x=0$ or $x+2=0$	*ZERO PRODUCT PROPERTY*
$x=0$ or $x=-2$	
0 and -2 are the solutions to this quadratic equation $x^2+2x=0$	
Check:	In the future, please do the checking if you have time after solving equations.

Replace x with 0 in the equation $x^2+2x=0$, we get $0^2+2(0)$ $=0+0$ $=0$ Also Replace x with -2 in the equation $x^2+2x=0$, we get $(-2)^2+2(-2)$ $=4+(-4)$ $=0$ Both 0 and -2 are the solutions to the equation $x^2+2x=0$ **Attention**: please do the checking after solving each equation whenever you have time. We won't show the checking process here anymore.	
2. $4x^2=100x$	
$4x^2=100x$	4 times x squared equals 100 times x First, we noticed that this equation is not in a general form.
$4x^2-100x=0$ $4x(x-25)=0$	Rewrite it as $4x^2-100x=0$ Factor out the common factor 4x
$\Rightarrow x=0$ or $x-25=0$ $\Rightarrow x=0$ or $x=25_{\#}$	*ZERO PRODUCT PROPERTY* (我想這裏我們應該不用再用英文解釋解一元一次方程式的過程了吧)
Another method that we mostly do:	
$4x^2=100x$ $x^2=25x$ $x(x-25)=0$ $\Rightarrow x=0, 25_{\#}$	Divide both sides by 4, and get x squared equals 25x Simplify and factor the common factor x Please do the checking when available.
ATTENTION:(extra important)	

Whenever we solve a quadratic equation, we always try to keep the equation in the *general standard form* **$ax^2+bx+c=0$**, This means we keep all the terms on the left side of the equation. Because we might make the mistake like this: $4x^2=100x$ Divide both sides by the common factor 4x and get: $x=25$ A quadratic equation always has two roots, if we solve the equation as above, we have only one root, and it's not correct! The reason why it happens: When we divide both sides by 4x , we don't know what the value of x is, and if x=0, then we make a big mistake by dividing the equaiton by 0. So please start to solve the quadratic equations in gerneral standard form to avoid the mistake.	
Ex2: solving the quadratic equations by using the *polynomial identities* 1. $x^2-4=0$	
$a^2-b^2=(a+b)(a-b)$	x squared minus 4 equals 0 Apply *polynomial identity* a squared minus b squared equals a plus b times a minus b (假設學生眼睛都有看著我們的手，括號我們就不唸出來了)
$x^2-4=0$ $x^2-2^2=0$ $(x+2)(x-2)=0$ $x+2=0$ or $x-2=0$ $x=-2,2_{\#}$	Replace a with x and b with 2 *ZERO PRODUCT PROPERTY*
2. $18x^2+98=84x$	

$18x^2-84x+98=0$ $9x^2-42x+49=0$ $(3x)^2-42x+7^2=0$  $(3x-7)^2=0$ $\Rightarrow 3x-7=0$ $X=\frac{7}{3},\frac{7}{3}$ Or $x=\frac{7}{3}$ (repeated roots) #	18 x squared plus 98 equals 84x Rewrite it in the standard form Divide both sides by their GCF 2 Rewrite the square terms Apply the polynomial identity $(a-b)^2=a^2-2ab+b^2$ Replace a with 3x and b with 7 0 is the only number that multiplying itself is still 0. **A quadratic equation** always has **two** roots. But there are certain kinds of equations that we don't know the roots yet. You will learn that when you get into grade 12.

Ex3: solve the following quadratic equations by using *our cross method*. 1. $x^2+3x+2=0$ $x^2+3x+2=0$  $x^2+x+2x+2=0$ $(x^2+x)+(2x+2)=0$ $x(x+1)+2(x+1)=0$ $(x+1)(x+2)=0$ $\Rightarrow x+1=0$ or $x+2=0$ $\Rightarrow x=-1$ or $x=-2$ $\Rightarrow x=-1, -2$ #	x squared plus 3x plus 2 equals 0 Since 2 equals 1·2 or (-1)(-2) But we need positive 3 for the middle term, so we choose 2 equals 1·2 We split the middle term 3x to x plus 2x Grouping and factoring the GCF x from the first parentheses and 2 from the second parentheses. Factoring the common factor (x+1)

2 $3x^2-22x=16$. $3x^2-22x=16$ $3x^2-22x-16=0$ 3x -1 -2 -4 -8 -16 x 16 8 4 2 1 or x -1 -2 -4 -8 -16 3x 16 8 4 2 1 $2x-24x=-22x$ $3x^2-22x=16$ $3x^2-22x-16=0$ $(x-8)(3x+2)=0$ $\Rightarrow x-8=0$ or $3x+2=0$ $X=8, -\frac{2}{3}$	3 times x squared minus 22x equals 16 Rewrite the equation in a standard form Factor the x^2 term $3x^2$ and the constant term -16 We found that x times 2 plus 3x times negative 8 is negative 22x And we write down the factoring from left to right. 8 and $-\frac{2}{3}$ are the solutions to the equation.

We can use all kinds of polynomial factoring methods to solve quadratic equations. For some quadratic equations that cannot be factorized, we will introduce how to complete the square with x^2 term and x term and quadratic formula to solve any kinds of quadratic equations.
Practicing more would build up the fluency on doing math.

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