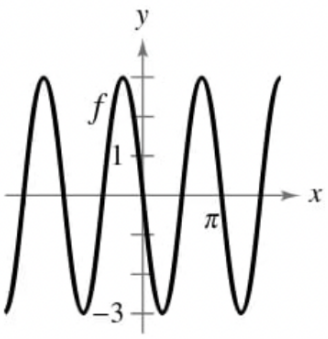
Topic: Fitting Data to a Sine Function and Graphs of the Cosine and the Tangent Functions

1. Graphical Reasoning



Follow the steps to find the value of a, b, and c for the function $y = a\sin(b(x-c)) + d$ to match the figure.

a. Find the maximum and the minimum then find the value of a and d.

b. Find the period of this function then find the value of b.

c. Given that a point $(\pi,0)$ on the graph, find the value of c

d. Use your graphing utility to check your answer.

使用建議	
教學活動安排	從圖形長相回推sine函數係數以銜接後續fit data to a sine function 所需的技巧
英文提問/開場	What periodic phenomena have you observed in your daily life? • The phases of the moon • The number of daylight hours at a particular location • Tidal variations in the depth of water in a harbor These phenomena demonstrate variable behaviors that repeats over time. So, we can use a sine function to model and predict periodic phenomena. Let's investigate the skills of using sine functions to model these periodic phenomena. Take a look at question 1. You have a graph of the sine function with unknown coefficients. Follow the instructions to determine which sine function matches the given graph. Let's do it together. Now let's wrap up what we've been doing. How do we fit data to a sine function when given a periodic graph?

使用建議	
參考答案	1. a. The maximum is 3, the minimum is −3. The amplitude a is equal to 3. The principal axis is $y = 0$, so d is equal to 0. b.The period of this graph is π, so b is equal to 2 c. $0 = 3\sin(2\pi - c), c = \pi$ d. Insert $y = 3\sin(2x - \pi)$ to check the graph. f. For $f(x) = a\sin[b(x + c)] + d$: • Calculate the amplitude $a = \frac{\text{maximum value} - \text{minimum value}}{2}$ • Calculate the vertical shift $d = \frac{\text{maximum value} + \text{minimum value}}{2}$ • Find the value of $b = \frac{2\pi}{\text{period}}$ • Find the phase shift c. • Calculate the horizontal shift by choosing given coordinates of a data point.
參考資料	Oxford Ib Diploma Programme: Mathematics: Analysis and Approaches, Higher Level

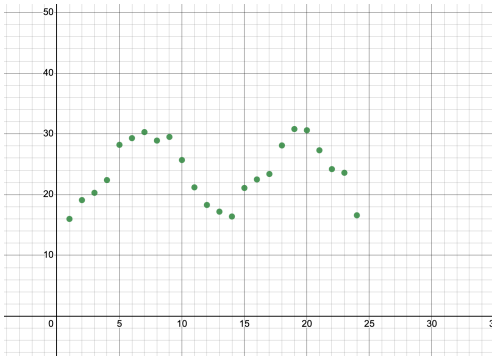
2. Fitting data to a sine function

MEAN MONTHLY TEMPERATURE **https://reurl.cc/K0L96e**

The mean monthly temperature for Taipei City over a 24–month (the years 110–111) period:

Month	1	2	3	4	5	6	7	8	9	10	11	12
T(°C)	16.0	19.1	20.3	22.4	28.2	29.3	30.3	28.9	29.5	25.7	21.2	18.3
Month	13	14	15	16	17	18	19	20	21	22	23	24
T(°C)	17.2	16.4	21.1	22.5	23.4	28.1	30.8	30.6	27.3	24.2	23.6	16.6

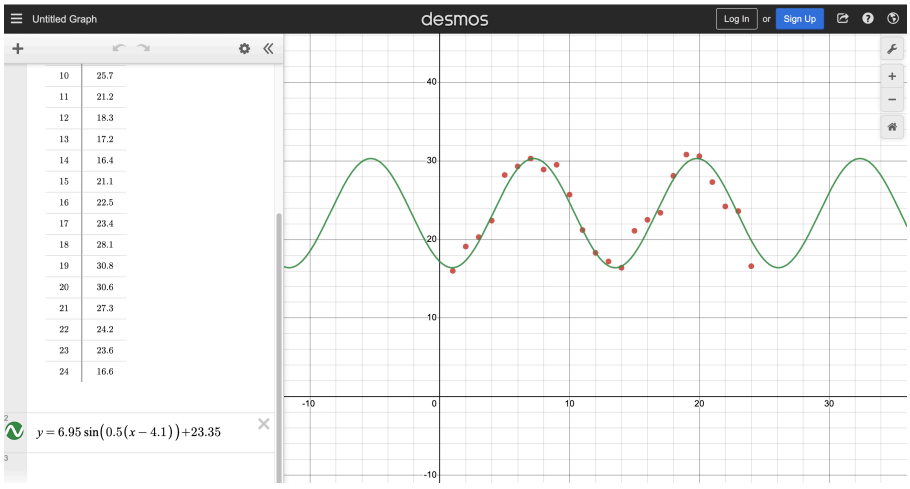
After inputting the data into the graphing calculator, we obtain the graph:



a. Find a sine model for this data in the form $T = a \sin\left(b\,(t - c)\right) + d$ without using a graphing utility, where T represents the mean monthly temperature, and t represents the months (Jan=1, Feb=2, and so on).

b. Use your graphing utility to check your answer to a.

c. Compare your answer with your classmates and see how well your model fits the data.

使用建議	
教學活動安排	用一個實際的例子（110年–111年台北的月均溫）去Fitting data to a sine function 資料出處：https://reurl.cc/K0L96e
英文提問/開場	Let's move to fit data to a sine function. It's an example about the mean monthly temperature. Now it's your turn to demonstrate your skill in fitting data to a sine function. Show us how well you've grasped this technique! We'll check it in five minutes.
參考答案	

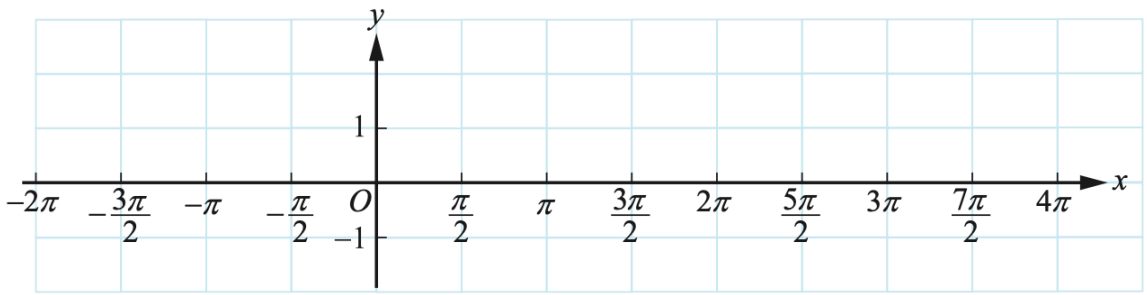
3. The graph of $y = \cos x$

a. Find the value of $\cos \frac{\pi}{6}$

b. Complete the following table

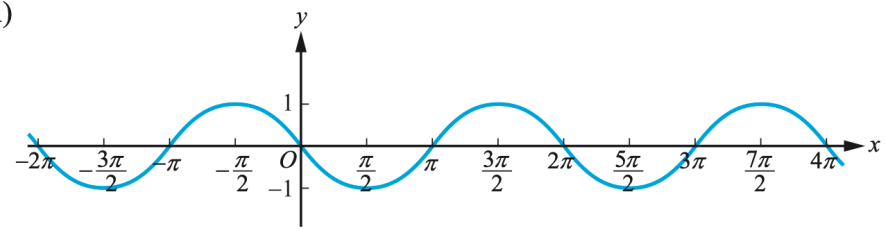
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
$\cos x$									

c. Use the table to sketch the graph of $y = \cos x$

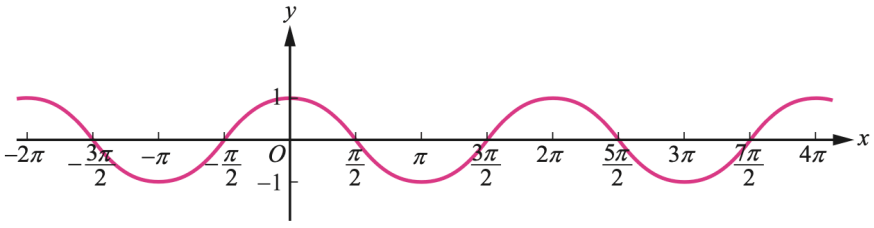


d. Which of the following options could be the graph of $y = \cos x$?

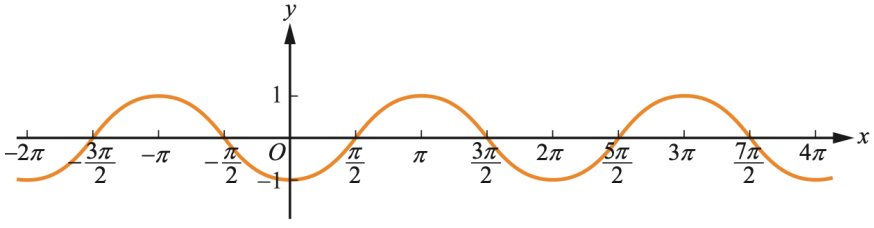
(A)



(B)



(C)



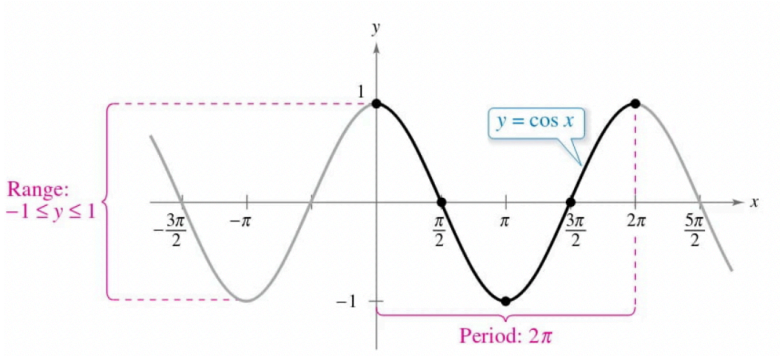
e. Is $y = \cos x$ a periodic function? If so, what is the period of $y = \cos x$?

f. Is the graph of $y = \cos x$ symmetrical about the x–axis or y–axis?

g. What is the range of $y = \cos x$?

h. Can we translate the graph of $y = \sin x$ to obtain the graph of $y = \cos x$? If so, state the translation and explain why we can perform this transformation.

使用建議	
教學活動安排	1. 畫出$y = \cos x$的圖形 2. 先讓學生求出當自變數是弧度量的角度時的$\cos x$的值 3. 接著讓學生完成表格並畫出圖形，表格中的角度都是以90度為基準的理由是考量學生會比較沒有耐心完成太多。且讓學生嘗試出大概的樣子即可。 4. 接著讓學生去選可能的$y = \cos x$圖形並回答其函數圖形的性質及與$y = \sin x$的關係
英文提問/開場	We've learned about the sine function and its graph. What about the other trigonometric functions and their graphs? Let's investigate the cosine function $y = \cos x$ and its graph. Follow the instructions on the worksheet. We'll check it in ten minutes. Anyone want to share your work? After answering these questions, what are the properties of the cosine function?

使用建議	
參考答案	The basic cosine function $y = \cos x$ and its graph.  • It is a periodic function with a period 2π. • The maximum value is 1 and the minimum is -1. The range is $-1 \le y \le 1$. • The amplitude is 1. • Since $\sin(x + \frac{\pi}{2}) = \cos x$, the cosine function is $\frac{\pi}{2}$ units left of the sine function under a horizontal translation. (Or The graph of the cosine function can be obtained by shifting the graph of the sine function two units to the left.) • Since $\cos(-x) = \cos x$, the graph of the cosine function is symmetric to the y −axis.
參考資料	高中數學三（A）,泰宇出版社 Precalculus with Limits: Larson, Ron –4th Edition

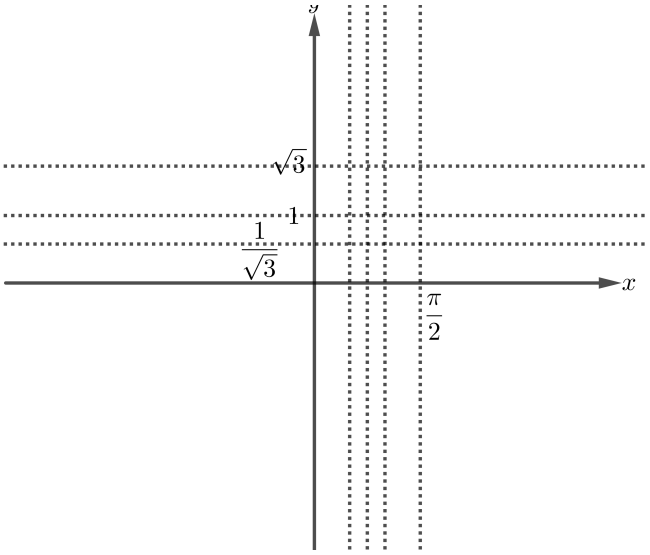
4. The graph of $y = \tan x$

a. Recall the definition of the tangent function is $\tan x = \dfrac{\sin x}{\cos x}$, as x increases from 0 to $\dfrac{\pi}{2}$, how do the values of $y = \tan x$ change?

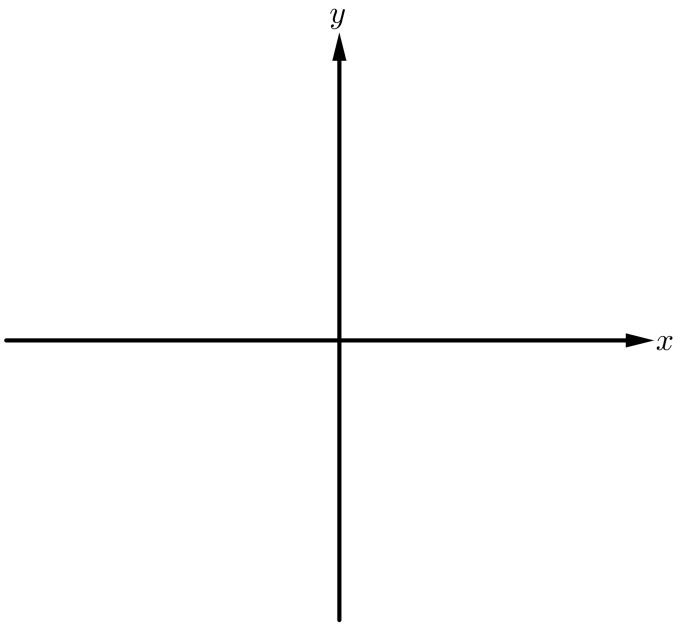
b. Complete the following table

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan x$					Undefined

c. Since $\tan(-x) = -\tan x$, the tangent function is odd. That is the graph of $y = \tan x$ is symmetric with respect to the origin. Use this property and the table to sketch the graph of $y = \tan x$ within the domain $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.



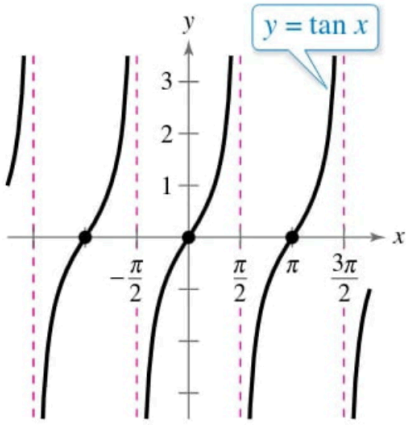
d. Another property is the identity $\tan(x + \pi) = \tan x$, use this property to sketch the graph of $y = \tan x$.



e. Use the graphing utility to check the graph of $y = \tan x$.

f. What are the properties of the graph of $y = \tan x$?

使用建議	
教學活動安排	$y = \tan x$ 的圖形跟前面學過的sine函數及cosine函數不一樣。所以這裡有多花一點時間以及多一點引導讓學生探索。
英文提問/開場	Now, let's move on to the graph of the tangent function. Its graph is really different from the sine function. So, take your time and follow the questions on the worksheet to explore what the graph looks like. Let's do this together. For question a, anyone wants to share your idea? What makes you think the graph is increasing within the domain 0 to $\frac{\pi}{2}$? Great! Well done! Now, let's create a table to assist us in sketching the graph of this function. Who's up for sharing their work on question b? Fantastic job! Then let's use the property that the graph is symmetric with respect to the origin to sketch the graph within the domain $-\frac{\pi}{2}$to $\frac{\pi}{2}$. Grab your graphing utilities and draw out what we've discussed. Let's see how close we get to the actual graph. Take a close look at your sketches and the graphing utility results. Do you observe anything special about the graph? Any unique features that catch your eye? （希望學生觀察到圖形從原點開始是比較平，後面慢慢遞增速度很快。） Since the $y-$value is undefined at $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$, we have asymptotes $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$. Now, let's move on to sketching the graph of the tangent function across the entire domain. To make things easier, let's use the helpful identity $\tan(x + \pi) = \tan x$, which reveals that the period of the tangent function is π. This little trick will save us some time and effort while sketching. Again grab your graphing utilities and see how close we get to the actual graph. Excellent work so far, everyone! Let's sum it up, what are the properties of the tangent function? Who would like to step up and tell us about the properties of the tangent function?

使用建議	
參考答案	The basic cosine function $y = \tan x$ and its graph.  • It is a periodic function with a period π. • The range is the set of all real numbers. • It has vertical asymptotes $x = \frac{\pi}{2} + k\pi$, where k is an integer. • It is an odd function, the graph of the cosine function is symmetric with respect to the origin.
參考資料	高中數學三（A）,泰宇出版社 Precalculus with Limits: Larson, Ron –4th Edition

製作者：台北市立育成高中 林玉惇老師