

根式的運算-1(Multiplication and Division)

Operations with Square Roots-1(Multiplication and Division)

課本內容：

帶有根號的數也可以作加、減、乘、除的運算，例如： $\sqrt{2} + 3$ 、 $\sqrt{2} - 1$ 、 $(-2) \times \sqrt{2}$ 、 $1 \div \sqrt{2}$ ，像這樣含有根號的式子，我們稱為**根式**。

英文翻譯參考：

Numbers with square roots can be used for addition, subtraction, multiplication, and division operations. For example: $\sqrt{2} + 3$, $\sqrt{2} - 1$, $(-2) \times \sqrt{2}$, $1 \div \sqrt{2}$.

An expression containing square roots is known as a radical expression.

☆1 根式的表示 Radical expression

課本內容：

【根式的簡記】 Simplifying square roots

若 $a \neq 0$ 且 b 為正數，則：If $a \neq 0$ and b is a positive integer, then

$$(1) a \times \sqrt{b} = a\sqrt{b}$$

$$(2) \sqrt{b} \div a = \frac{\sqrt{b}}{a} \text{ 或 } \frac{1}{a}\sqrt{b}$$

整數與根式的乘法 $3 \times 2\sqrt{2} = 3 \times 2 \times \sqrt{2} = 6 \times \sqrt{2} = 6\sqrt{2}$ ，利用同樣的方法，我們可以做整數或分數與根式的運算。

英文翻譯參考：

Multiplication of integers and radicals: $3 \times 2\sqrt{2} = 3 \times 2 \times \sqrt{2} = 6 \times \sqrt{2} = 6\sqrt{2}$. Using the same method, we can perform operations with integers, fractions, and radicals.

練習：

Calculate the following radical.

$$(1) 3 \times \sqrt{2} \quad (2) -2 \times \sqrt{3} \quad (3) \sqrt{5} \times \frac{1}{2}$$

☆2 根式的乘法 Multiplication with radicals

課本內容：

【根式的乘法運算】 Multiplication with radicals

若 a 、 b 為正數或 0，則 $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ 。 If $a \geq 0$ and $b \geq 0$, then $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$.

雙語使用參考範例：

$\sqrt{a} \times \sqrt{b}$ 會不會等於 $\sqrt{a \times b}$

Is $\sqrt{a} \times \sqrt{b}$ equal to $\sqrt{a \times b}$?

(先給學生數據嘗試)

T : Is $\sqrt{3} \times \sqrt{3}$ equal to $\sqrt{3 \times 3}$?

Because $\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$.

And $\sqrt{3 \times 3} = \sqrt{9} = 3$.

So, $\sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3}$

T : Let's try $\sqrt{4} \times \sqrt{4}$, and so on.

T : 問學生如果是 $\sqrt{3} \times \sqrt{5}$ 會不會等於 $\sqrt{3 \times 5}$

Is $\sqrt{3} \times \sqrt{5}$ equal to $\sqrt{3 \times 5}$ ($=\sqrt{15}$)?

Because $(\sqrt{3} \times \sqrt{5})^2 = (\sqrt{3} \times \sqrt{5}) \times (\sqrt{3} \times \sqrt{5})$
 $= (\sqrt{3} \times \sqrt{3}) \times (\sqrt{5} \times \sqrt{5})$
 $= (\sqrt{3})^2 \times (\sqrt{5})^2$
 $= 3 \times 5 = 15$

Also, $(\sqrt{3 \times 5})^2 = \sqrt{15}^2 = 15$

Since $\sqrt{3} \times \sqrt{5}$ and $\sqrt{3 \times 5}$ are positive numbers, $\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5}$.

之後可以多試幾組讓學生觀察與歸納下列結論(也可以透過計算機)

In fact, if $a \geq 0$ and $b \geq 0$, then $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$.

練習：

Circle the equivalent expressions.

(1) $\sqrt{3} \times \sqrt{2}$

$3 \times \sqrt{2}$ $\sqrt{3 \times 2}$ $3\sqrt{2}$ $\sqrt{6}$

☆3 根式的除法 Division with radicals

課本內容：

【根式的除法運算】 Division with radicals

$$\text{若 } a \geq 0, b > 0, \text{ 則 } \sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} = \sqrt{a \div b} .$$

$$\text{If } a \geq 0 \text{ and } b > 0, \text{ then } \sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} = \sqrt{a \div b} .$$

雙語使用參考範例：

T : We have learned that $\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5}$.

How to apply this rule in division with radicals?

Is $\sqrt{3} \div \sqrt{5}$ equal to $\sqrt{3 \div 5}$?

先讓學生自己試試看

引導學生寫出下列式子

$$\text{Because } \sqrt{3} \div \sqrt{5} = \frac{\sqrt{3}}{\sqrt{5}}$$

$$(\sqrt{3} \div \sqrt{5})^2 = \left(\frac{\sqrt{3}}{\sqrt{5}}\right)^2 = \left(\frac{\sqrt{3}}{\sqrt{5}}\right) \times \left(\frac{\sqrt{3}}{\sqrt{5}}\right) = \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{5} \times \sqrt{5}} = \frac{3}{5} .$$

$$\sqrt{3 \div 5} = \sqrt{\frac{3}{5}} .$$

$$(\sqrt{3 \div 5})^2 = \left(\sqrt{\frac{3}{5}}\right)^2 = \left(\sqrt{\frac{3}{5}}\right) \times \left(\sqrt{\frac{3}{5}}\right) = \frac{3}{5} .$$

之後可以多試幾組讓學生觀察與歸納下列結論(也可以透過計算機)

$$\text{In fact, if } a \geq 0 \text{ and } b > 0, \text{ then } \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} .$$

練習：

Circle the equivalent expressions.

(1) $\sqrt{3} \div \sqrt{2}$

$3 \div \sqrt{2}$ $\sqrt{3 \div 2}$ $3\sqrt{2}$ $\sqrt{\frac{3}{2}}$

☆4 比較根式的大小 Order radicals

練習：

Q1. Compare $2\sqrt{6}$ and $2\sqrt{7}$

Because $\sqrt{7} > \sqrt{6}$.

(Because the square root of 7 is greater than the square root of 6.)

So, $2 \times \sqrt{7} > 2 \times \sqrt{6}$.

(So, 2 times the square root of 7 is greater than 2 times the square root of 6.)

$2\sqrt{7} > 2\sqrt{6}$.

(Therefore, 2 times the square root of 7 is greater than 2 times the square root of 6.)

Q2. Compare $3\sqrt{5}$ and $5\sqrt{3}$

Because

$3\sqrt{5} = 3 \times \sqrt{5} = \sqrt{9} \times \sqrt{5} = \sqrt{45}$

(Because 3 times the square root of 5 is equal to the product of the square root of 9 and the square root of 5 and is equal to the square root of 45.)

$5\sqrt{3} = 5 \times \sqrt{3} = \sqrt{25} \times \sqrt{3} = \sqrt{75}$

(Because 5 times the square root of 3 is equal to the product of the square root of 25 and the square root of 3 and is equal to the square root of 75.)

$\sqrt{75} > \sqrt{45}$

(The square root of 75 is greater than the square root of 45.)

So, $5\sqrt{3} > 3\sqrt{5}$.

(Therefore, 5 times the square root of 3 is greater than 3 times the square root of 5.)

Q3. Compare $\frac{\sqrt{6}}{2}$ and $\frac{\sqrt{7}}{2}$ (提供 $\frac{\sqrt{6}}{2}$ 的三種念法)

(Compare **the quotient of the square root of 6 and 2** and the quotient of the square root of 7 and 2.)

Because $\sqrt{7} > \sqrt{6}$.

(Because the square root of 7 is greater than the square root of 6.)

So, $\frac{1}{2} \times \sqrt{7} > \frac{1}{2} \times \sqrt{6}$.

(So, one-half times the square root of 7 is greater than **one-half times the square root of 6**.)

$\frac{\sqrt{7}}{2} > \frac{\sqrt{6}}{2}$.

(The square root of 7 divided by 2 is greater than **the square root of 6 divided by 2**.)

Q4. Compare $\frac{\sqrt{5}}{3}$ and $\frac{\sqrt{3}}{5}$

Because

$$\frac{\sqrt{5}}{3} = \frac{1}{3} \times \sqrt{5} = \sqrt{\frac{1}{9}} \times \sqrt{5} = \sqrt{\frac{5}{9}}$$

(Because the quotient of the square root of 5 and 3

is equal to one-third times the square root of 5

is equal to the square root of one over nine times the square root of 5

is equal to the square root of five over nine.)

$$\frac{\sqrt{3}}{5} = \frac{1}{5} \times \sqrt{3} = \sqrt{\frac{1}{25}} \times \sqrt{3} = \sqrt{\frac{3}{25}}$$

(Because the quotient of the square root of 3 and 5

is equal to one-fifth times the square root of 3

is equal to the square root of one over twenty-five times the square root of 3

is equal to the square root of three over twenty-five.)

$$\sqrt{\frac{5}{9}} > \sqrt{\frac{3}{25}}$$

(So, the square root of five over nine is greater than the square root of three over twenty-five.)

So, $\frac{\sqrt{5}}{3} > \frac{\sqrt{3}}{5}$.

(So, the quotient of the square root of 5 and 3 is greater than the quotient of the square root of 3 and 5.)

參考資料來源

1. 111 國中數學翰林版課本
2. Into Math Advanced2
3. <https://byjus.com/maths/radical/>

☆老師們可以自己從中選擇以做出适合自己學生程度的學習單或是在課堂中適時補充這些英文。

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