

## 根式的運算-2 最簡根式與有理化分母

# Simplest radical forms and rationalizing denominator

### ☆最簡根式

雙語使用參考範例：

(由教材 16  $\sqrt{6} = \sqrt{3 \times 2} = \sqrt{3} \times \sqrt{2}$  換成  $\sqrt{12} = \sqrt{3 \times 4} = \sqrt{3} \times \sqrt{4} = \sqrt{3} \times 2 = 2\sqrt{3}$  連結到最簡根式)

T : Which expression is equivalent to  $\sqrt{6}$ ?

S :  $\sqrt{3 \times 2}$  and  $\sqrt{3} \times \sqrt{2}$

(The square root of 3 times 2 and the square root of 3 times the square root of 2.)

T : Which expression is equivalent to  $\sqrt{12}$ ?

S :  $\sqrt{3 \times 4}$  and  $\sqrt{3} \times \sqrt{4}$

(The square root of 3 times 4 and the square root of 3 times the square root of 4.)

T : Please let me know if there are any other expressions that are also equivalent to  $\sqrt{12}$ .

等待學生討論與回答

T : Because  $\sqrt{4} = \sqrt{2} \times \sqrt{2} = (\sqrt{2})^2 = 2$ .

(Because the square root of 4 is equal to the square root of 2 times the square root of 2 is equal to the square root of 2 all to the power of 2 is equal to 2.)

So,  $\sqrt{12} = \sqrt{3 \times 4} = \sqrt{3} \times \sqrt{4} = \sqrt{3} \times 2 = 2\sqrt{3}$ .

(So, the square root of 12 is equal to the square root of 3 times the square root of 4 is equal to the square root of 3 times 2 is equal to 2 times the square root of 3.)

T : Let's find the equivalent expressions of  $\sqrt{18}$ .

(讓學生去思考 如果是 $\sqrt{18}$  拆成 $\sqrt{6 \times 3}$ 和 $\sqrt{9 \times 2}$  會有甚麼差異?)

$$S : \sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3 \times \sqrt{2} = 3\sqrt{2}.$$

(The square root of 18 is equal to the square root of 9 times the square root of 2  
is equal to the square root of 2 times 3  
is equal to 3 times the square root of 2.)

$$\sqrt{18} = \sqrt{6 \times 3} = \sqrt{6} \times \sqrt{3}.$$

(The square root of 18 is equal to the square root of 6 times the square root of 3.)

T : What is the difference between  $\sqrt{6 \times 3}$  and  $\sqrt{9 \times 2}$ ?

等待學生討論與回答後利用課本說明最簡根式

最簡根式為 $r\sqrt{n}$ ，其中：

$r$  是一個整數或分數，表示根式的係數。

$n$  是一個正整數，表示根式內的數字。

如果我們將  $n$  分解成標準的質因數分解式，其中每個質因數的次方皆為 1，則表示這個根式已經被化簡至最簡形式。

例如，如果  $n = 18$ ，其質因數分解為  $18 = 3^2 \times 2$ 。那麼最簡根式就可以表示為  $3\sqrt{2}$ ，因為根號內的質因數次方已經都是 1。

英文翻譯參考：

The simplest radical form is  $r\sqrt{n}$ , where:

$r$  is an integer or a fraction, representing the coefficient of the radical.

$n$  is a positive integer, representing the number within the radical.

If we factorize  $n$  into its standard prime factorization, with each prime factor having an exponent of 1, then this radical has been simplified to its simplest form.

For example, if  $n = 18$ , its prime factorization is  $18 = 3^2 \times 2$ .

In this case, the simplest radical form can be expressed as  $3\sqrt{2}$ , since the exponents of the prime factors inside the square root are already 1.

一個根式有下列任何一種情形時，就不是最簡根式：

- (1) 根號內的正整數，化成標準分解式後，有任何一個質因數的次方大於 1。
- (2) 根號內有分數或小數。
- (3) 分母有根式。

英文翻譯參考：

A radical expression is not in its simplest form if it falls into any of the following cases:

- (1) The positive integer **inside** the square root, when expressed in standard prime factorization form, has at least one prime factor raised to a power greater than 1.
- (2) There are fractions or decimals **inside** the square root.
- (3) The denominator contains a radical expression.

Q1. Which of the following radical expressions are in their simplest form?

$$\sqrt{3} \cdot 2\sqrt{3} \cdot \sqrt{18} \cdot \sqrt{19} \cdot \sqrt{\frac{3}{2}} \cdot \sqrt{3.2} \cdot \frac{2}{\sqrt{3}}$$

Q2. Simplify the following expressions into their simplest radical forms:

a.  $\sqrt{24}$

b.  $\sqrt{32}$

c.  $\sqrt{128}$

d.  $\sqrt{6} \times \sqrt{15}$

e.  $\frac{\sqrt{50}}{\sqrt{10}}$

f.  $\frac{\sqrt{20} \times \sqrt{5}}{\sqrt{2}}$

# ☆ 有理化分母 Rationalizing denominator

雙語使用參考範例：

(由 Q1  $\frac{2}{\sqrt{3}}$  連結到有理化分母)

T：我們都知道  $\frac{2}{\sqrt{3}}$  不是最簡根式，那你們有沒有想過可以利用一些根號的運算性質使得  $\frac{2}{\sqrt{3}}$  變成最簡根式呢？

We all know that  $\frac{2}{\sqrt{3}}$  is not in its simplest radical form. **Have you ever thought about using some operations of square root to simplify  $\frac{2}{\sqrt{3}}$  into the simplest radical form?**

T： $\frac{2}{\sqrt{3}}$  因為  $\sqrt{3}$  在分母所以不是最簡根式

Because  $\sqrt{3}$  is in the denominator of  $\frac{2}{\sqrt{3}}$ , it is not in its simplest radical form.

學生可能使用的方法：將  $\frac{2}{\sqrt{3}}$  平方

T：如果將  $\frac{2}{\sqrt{3}}$  平方， $(\frac{2}{\sqrt{3}})^2 = \frac{4}{3}$ ，根號消失了，但還是原本的  $\frac{2}{\sqrt{3}}$  嗎？

If we square  $\frac{2}{\sqrt{3}}$ ,  $(\frac{2}{\sqrt{3}})^2 = \frac{4}{3}$ , the square root disappears. However, is it still the same as  $\frac{2}{\sqrt{3}}$ ?

需要保持值不變 → 分子分母同乘一個數 → 分子分母同乘一個根號的數

**Multiply Both Numerator and denominator by a Root**

T： $\frac{2}{\sqrt{3}}$  分子分母同時乘上哪一個根號的數會使得分母沒有根號？

Which square root value should be multiplied to both the numerator and the

denominator of  $\frac{2}{\sqrt{3}}$  to eliminate the radical from the denominator?

$$S : \sqrt{3}$$

$$T : \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

T : 這個步驟即為使分母有理化 (被稱為「有理化分母」)。

This step is the process of rationalizing the denominator (referred to as "denominator rationalization").

Q3. Please rationalizing the denominator of the following radical expressions.

a.  $\frac{1}{\sqrt{24}}$

b.  $\frac{50}{\sqrt{10}}$

c.  $\frac{\sqrt{20} \times \sqrt{5}}{\sqrt{2}}$

## 參考資料來源

1. 111 國中數學翰林版課本
2. Into Math Advanced2
3. <https://byjus.com/maths/radical/>

☆老師們可以自己從中選擇以做出适合自己學生程度的學習單或是在課堂中適時補充這些英文。

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