二次方根的近似值 Approximating a Square root

課本內容：

　　若一個整數 m 是某個整數 a 的平方，即 $m＝a^2$，則 m 稱為 **完全平方數**。例如：$81＝9^2$，$289＝17^2$，因此 81 及 289 都是完全平方數。

> 英文版定義：
>
> In mathematics, a square number or perfect square is an integer that is the square of an integer.
>
> m is an integer that is the square of an integer a.
>
> $m＝a^2$
>
> So, m is a perfect square.

> 翻譯示例：
>
> 81 is a perfect square because $81＝9^2$
>
> 289 is also a perfect square because $289＝17^2$
>
> Both 81 and 289 are perfect squares.

　　將 1 至 400 的完全平方數，列表如下：

整數 a^2	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
完全平方數 m	1	4	9	16	25	36	49	64	81	100
整數 a^2	11^2	12^2	13^2	14^2	15^2	16^2	17^2	18^2	19^2	20^2
完全平方數 m	121	144	169	196	225	256	289	324	361	400

　　熟悉完全平方數後，如果根號內是一個完全平方數時，就可以很容易求得它的值，例如：$\sqrt{169} = \sqrt{13^2} ＝13$。

練習：

Find the following values:

(1) $\sqrt{4}$

　　 $＝2$

(2) $\sqrt{16}$

　　 $＝4$

(3) $\sqrt{225}$

　　 $＝15$

(4) $\sqrt{324}$

　　 $＝18$

課本內容：

　　當根號內的數無法直接看出它是否為某正數的平方時，可利用下面的方法求出它的值。

翻譯示例：

To determine if a radicand (the number inside the radical symbol) is a perfect square, we can use the following method:

練習：求 $\sqrt{a^2}$ 的值 (引導學生利用質因數分解)

Use prime factorization to find the value of $\sqrt{a^2}$.

雙語使用參考範例：

☆可以先給學生較大的完全平方數（需要計算較久）

例如：

T: How can you find the square root of $\sqrt{2116}$?

S: Use the perfect square table.

T: You can try it. (讓學生自己試試 但會花較多時間)

S: It is 46. Because $46 \times 46 = 2116$

T: Do you think the perfect square table is useful / convenient? (問一下學生的解法)

介紹用質因數分解

Let's use Prime Factorization.

$$\sqrt{2116} = \sqrt{2^2 \times 23^2} = \sqrt{(2 \times 23)^2} = \sqrt{(46)^2} = 46$$

引入分數的例子：

How can you find the square root of $\sqrt{\frac{4}{9}}$?

$$\sqrt{\frac{4}{9}} = \sqrt{\frac{2^2}{3^2}} = \sqrt{(\frac{2}{3})^2} = \frac{2}{3}$$

課本內容：

$\sqrt{a}$ 的近似值　**Approximate the value of** $\sqrt{a}$

▶十分逼近法　digit-by-digit calculation

　　正方形的面積為 3，其邊長可記為 $\sqrt{3}$，那麼 $\sqrt{3}$ 的近似值應該是多少呢？

　　因為 $1^2＝1＜3$，且 $2^2＝4＞3$，所以 $1＜\sqrt{3}＜2$，故 $3＝1....$，即 $\sqrt{3}$ 的整數部分為 1，下面將進一步推導 $\sqrt{3}$ 的近似值。

> 翻譯示例：
>
> 　　The area of a square is 3, and its side length can be recorded as $\sqrt{3}$, so what should be the approximate value of $\sqrt{3}$?
>
> Because $1^2＝1＜3$ and $2^2＝4＞3$, so $1＜\sqrt{3}＜2$.
>
> $3＝1....$
>
> The integer part of $\sqrt{3}$ is 1.
>
> A closer approximation of $\sqrt{3}$ will be derived by conducting the following exploration activities.

Make a table of numbers whose squares are close to 3.

Number	1.4	1.5	1.6	1.7	1.8	1.9
Square of Number	1.96	2.25	2.56	2.89	3.24	3.61

The table shows that 3 is between 2.89 and 3.24.

Because $(\sqrt{3})^2＝3$ and $(1.7)^2＜(\sqrt{3})^2＜(1.8)^2$.

So, $1.7＜\sqrt{3}＜1.8$.

補充：如果學生對於 $(1.7)^2＜(\sqrt{3})^2＜(1.8)^2$

則 $1.7＜\sqrt{3}＜1.8$ 的理解有困難，可以讓學生回頭看看或多試幾組數字，例如：

T: Which consecutive integers the square root was between?

補充：$y＝\sqrt{x}$ 的圖形，可以藉此跟學生簡單說明為何 $a＜b＜c$，則 $\sqrt{a}＜\sqrt{b}＜\sqrt{c}$

那麼 $\sqrt{3}$ 到底比較靠近 1.7 還是 1.8 呢？我們知道 $(1.75)^2 = 3.0625$，所以 $1.7 < \sqrt{3} < 1.75$。

翻譯示例：
So is $\sqrt{3}$ closer to 1.7 or 1.8?
We know that $(1.75)^2 = 3.0625$,

So $1.7 < \sqrt{3} < 1.75$.

如果用四捨五入法求得 $\sqrt{3}$ 的近似值到小數點後第一位，即 $\sqrt{3} \fallingdotseq 1.7$。（$\fallingdotseq$ 讀作「近似於」）按照此方法，可以求出 $\sqrt{3}$ 的近似值到任意小數位數，這個方法稱為**十分逼近法**。

翻譯示例：

If the approximate value of $\sqrt{3}$ is rounded to the first decimal place, i.e. $\sqrt{3} \fallingdotseq$ 1.7. ($\fallingdotseq$ pronounced as "approximately").

According to the same way, the approximate value of $\sqrt{3}$ can be obtained to any decimal place. This method is called the digit-by-digit calculation.

練習：

Estimate $\sqrt{23}$ to the nearest tenth.

即利用十分逼近法求$\sqrt{23}$的近似值，並以四捨五入法求到小數點後第一位。

Please use the digit-by-digit calculation to approximate the value of $\sqrt{23}$ and round to the nearest tenth.

Step 1

Find the two perfect squares closet to 23, one greater than 23 and one less than 23.

16 and 25

Step 2

What are their square roots?

4 and 5

Step 3

Hence we know that the whole number part of square root of 23 is 4.

Step 4

$\sqrt{23}$ is between which two consecutive numbers up to tenths place?

4.7 and 4.8

Step 5

$\sqrt{23}$ is between which two consecutive numbers up to hundredths place?

4.79 and 4.80

$\sqrt{23} \approx 4.8$

計算機

雙語使用參考範例：

How to use a calculator to find the square root?

Some calculators require you to input the number first and then push the square root button.

Others require you to push the square root button first, followed by your number.

Find the square root of 5.

You need to push 5 if your calculator requires you to enter the number first and then push the square root button.
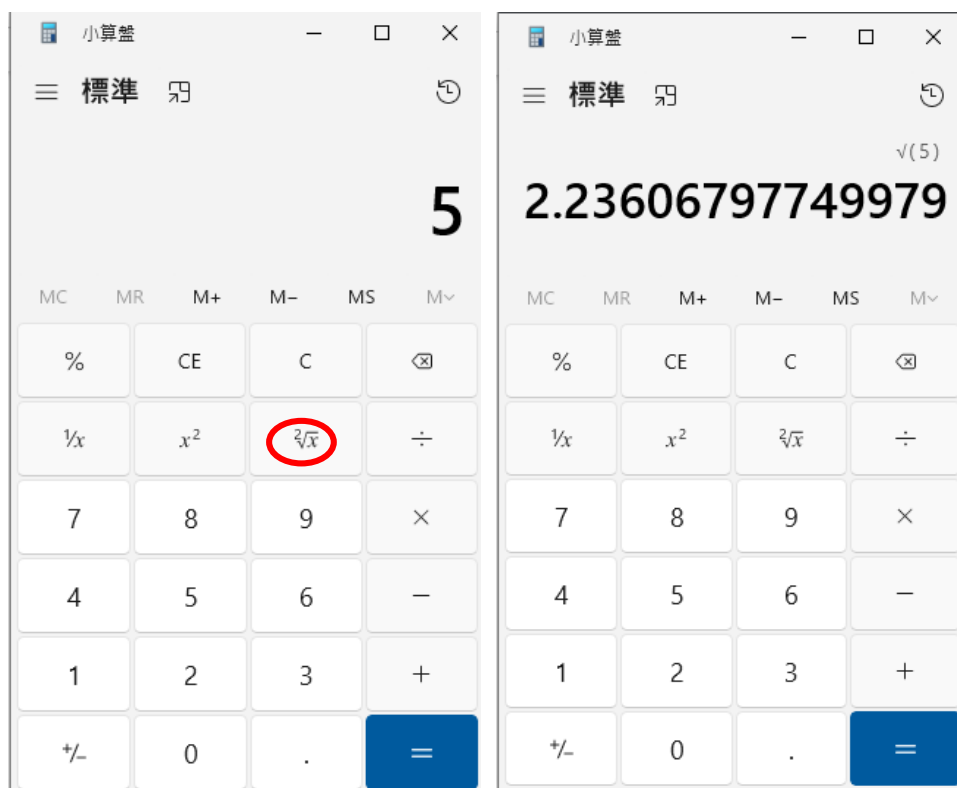
參考資料來源

1. 111 國中數學翰林版課本
2. Into Math Advanced2
3. https://byjus.com/maths/radical/

☆老師們可以自己從中選擇以做出適合自己學生程度的學習單或是在課堂中適時補充這些英文。

製作者:康橋國際學校 陳怡伶