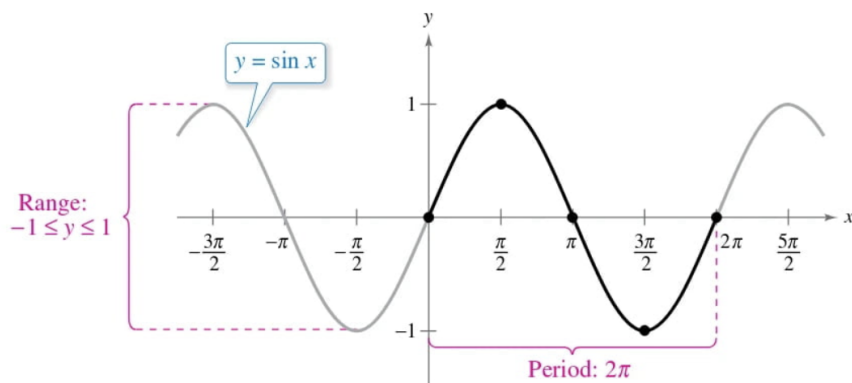


Topic: Graphing the Sine Function and Its Transformations

1. The basic sine function $y = \sin x$ and its graph.

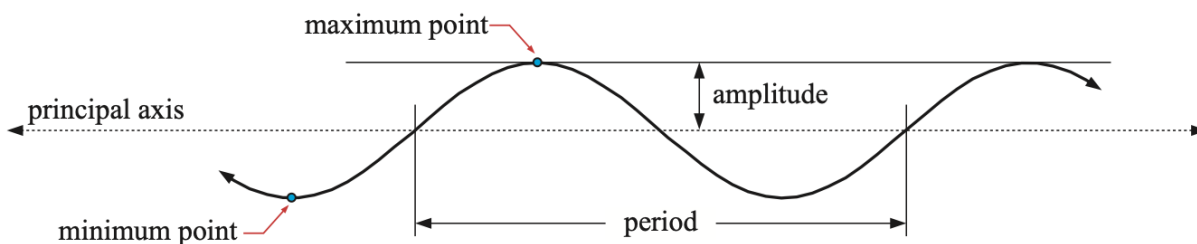


–It is a periodic function with a period of 2π .

–The maximum value is 1 and the minimum value is -1 .

–The amplitude is 1.

Note: The amplitude is the distance between a maximum (or minimum) point and the principal axis.



使用建議

[教學活動安排]

1. 延續上一節認識 $y = \sin x$ 的圖形並統整 $y = \sin x$ 的特徵 (週期、最大最小值、振幅)
2. 教師可考慮這裡用填空或問答讓學生去觀察 $y = \sin x$ 的特徵

[英文開場\提問]

We've learned the graph of the sine function. Does anyone remember what the graph looks like? Anyone want to sketch it for us? (Or Does anyone want to sketch it for us?)

Does it have a maximum value? Does it have a minimum value?

Let's choose five points to help us sketch the graph.

Let's create a table with the given y -values $0, 1, 0, -1, 0$. What are the corresponding x -values?

x					
y	0	1	0	-1	0

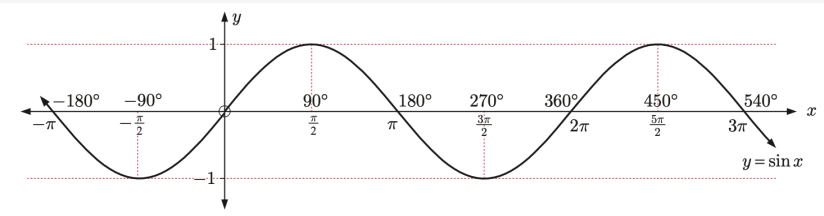
Let's plot the points on the Cartesian plane.

Connect them to form a smooth curve.

What the graph looks like? A wave curve!

Is it done? Not yet!

We can continue to draw this wave curve beyond $0 \leq x \leq 2\pi$ in both directions.

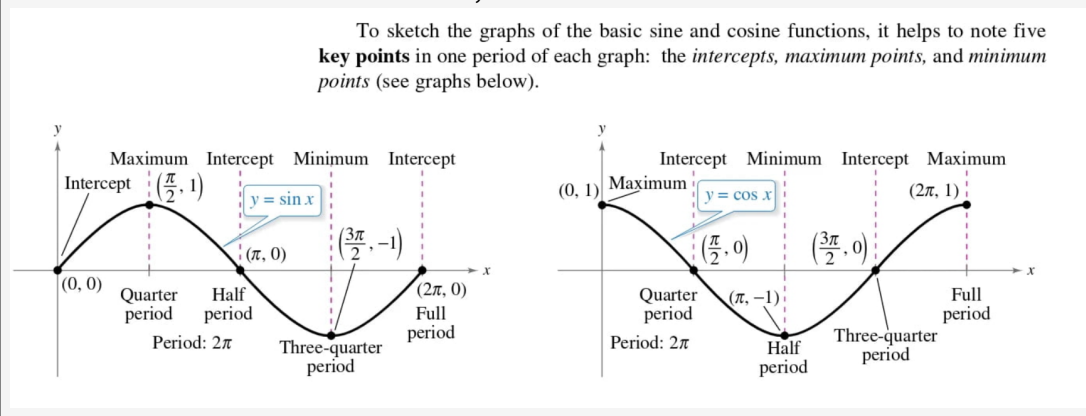


Now let's summarize the important characteristics of the sine function.

See the worksheet.

[參考資料]

Precalculus with Limits: Larson, Ron -4th Edition

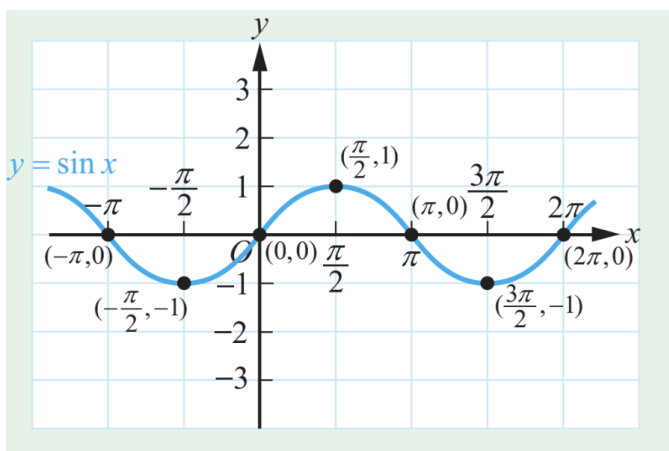


2. Investigate the Vertical Shrinking and Stretching of the sine function

Using key points to sketch the following sine function on the graph of $y = \sin x$, then answer the questions below.

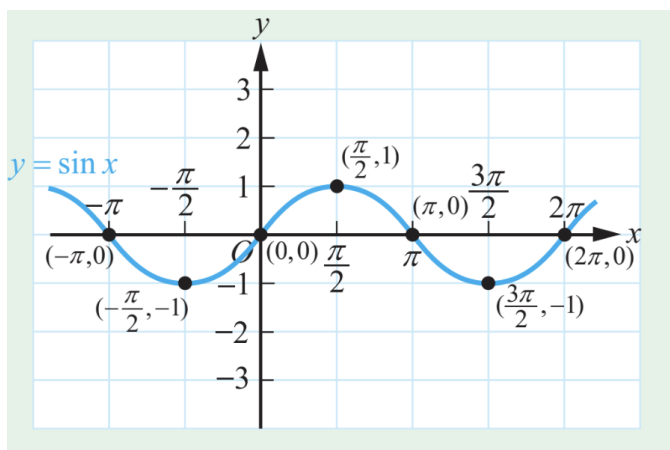
a. $y = 2 \sin x$

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y							



b. $y = \frac{1}{3} \sin x$

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y							



c. How does a affect the function $y = a \sin x$? When $0 < a < 1$, the graph_____.
 When $a > 1$, the graph_____.When $a = -1$, the graph_____.Use
 Desmos to confirm your thoughts.

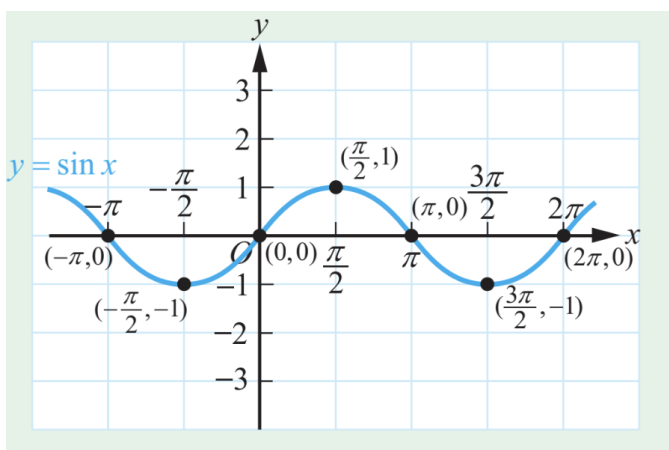
使用建議
<p>[教學活動安排] 讓學生用最大值、最小值及截距畫圖去觀察係數對圖形的影響</p> <p>[英文開場\提問] We've learned the transformation of functions. Let's start to investigate the vertical shrinking and stretching of the sine function. Complete the investigation 2. We'll check-in in five minutes. Anyone wants to share your work?</p>

3. Investigate the Horizontal Shrinking and Stretching of the sine function

Using key points to sketch the following sine function on the graph of $y = \sin x$, then answer the questions below.

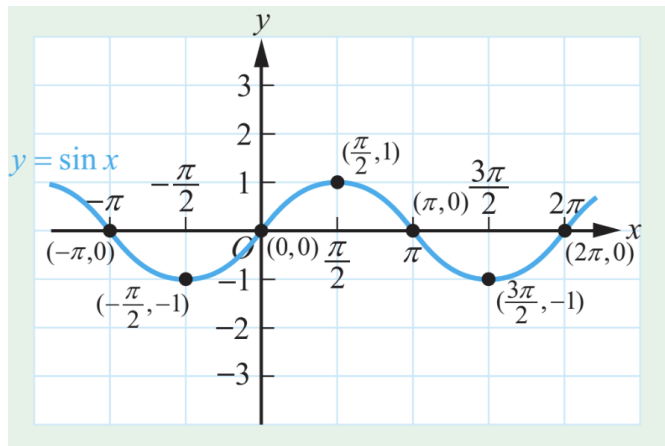
a. $y = \sin 2x$

x			0				
y	0	-1	0	1	0	-1	0



b. $y = \sin \frac{x}{2}$

x			0				
y	0	-1	0	1	0	-1	0



c. Given that $b > 0$, how does b affect the function $y = \sin bx$? Use Desmos to confirm your ideas.

d. Find the following function's maximum, minimum, and period without using Desmos or graphing utilities.

(1) $y = 3 \sin x$ (2) $y = \sin \frac{1}{3}x$ (3) $y = 3 \sin 4x$

使用建議

[教學活動安排]

讓學生用最大值、最小值及截距畫圖去觀察係數對圖形的影響

[英文開場\提問]

Let's move to investigate the horizontal shrinking and stretching of the sine function.

Complete the investigation 3.

We'll check-in in five minutes.

Anyone wants to share your work?

4. The summary of the family $y = a \sin x, a \neq 0$ and $y = \sin bx, b > 0$

From the previous **Investigations** you should have found:

Family $y = a \sin x, a \neq 0$

- a affects the amplitude of the graph; amplitude = $|a|$
- The graph is a vertical stretch of $y = \sin x$ with scale factor $|a|$.
- If $a < 0$, the graph of $y = \sin x$ is also reflected in the x -axis.

Family $y = \sin bx, b > 0$

- The graph is a horizontal stretch of $y = \sin x$ with scale factor $\frac{1}{b}$.
- period = $\frac{2\pi}{b}$

使用建議

[教學活動安排]

統整 Investigate 2、3

[英文開場\提問]

Let's wrap up what we've done.

How does a affect the function $y = a \sin x$?

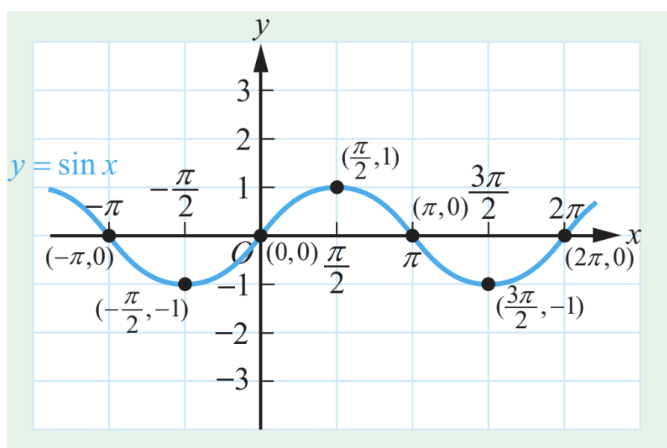
Given that $b > 0$, how does b affect the function $y = \sin bx$?

5. Investigate the translations of the sine function

Using key points to sketch the following sine function on the graph of $y = \sin x$, then answer the questions.

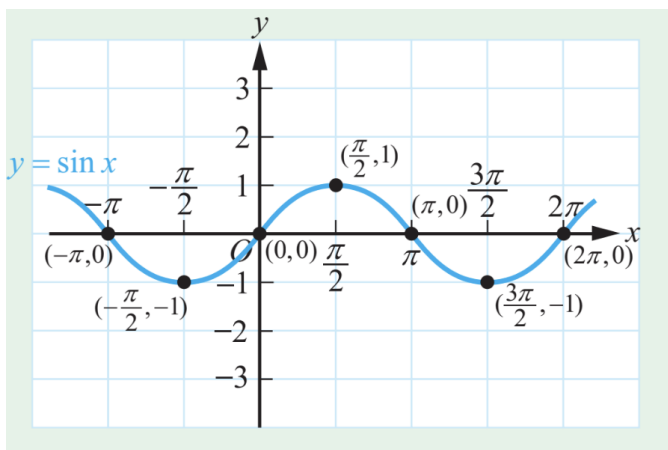
a. $y = \sin x + 1$

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y							



b. $y = \sin(x + \frac{\pi}{2})$

x			0				
y	0	-1	0	1	0	-1	0



c. What transformation move $y = \sin x$ to $y = \sin x + c$?

d. What transformation move $y = \sin x$ to $y = \sin(x - d)$?

f. Find the following function's maximum, minimum, and period without using Desmos or graphing utilities.

(1) $y = \sin(x + 1)$ (2) $y = \sin x - 3$ (3) $y = \sin(2x - 3)$

使用建議

[教學活動安排]

$y = \sin x$ 圖形的平移

[英文開場\提問]

Let's move to the transformations that you should be familiar with.

Complete the investigation 5.

We'll check-in in five minutes.

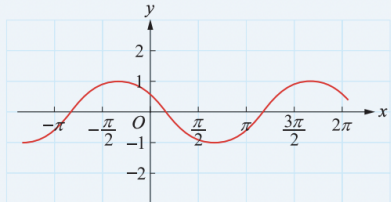
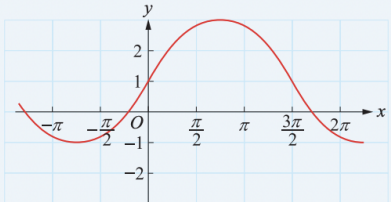
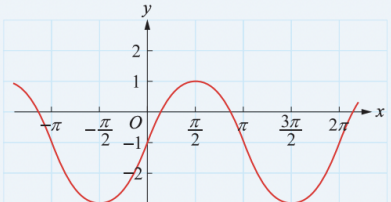
Anyone wants to share your work?

[參考答案]

$y = \sin(x - c)$ is a horizontal translation of $y = \sin x$ by c units.

$y = \sin x + d$ is a vertical translation of $y = \sin x$ by d units.

6. Use the concept of transformations to match the following functions with the

$y = 2 \sin x - 1$	$y = -\sin(x - \frac{\pi}{3})$	$y = 2 \sin \frac{2x}{3} + 1$
•	•	•
•	•	•
		

graphs.

使用建議

[教學活動安排]

練習運用函數圖形伸縮平移的概念

[英文開場\提問]

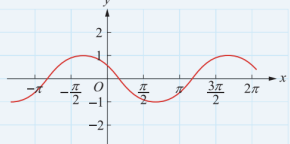
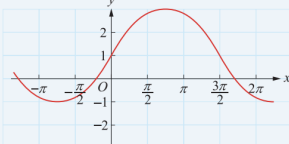
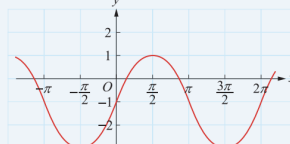
Let's dive into a mix of transformation exercises.

Match the following functions with their corresponding graphs.

We'll check-in in five minutes.

What's the corresponding graph of $y = 2 \sin x - 1$? Why?

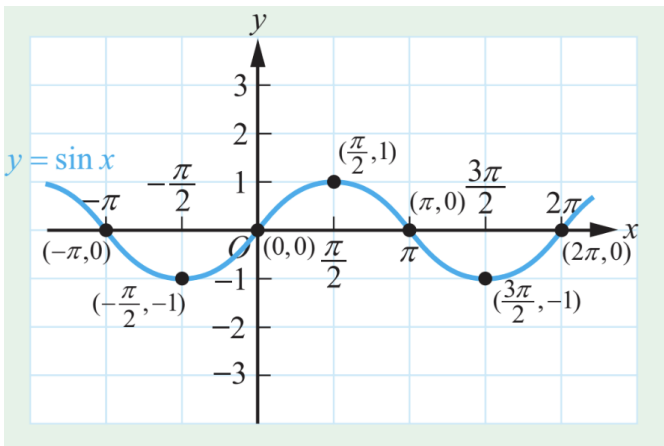
[參考答案]

$y = 2 \sin x - 1$	$y = -\sin(x - \frac{\pi}{3})$	$y = 2 \sin \frac{2x}{3} + 1$
		

7. Follow the instructions to sketch $y = \frac{1}{2} \sin x + 1$ by using the concept of transformations on the graph of $y = \sin x$.

Step1. Sketch $y = \frac{1}{2} \sin x$

Step2. Sketch $y = \frac{1}{2} \sin x + 1$



使用建議

[教學活動安排]

練習運用函數圖形伸縮平移的概念畫出函數圖形

[英文開場\提問]

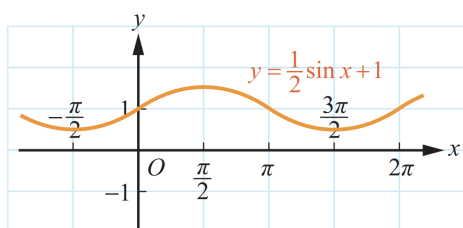
Let's dive into another mix of transformation exercises.

Sketch the graph of the function by using the concept of transformations.

We'll check-in in five minutes.

Anyone wants to share your work?

[參考答案]



8. Investigate the difference between $y = \sin(2x - \frac{\pi}{2})$ and $y = \sin[2(x - \frac{\pi}{2})]$
- a. Use Desmos to determine whether $y = \sin(2x - \frac{\pi}{2})$ or $y = \sin[2(x - \frac{\pi}{2})]$ represents a horizontal translation of $\frac{\pi}{2}$ units from $y = \sin 2x$.
- b. 觀察原點(0,0)的位置變化來說明 $y = \sin(2x - \frac{\pi}{2})$ 及 $y = \sin[2(x - \frac{\pi}{2})]$ 的平移情形。

使用建議

[教學活動安排]

更仔細的探討sin函數的平移

[英文開場\提問]

Let's dive into more detail about the translation of the sine function.

Follow the instructions of investigate 8.

We'll check-in in five minutes.

Anyone wants to share your work?

[參考答案]

$y = \sin[2(x - \frac{\pi}{2})]$ is the horizontal translation by $\frac{\pi}{2}$ units from $y = \sin 2x$.

$y = \sin(2x - \frac{\pi}{2})$ is the horizontal translation by $\frac{\pi}{4}$ units from $y = \sin 2x$.

9. Summary of the sine function and its transformations

THE GENERAL SINE FUNCTION

The **general sine function** is

$$y = a \sin(b(x - c)) + d \quad \text{where } b > 0.$$

affects amplitude affects period affects horizontal translation affects vertical translation

The **principal axis** of the general sine function is $y = d$.

The **period** of the general sine function is $\frac{2\pi}{b}$.

The **amplitude** of the general sine function is $|a|$.

使用建議

[教學活動安排]

統整這堂課的內容

[英文開場\提問]

Let's summarize what we've learned today.

$$y = a \sin(b(x - c)) + d$$

教師可在黑板上寫“ $y = a \sin(b(x - c)) + d$ ”

What does the value of a, b, c and d affect in the equation ?

[參考資料]

MATHEMATICS FOR THE INTERNATIONAL STUDENT Mathematics HL(Core) third edition, Haese Mathematics.

製作者：台北市立育成高中 數學科林玉惇