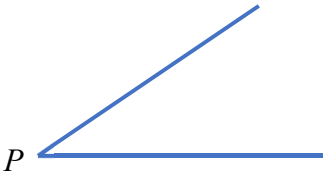
尺規作圖 2
Ruler and compass constructions 2

Class:_____ Name: _____

In this section, we will learn 6 basic ruler and compass constructions. We have already learned 3 of them in the last worksheet. Now, we are going to learn the remaining 3 constructions.

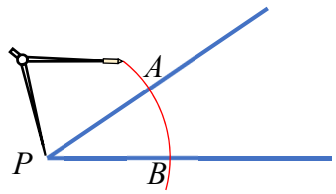
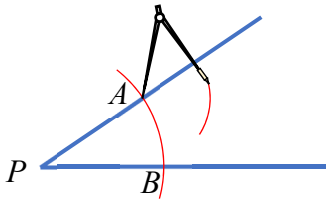
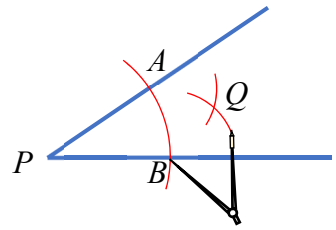
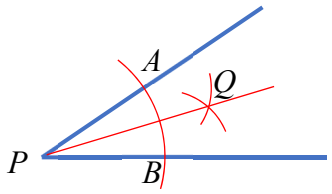
1. Construct an angle bisector (角平分線作圖)

Given an angle $\angle P$. Construct the angle bisector of angle $\angle P$.



尺規作圖前半段(Constructing step by step)逐步針對每個步驟教學，後半部(Try it by yourself)則讓學生練習類題

Constructing step by step

(1) 	(2) 
(3) 	(4) 

(1) Draw an arc centered at P and intersects at two sides of $\angle P$ at points A and B.

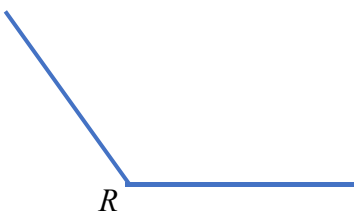
(2) Draw an arc centered at A, with the radius greater than half the length of $\overline{AB}$.

(3) Keep the same width and draw an arc centered at B. Let two arcs intersect at point Q.

(4) Connect a ray $\overrightarrow{PQ}$. Then, $\overrightarrow{PQ}$ is the angle bisector of $\angle P$.

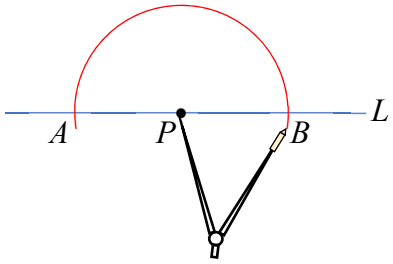
Try yourself

Given an angle $\angle R$. Draw the angle bisector of angle $\angle R$.



2. Construct a perpendicular line through a point on a line (過線上一點作垂線)

Given a line L and a point P on L. Draw a perpendicular line of line L through point P.

<table>
<tr>
<td>(1)
 
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 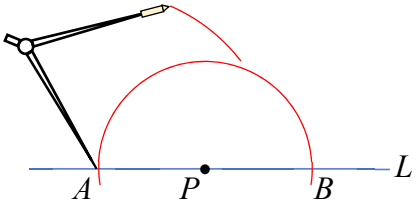
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 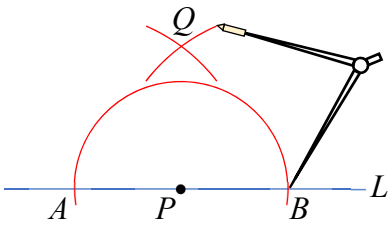
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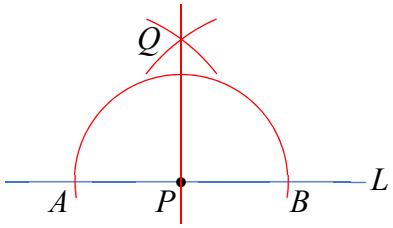
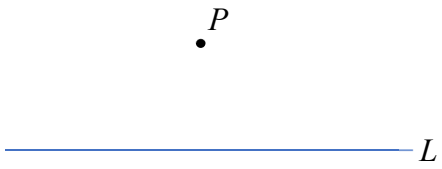
<td>(4)
 
</td>
</tr>
<tr>
<td colspan="2">(1) Draw an arc centered at P which intersects line L at points A and B.
(2) Draw an arc centered at A, with the radius greater than half the length of $\overline{AB}$.
(3) Keep the same width and draw an arc centered at B. Let two arcs intersect at Q.
(4) Connect line $\overleftrightarrow{PQ}$. Then, $\overleftrightarrow{PQ}$ is the required line.</td>
</tr>
</table>

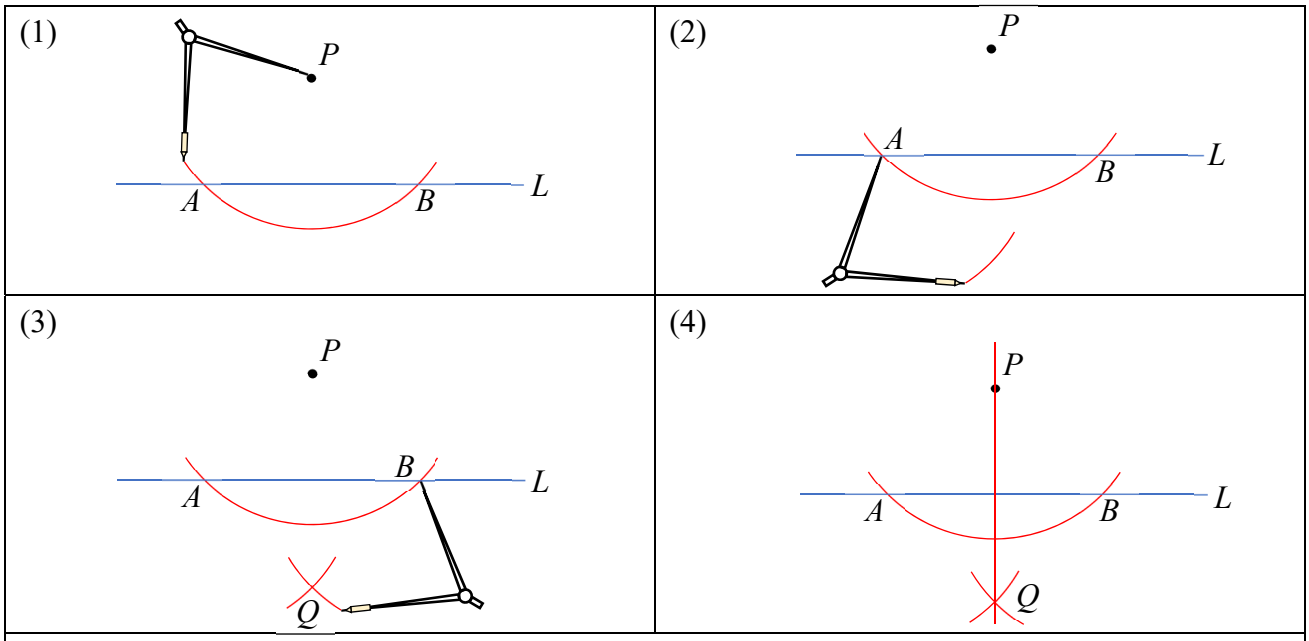
Try yourself

Given a line L and a point Q as shown below. Draw a perpendicular line of line L through point P.

3. Construct a perpendicular line through a point not on a line (過線外一點作垂線)

Given a line L and a point P not on L. Draw a perpendicular line of line L through point P.





(1) Draw an arc centered at P which intersects line L at points A and B.

(2) Draw an arc centered at A, and adjust the compass such that its width is greater than half the length of $\overline{AB}$.

(3) Keep the same width and draw an arc centered at B. Let two arcs intersect at point Q.

(4) Connect line $\overleftrightarrow{PQ}$. Then, $\overleftrightarrow{PQ}$ is the required line.

Try yourself

Given a line L and a point Q as shown below. Draw a perpendicular line of line L through point Q.

一、設計理念：

1. 尺規作圖又可譯為 staightedge and compass construction 或 geometric construction。
2. 尺規作圖教師需解說與操作互相搭配，故同學可藉由教師的操作理解每個過程的操作方式。

二、英文詞彙：

中文	英文
直尺	ruler
圓規	compass
線段	line segment
圓弧	arc
角	angle
半徑	radius
中垂線	perpendicular bisector
角平分線	angle bisector
垂線	perpendicular line
相交	intersect
大於	greater than

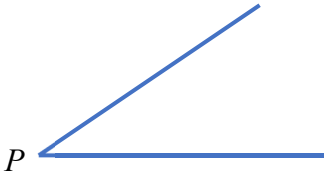
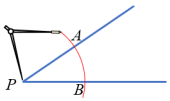
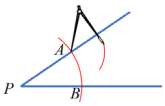
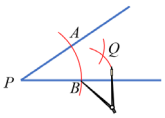
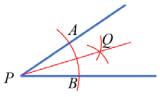
三、數學英文用法：

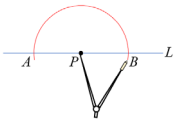
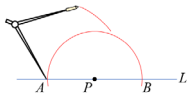
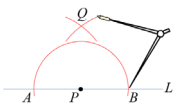
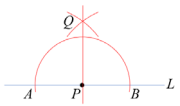
數學表示法	英文
$\overline{AB}$	line segment AB
$\overrightarrow{AB}$	ray AB
$\overleftrightarrow{AB}$	line AB
$\angle A$	angle A
$\overline{AB}$ 與 L 垂直	line segment AB is perpendicular to L
以 A 點為圓心畫弧	draw an arc centered at A

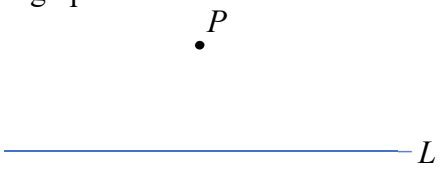
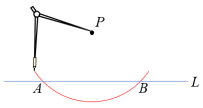
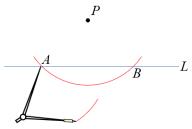
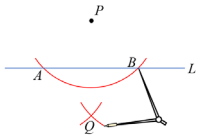
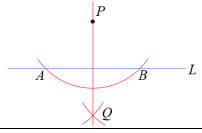
四、六種尺規作的參考英譯：

尺規作圖	英文
等線段作圖	Copy a line segment
等角作圖	Copy an angle
中垂線作圖	Construct a perpendicular bisector
角平分線作圖	Construct an angle bisector
過線上一點作垂線	Construct a perpendicular line through a point on a line
過線外一點作垂線	Construct a perpendicular line through a point not on a line

四、教學參考範例：

<table>
<tr><td rowspan="6">1
【角平分線作圖】</td><td colspan="2">Construct an angle bisector (角平分線作圖)
Given an angle $\angle P$. Construct the angle bisector of angle $\angle P$.
 
</td></tr>
<tr><td></td><td>Next, we are going to draw the angle bisector of a given angle. The angle bisector is a ray in the interior of the angle that divides the angle into equal halves. Just as constructing the perpendicular bisector of a given line segment, we can directly construct the angle bisector on angle P.</td></tr>
<tr><td>(1)
 
</td><td>The first step is to draw an arc centered at the vertex of the angle. Take out your compass and put the pointed end on P. Draw an arc that intersects both sides of the angle at points A and B respectively.</td></tr>
<tr><td>(2)
 
</td><td>Take out the compass and put the pointed end on A. Then, draw a little arc in the middle of the angle P. Notice that the radius of the arc cannot be too short. It should be longer than half of the length of AB.</td></tr>
<tr><td>(3)
 
</td><td>Keep the same setting and draw an arc centered at B. The arc intersects the previous one at point Q.</td></tr>
<tr><td>(4)
 
</td><td>Take out your ruler and connect a ray from point P to point Q. We can see the ray divides angle P into two angles that are congruent to each other. Then, it is the angle bisector of angle P.</td></tr>
</table>

<table>
<tr><td rowspan="5">2
【過線上一點作垂線】</td><td colspan="2">Construct a perpendicular line through a point on a line
(過線上一點作垂線)
Given a line L and a point P on L. Draw a perpendicular line of line L through point P.
 
</td></tr>
<tr><td></td><td>Next, line L and point P are given, and P lies on L. We are trying to draw a line through P which will be perpendicular to L. We have done a similar construction before. We have constructed the perpendicular bisector of a segment. What we want to copy the process of the previous construction.</td></tr>
<tr><td>(1)
 
</td><td>First, we construct an arbitrary segment AB so that P is the midpoint. Swing the compass and draw a semicircle centered at P and intersects the line on either side of P. The semicircle should intersect L on both sides. The two intersections are points A and B, both of which are equidistant from P.</td></tr>
<tr><td>(2)
 
</td><td>Next, we can construct the perpendicular bisector of line segment AB. Because P is the midpoint of AB, P is on the perpendicular bisector of AB. Through any two points, there exactly one line, so we only need to find another point equidistant from A and B. The point can be either above or below P. Therefore, make the compass a little wider and draw an arc centered at A above L. Of course, you can draw an arc below L.</td></tr>
<tr><td>(3)
 
</td><td>Then, keep the same setting and draw an arc centered at B. Make sure these two arcs ~~should~~ intersect. Now we have the intersection point Q which is the same distance from A and B. Then, Q is also on the perpendicular bisector of AB.</td></tr>
<tr><td>(4)
 
</td><td>Finally, use the ruler to join P and Q. Line PQ is the perpendicular bisector of AB, in particular, is perpendicular to L through P as required.</td></tr>
</table>

<table>
<tr><td rowspan="5">3
【過線外一點作垂線】</td><td colspan="2">Construct a perpendicular line through a point not on a line
(過線外一點作垂線)
Given a line L and a point P not on L. Draw a perpendicular line of line L through point P.
 
</td></tr>
<tr><td></td><td>The final construction is to construct the perpendicular line through the point not lying on the line. The process is similar to drawing a perpendicular line through the point on the line. We need to find a line segment and construct the perpendicular bisector of the line segment.</td></tr>
<tr><td>
 
</td><td>Now we will start our construction. We want to draw a line segment on L, but we ~~can not~~ randomly choose the line segment. The perpendicular bisector of the line segment should pass through P. Therefore, P should be equidistant from A and B. We draw an arc centered at P and intersects L at A and B. Notice that the radius of the arc should be long enough to cross line L twice. If the arc does not intersect L at two points, adjust the width of the compass slightly greater.</td></tr>
<tr><td>
 
</td><td>We have the line segment AB now. Next, we are going to construct the perpendicular bisector of AB. We already have point P on perpendicular line which will bisect AB. Therefore, we only need to find another point. We can directly keep the compass in the same setting. Then, draw an arc centered at A below line L.</td></tr>
<tr><td>
 
</td><td>Keep the same setting again, and move the pointed end to B. Draw an arc that intersects the previous one at point Q. Q is equidistant from A and B, so it is also lies on the perpendicular bisector of AB.</td></tr>
<tr><td>
 
</td><td>Connect P and Q. Line PQ is the perpendicular bisector of AB. Therefore, it is perpendicular to L and passes through P as required.</td></tr>
</table>

製作者：臺北市雙園國中 劉家宇