雙語教學主題(國中九年級上學期教材): 連比例

Topic: continued proportion

Vocabulary

Ratio, continued ratio, continued proportion,

We learned what ratio is in grade seven when we compared two quantities. Now if we want to compare three or more quantities, we use the continued ratio.

For example:

There are 3 ants, 2 bees, and 5 cats, we can show the expression as follows:

Let a stand for ant, b stand for bee, and c stand for cat, then

The ratio of two quantities a and b is

　　　　a to b equals 3 to 2 also written as a:b=3:2

　　　　b to c equals 2 to 5 also written as b:c=2:5

Here we have the same portion of b which is 2,

　　　　a to b to c equals 3 to 2 to 5

also written as the form of a continued proportion

　　　　　　a:b:c=3:2:5

But in general situations, we need to do a little more work to get the continued proportion needed. Let's look at the example below:

Example:

When a:b=1:2, b:c=3:4, what is a:b:c?

We can get the continued ratio by using the method

shown on the right side

	a	b	c
a:b=1:2	1	2	
b:c=3:4		3	4

In this case, b is the only common item in two given ratios.

But b has different portions in these two ratios. If we want

to get the continued ratio with these two ratios, we need to have

the same portion of b. The easier way is to get the LCM　　　　　6

of 2 and 3(of course you can pick any number you like b to be, but we always get

the LCM of different portion of b.), $[2,3]$=6. When b equals 2x3, which is

a:b=3:2=3x3:2 x3 and

b:c=3:4=3x2:4 x2, then we get

　　　　a:b:c=3:6:8#

	a	b	c
	1x3	2x3	
		3 x2	4 x2
	3	6	8

Let's look at another example.

> If $a:c = \dfrac{1}{3} : \dfrac{1}{2}$, $b:c = \dfrac{1}{5} : \dfrac{1}{4}$, please find the continued proportion a:b:c
>
> The first thing we do is to simplify ratios a:c and b:c.
>
> (Of course, if you're not afraid of working with fractions, you can skip this part and work on the continued proportion directly.)
>
> To simplify ratio a:c, we need to get the LCM of two denominators 3 and 2.
>
> [3,2]=6. $a:c = \dfrac{1}{3} \times 6 : \dfrac{1}{2} \times 6 = 2:3$
>
> Similarly, for ratio b:c, we get LCM of 5 and 4, [5,4]=20
>
> So $b:c = \dfrac{1}{5} \times 20 : \dfrac{1}{4} \times 20 = 4:5$
>
> Now we get the continued proportion a:b:c with
>
> a:c=2:3 and b:c=4:5.
>
> Since c is the common item in these two ratios, we have the same portion of c by getting the LCM of 3 and 5, [3,5]=15.
>
> When c has the same portion 15 in these two ratios, we can then get the continued proportion of a:c and b:c.
>
> | | a | b | c |
> |---|---|---|---|
> | a:b=2:3=2x5:3x5 and | 2x5 | | 3 x5 |
> | b:c=5:4=5x3:4 x3, then we get | | 4 x3 | 5 x3 |
> | | 10 | 12 | 15 |
>
> a:b:c=10:12:15#

Continued proportion is very useful when solving the real-life problems. When we have a continued proportion like:

a:b:c=3:6:8, we know that a:b=3:6, and b:c=6:8. From what we learned before,

6a=3b or $\dfrac{a}{3} = \dfrac{b}{6}$ and 6c=8b or $\dfrac{c}{8} = \dfrac{b}{6}$, combine the equivalent fractions, we get

$$\dfrac{a}{3} = \dfrac{b}{6} = \dfrac{c}{8}$$

Let $\dfrac{a}{3} = \dfrac{b}{6} = \dfrac{c}{8} = k$, k¹0, then a=3k, b=6k, c=8k

This expression makes it easier for us to solve lots of problems.

Let's look at some examples.

Ex 1:

Let x, y, and z be the price of a pair of socks, a t-shirt, and a pair of pants respectively and x, y, and z are in proportion of x:y:z=3:6:8. If Jojo pays 6400 for 2 pair of socks, 3 t-shirts and a pair of pants, please find out how much a pair of socks costs.

Sol:

Since x:y:z=3:6:8, let x=3k, y=6k, and z=8k, $k \neq 0$

According to the quantities of Jojo's purchase, we get

2x+3y+z=6400,

Replace x, y, and z with 3k, 6k, and 8k

2×3k+3×6k+8k=6400

32k=6400   divide both sides by 32

k=200

Therefore, a pair of socks costs

x=3k=3×200

x=600

So a pair of socks costs 600 dollars.

Ex 2:

Given a:b=3:2, a:c=1:2. Please find out the following continued ratios:

(1)4a:3b:2c

(2)(a-b):b:(a-c)

Sol:

a:b=3:2, a:c=1:2, we take a=$\begin{bmatrix}3,1\end{bmatrix}$ =3

a	b	c
3x1	2 x1	
1x3		2 x3
3	2	6

We get a:b:c=3:2:6

Let a=3k, b=2k, c=6k, $k \neq 0$

4a:3b:2c=4×3k:3×2k:2×6k

=12k:6k:12k   divided by 6k

=2:1:2

(2) (a-b):b:(a-c)= (3k-2k):2k:(3k-6k)

=k:2k:(-3k)

=1:2:(-3) #

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