雙語教學主題(國中九年級上學期教材): 縮放與多邊形相似
Topic: introducing dilation and polygon similarity

Vocabulary
Similar, polygon, corresponding sides, corresponding angles, dilate, shrink, preserved, rigid transformation, proportional sides, congruent angles, preserved, coordinates, measure,

Let's watch some videos for the basic concepts of dilation and similarity, And we have a discussion afterward.

A. Dilation— basic concept
https://youtu.be/y3NdTcEgGlk
After watching the video, let's answer the questions below and ~~maybe do~~ some discussion.
Questions:
1. When you dilate a figure, will the shape remain the same?

   ____

   (YES)
2. This video mentions scale factor "k", what do you get if you dilate a shape with $0<k<1$?

   ____

   (THE FIGURE SHRINKS)
3. Is the measure of an angle or the measure of a side length always the same on a dilated figure?

   ____

   (THE SIDE LENGTH IS CHANGED, BUT THE MEASURE OF THE ANGLE REMAINS THE SAME)
4. What is the relation between the side lengths in an original figure and a dilated figure?

   ____

   (THEY ARE PROPORTIONAL)
5. What is the difference between rigid transformation and non-rigid transformation?

   ____

   (FOR RIGID TRANSFORMATION, ALL THE MEASURES OF ANGLES AND SIDE LENGTHS REMAIN THE SAME, BUT NON-RIGID TRANSFORMATION DOESN'T)

6. Dilated figures are not congruent, they are __________.

(SIMILAR)

B. Quadrilateral dilation and preservation

https://youtu.be/eu4qSv9Vex0

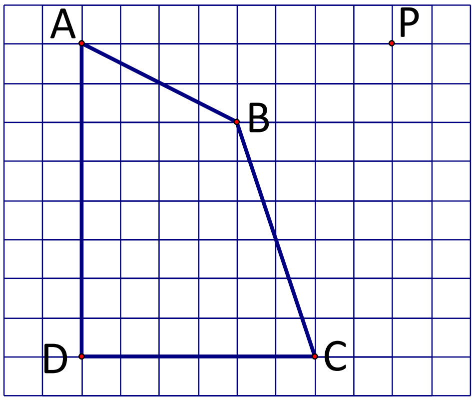
Worksheet:

Quadrilateral ABCD is dilated about point P.

Please prepare a ruler, a protractor, and pens of course.

Choose your scale factor: k= ____. And complete your work on the chart below.

(Because of the space we have here, I recommend that you choose a scale factor k that is smaller than 1, as it will be easier to work with.)



Please measure all the angles and all the side lengths.

Side length: $\overline{AB}$ = ________ $\overline{BC}$ = ________ $\overline{CD}$ = ________ $\overline{DA}$ = ________

$\overline{A'B'}$ = ________ $\overline{B'C'}$ = ________ $\overline{C'D'}$ = ________ $\overline{D'A'}$ = ________

The ratio of the corresponding side lengths:

$\frac{\overline{A'B'}}{\overline{AB}}$ = ______ $\frac{\overline{B'C'}}{\overline{BC}}$ = ______ $\frac{\overline{C'D'}}{\overline{CD}}$ = ______ $\frac{\overline{D'A'}}{\overline{DA}}$ = ______

Angle measurement:

m∠A=_____, m∠B=_____, m∠C=_____, m∠D=_____,

m∠A'=_____, m∠B'=_____, m∠C'=_____, m∠D'=_____,

Questions:

1. Are the corresponding line segments on the same line?

____

(NOT ALWAYS)

2. Are the measures of all angles preserved?

____

(YES)

3. Are side lengths and perimeter preserved?

____

(NO)

4. What are the relations between corresponding side lengths?

   ____

   (THEY DON'T HAVE THE SAME LENGTHS ANYMORE, BUT THEY ARE PROPORTIONAL.)

5. Is the area preserved?

   ____

   (NO)

We can see that a figure and its dilated figure are similar, and their corresponding angles are congruent, but their corresponding side lengths are not necessarily the same

The corresponding side lengths are proportional.

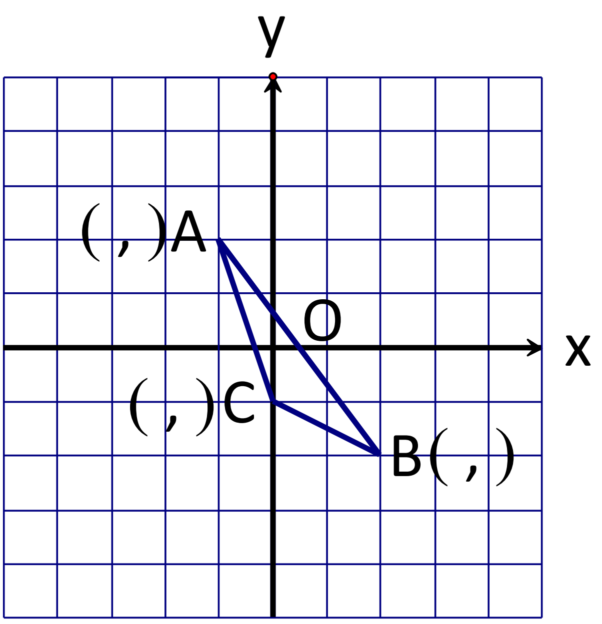
Let's work on some more examples.

Ex:

Look at the chart below. Please fill in the coordinates of three vertices.

Triangle ABC is dilated about point B. Let's set the scale factor k= $\frac{3}{2}$

Please draw your dilated triangle.



Do the coordinates of three vertices remain the same after dilation of triangle ABC?

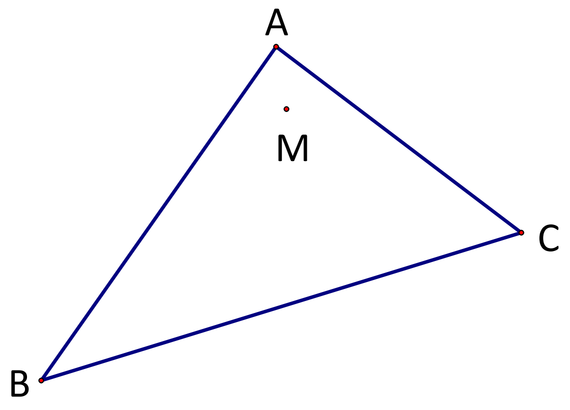
(NO)

C. Construct a dilation

[https://youtu.be/WCXN4qzrBTohttps://youtu.be/WCXN4qzrBTo](https://youtu.be/WCXN4qzrBTo)

Worksheet:

Please dilate triangle ABC about point M. Set your scale factor: K= _______



After the above discussion, we have an overall basic understanding of the dilation of shapes.

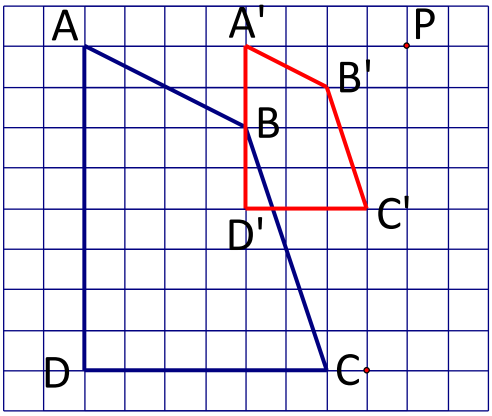
Conclusions:

1. Dilation is a non-rigid transformation
2. All measures of corresponding angles are preserved among dilated shapes.
3. All corresponding line segments are proportional.

And we say dilated polygons are **similar**.

For instance, let's look at the results from part B. Quadrilateral is dilated about point P. The scale factor k= 1/2

We get a graph like this:



Compare quadrilateral ABCD and quadrilateral A'B'C'D'

We get:

1. Dilation is a non-rigid transformation
2. All measures of corresponding angles are preserved among dilated shapes.
3. All corresponding line segments are proportional.

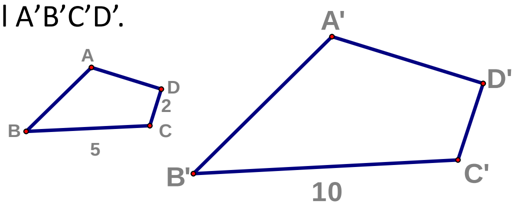
And we say dilated quadrilaterals are **similar**. When two polygons are similar, we denote it with the sign "∼". When we say △ABC is similar to △DEF, we express it as: △ABC∼△DEF.

Let’s do an example here.

Given quadrilateral ABCD～quadrilateral A’B’C’D’.

Points A, B, C, D are corresponding to points A’, B’, C’, D’ respectively.

Please answer the questions:



(1) if $\overline{BC}$=5, $\overline{CD}$=2, and $\overline{B'C'}$=10, then $\overline{C'D'}$=?

(2) If the perimeter of quadrilateral ABCD is 14, then calculate the perimeter of quadrilateral A’B’C’D’.

(3) If $\angle$A=120° , $\angle$B:$\angle$C:$\angle$D=1:7:4 , then what is the measure of $\angle$B’?

Solution:

(1) since the side lengths are proportional between similar shapes, we have

$\frac{\overline{B'C'}}{\overline{BC}}=\frac{\overline{C'D'}}{\overline{CD}}$ (line segment B’C’ over line segment BC is equal to line segment C’D’ over line segment CD)

Replace the given information into the equation above, we get

$$\frac{10}{5}=\frac{\overline{C'D'}}{2}$$

Then $\overline{C'D'}$=4#

(2) quadrilateral ABCD～quadrilateral A’B’C’D’

we know $\frac{\overline{A'B'}}{\overline{AB}}=\frac{\overline{B'C'}}{\overline{BC}}=\frac{\overline{C'D'}}{\overline{CD}}=\frac{\overline{D'A'}}{\overline{DA}}$=2

$\Rightarrow$ $\overline{A'B'}=2\overline{AB}$, $\overline{B'C'}=2\overline{BC}$, $\overline{C'D'}=2\overline{CD}$, and $\overline{D'A'}=2\overline{DA}$ ……(a)

the perimeter of quadrilateral A’B’C’D’

$=\overline{A'B'}+\overline{B'C'}+\overline{C'D'}+\overline{D'A'}$

$=2\overline{AB}+2\overline{BC}+2\overline{CD}+2\overline{DA}$ …… (from (a))

$=2(\overline{AB}+\overline{BC}+\overline{CD}+\overline{DA})$

=2x(the perimeter of quadrilateral ABCD)

=2x14

=28#

(3) the sum of the four interior angles of a quadrilateral is always 360°.

i.e. $\angle A+\angle B+\angle C+\angle D=360°$

given $\angle A=120°$, then $\angle B+\angle C+\angle D=240°$

given $\angle B:\angle C:\angle D=1:7:4$

then the measure of $\angle B'$= the measure of $\angle B$

(we sometimes denote it as $m\angle B'=m\angle B$)

(Corresponding angles are congruent between similar quadrilaterals)

So $m\angle B'=m\angle B$

$$=240°\bullet\frac{1}{1+7+4}$$

$$=240°\bullet\frac{1}{12}$$

$$=20°$$ #

Hope you all get an understanding of polygon similarity from the introduction above. We will focus on discussing triangle similarity which is the most important issue among all polygon similarities to ~~we~~ high school students.

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