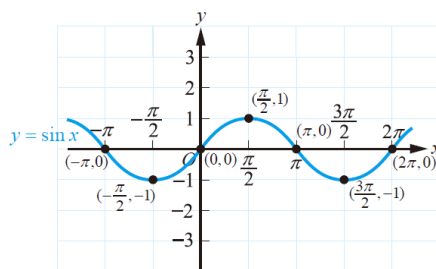## Topic: Harmonic Addition Theorem
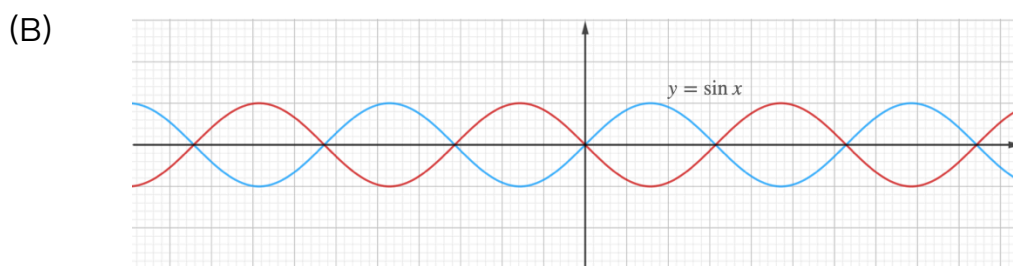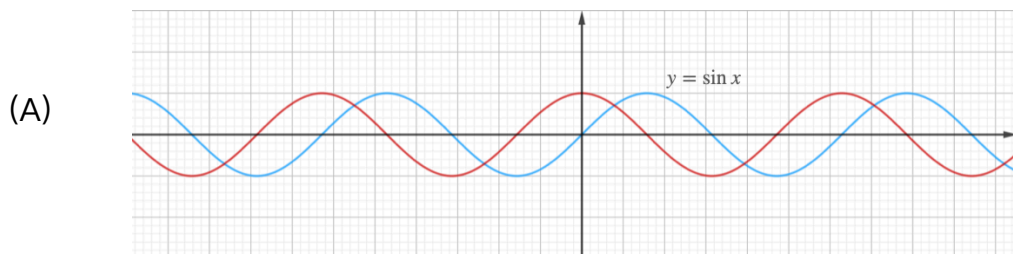
1. Noise–canceling earphones

Noise–canceling headphones work by using built–in microphones to detect external sounds. They then create an 'anti–noise' soundwave that overlaps with the original sound, effectively canceling it out and providing a quieter listening experience.

Now if a sound wave is modeled $y = \sin x$ in the diagram below.

Then which of the following diagrams demonstrates another sound wave that overlaps with the original sound wave $y = \sin x$, effectively canceling it out? Try to think of what makes those two waves overlap to eliminate each other.

(A)

(B)

	使用建議
教學活動安排	利用降躁耳機的原理來讓學生從函數圖形去感覺疊合在圖形上的含義。
英文提問／開場	Today we are going to learn harmonic addition theorem. Let's explore its idea through a real-life example. The concept behind noise-canceling headphones is to detect the sound waves of noise, then generate another sound wave that overlaps to eliminate the original sound wave. We know that sound waves can be represented by sine or cosine functions. But what do these functions "overlap" mean in math? Do the example on the worksheet and collaborate with your partner. We'll check it in five minutes. Anyone wants to share your thoughts? What makes you say those two waves overlap to eliminate each other? What do these functions "overlap" mean in math? "Overlap" means that when two functions are added together, the corresponding y-values for the same x-values are added. In this example, we added the negative y-values than the origin y-values to achieve a zero amplitude. So, discussing overlapping waves in math is to find the results of addition of sine fuctions and cosine functions, which is today's topic-the harmonic addition theorem.
參考答案	(B)
參考資料	1.https://www.sony.com.tw/zh/electronics/support/articles/00203389 2. https://www.soundguys.com/how-noise-cancelling-headphones-work-12380/

2. Harmonic addition–Explore the graph of $y = \sin x + \cos x$

a. Below the blue curve is part of the graph of $y_1 = \sin x$, the blue dotted curve is part of the graph of $y_2 = \cos x$, and the orange curve is part of graph $y = \sin x + \cos x$. However, some part of the orange curve has not been finished, finish it by using a ruler and a pen.
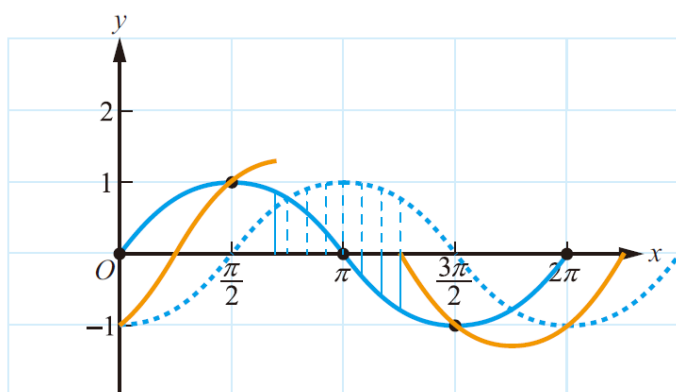
b. Plot $y = \sin x + \cos x$ by Desmos and find the function of the graph shown on Desmos.

c. If we add a sine function and a cosine function where these two functions have the same period(variable and its coefficient are the same) will the result still be a sine or cosine function?

使用建議
<table><tr><td>教學活動安排</td><td>1.讓學生手畫一次函數圖形相加是y坐標相加。 2.從最簡單的例子$y = \sin x + \cos x$ 讓學生用已熟悉的概念去找兩個函數疊合後的結果 3.疊合後的結果可以是sine函數也可以是cosine函數</td></tr></table>

	使用建議

| 英文提問 / 開場 | Let's look at option A as an example of noise–canceling earphone.
The waves of this diagram are $y = \sin x$ and $y = \cos x$.
What will the graph look like when a sine wave overlaps with a cosine wave?
The question also means what is result of a function $y = \sin x + \cos x$?
Work on question a to find the graph that a sine wave overlaps with a cosine wave.
Anyone wants to share?
What does the graph $y = \sin x + \cos x$ look like? Is it a sine wave or a cosine wave? What are its maximum and minimum values? What is its period?
Good job! Amazing! Great observers!
Now let's use technology to help us to find the accurate graph. Plot the function $y = \sin x + \cos x$ by Desmos. Then find the function of the graph.
Anyone wants to share your answer?
What is the result of $y = \sin x + \cos x$?
Good job.
So think about question c. Anyone wants to share your thoughts? |
| 參考答案 | $y = \sin x + \cos x = \sqrt{2}\sin(x + \dfrac{\pi}{4}) = \sqrt{2}\cos(x - \dfrac{\pi}{4})$ |

3. Explore the harmonic addition

Plot the following functions by Desmos and find the function of the graphs shown on Desmos.

a. $y = \sqrt{3}\sin x + \cos x$

b. $y = 3\sin x + 4\cos x$

c. $y = a\sin x + b\cos x$, choose any a and b you like.

d. In general, share your findings on the result of the addition of the two functions addition.

使用建議

教學活動安排	讓學生多嘗試幾個例子觀察規律。
英文提問 / 開場	Let's explore more examples to find the patterns of the result of the harmonic addition. Use desmos to finish questions a to d. Then we'll discuss them in ten minutes. Anyone wants to share your findings?

4. Scan the QR code to learn about the harmonic addition theorem.

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