

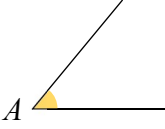

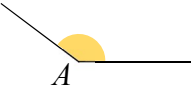


# 多邊形的內角

## The interior angles of a polygon

Class: \_\_\_\_\_ Name: \_\_\_\_\_

### 1. Types of angles(角的分類)

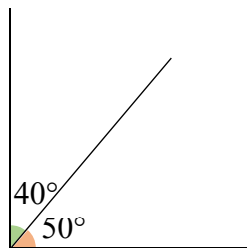
Let's review some names and relations among angles we have learned before. We have learned these 5 kinds of angles before. They are classified according to their measures.

acute angle 銳角	right angle 直角	obtuse angle 鈍角	straight angle 平角	full rotation angle 周角
				
$0^\circ < \angle A < 90^\circ$	$\angle A = 90^\circ$	$90^\circ < \angle A < 180^\circ$	$\angle A = 180^\circ$	$\angle A = 360^\circ$

### 2. Relationship between two angles(兩角關係)

#### (1) complementary angle(餘角)

Two angles are called complementary angles if the sum of their measures is  $90^\circ$ . Each angle is called the complement of the other. For example, angles of  $50^\circ$  and  $40^\circ$  are complementary.

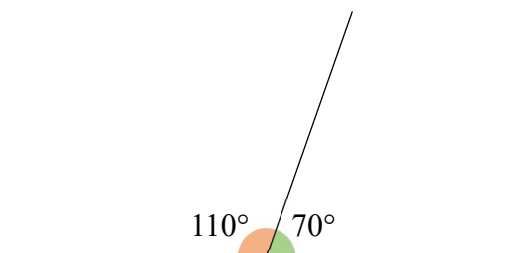


#### Exercise 1.

If  $\angle 1$  and  $\angle 2$  are complementary and  $\angle 1 = 52^\circ$ . What is the measure of  $\angle 2$ ?

#### (2) supplementary angle(補角)

Two angles are called supplementary angles if the sum of their measures is  $180^\circ$ . Each angle is called the supplement of the other. For example, angles of  $70^\circ$  and  $110^\circ$  are supplementary.

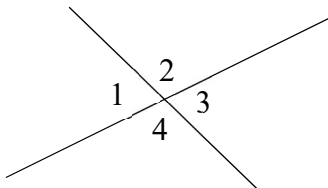


Exercise 2.

If  $\angle 1$  and  $\angle 2$  are supplementary and  $\angle 1 = 123^\circ$ . What is the measure of  $\angle 2$ ?

(3) vertical angle(對頂角)

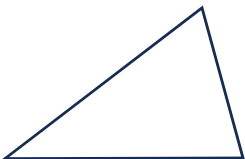
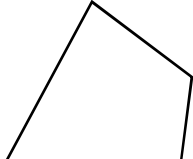
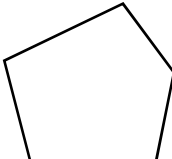
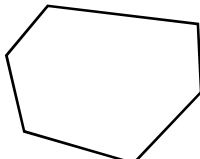
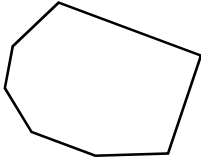
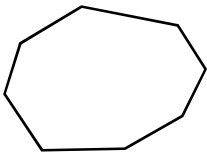
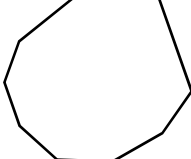
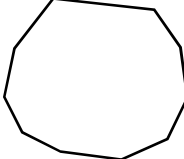
When two lines cross, the angles opposite each other are called vertical angles. In the following figure,  $\angle 1$  and  $\angle 3$  are one pair of vertical angles.  $\angle 2$  and  $\angle 4$  are the other pair of vertical angles.  $\angle 1 + \angle 2 = 180^\circ$  and  $\angle 3 + \angle 2 = 180^\circ$ , so  $\angle 1 = \angle 3$ . Similarly,  $\angle 2 = \angle 4$ .



★ Vertical angles are congruent.

3. Introduction to polygons

We have to learn the names of the following polygons because we will keep seeing these words in this chapter.

triangle	quadrilateral	pentagon	hexagon
			
heptagon	octagon	nonagon	decagon
			

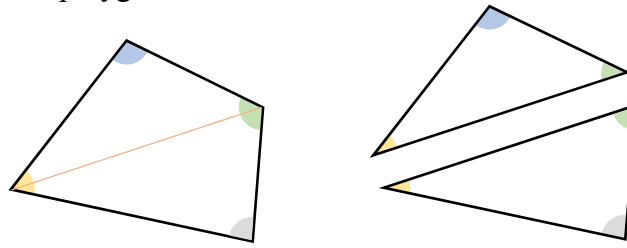
4. The sum of the interior angles of a triangle(三角形的內角和)

The sum of the interior angles of a triangle is  $180^\circ$ . In  $\triangle ABC$ ,  $\angle A + \angle B + \angle C = 180^\circ$ .

Exercise 3. In  $\triangle ABC$ ,  $\angle A = 52^\circ$ .  $\angle B = 75^\circ$ . Then,  $\angle C = ?$

5. The sum of the interior angles of a polygon(多邊形的內角和)

We can divide the quadrilateral into two triangles by connecting one diagonal, each of which has interior angles that add up to  $180^\circ$ . Therefore, we can use this method to find the sum of the interior angles of any convex polygons with  $n$  sides.



Draw all diagonals from one vertex and divide the polygons shown below into triangles. Then, complete the following table.

polygon	pentagon	hexagon	heptagon	octagon
shape				
number of sides				
numbers of triangles				
sum of the interior angles				

We can find that an  $n$ -gon can be divided into  $(n - 2)$  triangles. Therefore, the sum of the interior angles of an  $n$ -gon is  $(n - 2) \times 180^\circ$ .

Example 4.

Find the sum of the interior angles of a decagon (10-sided polygon).

All interior angles are equal in a regular polygon, so it is easy to find the measure of each interior angle in it.

Example 5.

Find the measure of an interior angle in a regular nonagon (9-sided polygon).

### 一、設計理念：

1. 本份學習單主要介紹三角形及多邊形的內角和，但由於課本中此章節亦須介紹角與兩角關係，故亦至於此學習單中。
2. 國外的教材在角度分類的部分，會介紹介於 180 度至 360 度之間的角稱為「reflex angle」，中文譯為優角。
3. 周角除使用「full rotation angle」外，亦有人使用「complete angle」一詞。
4. 「complement」來自於「complete」，因為兩個角互餘後會「完成」一個直角。
5. 多邊形的英文源自古希臘數字字首，因此變化不規則。有時亦可用其他方式表示多邊形，例如十七邊形可稱為「17-gon」或「17-sided polygon」。

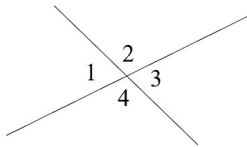
### 二、英文詞彙：

中文	英文
角	angle
銳角	acute angle
直角	right angle
鈍角	obtuse angle
平角	straight angle
周角	full rotation angle
餘角	complementary angle
補角	supplementary angle
對頂角	vertical angle
內角	interior angle
多邊形	polygon
三角形	triangle
四邊形	quadrilateral
五邊形	pentagon
六邊形	hexagon
七邊形	heptagon
八邊形	octagon
九邊形	nonagon
十邊形	decagon

### 三、數學英文用法：

數學表示法	英文
$90^\circ$	90 degrees
$\angle A$	angle A
$\angle A$ 和 $\angle B$ 互餘	Angle A and angle B are complementary.
$\angle A$ 和 $\angle B$ 互補	Angle A and angle B are supplementary.
$\angle A$ 和 $\angle B$ 為一組對頂角	Angle A and angle B are vertical angles.

四、教學參考範例：

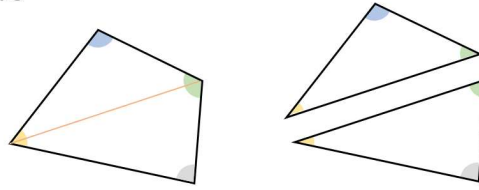
<p>1 【對頂角】 Vertical angle</p>	<p>(3) vertical angle(對頂角)</p> <p>When two lines cross, the angles opposite each other are called vertical angles. In the following figure, <math>\angle 1</math> and <math>\angle 3</math> are one pair of vertical angles. <math>\angle 2</math> and <math>\angle 4</math> are the other pair of vertical angles. <math>\angle 1 + \angle 2 = 180^\circ</math> and <math>\angle 3 + \angle 2 = 180^\circ</math>, so <math>\angle 1 = \angle 3</math>. Similarly, <math>\angle 2 = \angle 4</math>.</p>  <div style="border: 1px solid black; padding: 2px; margin-top: 10px;"><p>★ Vertical angles are congruent.</p></div>
	<p>When two lines cross, they form four angles. We mark these four angles as angle 1, angle 2, angle 3, and angle 4. Angle 1 and angle 3 are opposite each other, and we call them vertical angles. Similarly, angle 2 and angle 4 are also vertical angles.</p> <p>Next, we want to find the relationship between vertical angles. If angle 1 is 70 degrees, can you find the measure of the other three angles? Because angle 1 and angle 2 form a straight angle, so angle 2 equals 180 degrees minus 70 degrees, which is 110 degrees. Angle 1 and angle 4 also form a straight angle so angle 4 is also 110 degrees. Lastly, angle 2 and angle 3 form a straight angle, so angle 3 equals 180 degrees minus 110 degrees, which equals 70 degrees. From this example, we find angle 1 and angle 3 are vertical angles, and they are both 70 degrees. Angle 2 and angle 4 are vertical angles, and they are both 110 degrees.</p> <p>Vertical angles must be congruent, and we can prove this in general. We take angle 1 and angle 3 as example. Angle 1 plus angle 2 equals 180 degrees, and angle 3 plus angle 2 equals 180 degrees. Therefore, we have angle 1 plus angle 2 equals angle 3 plus angle 2. Subtract angle 2 on both sides, then we have angle 1 equals angle 3.</p> <p>In conclusion, vertical angles are congruent.</p>

must

2  
【多邊形內角和】  
The sum of the interior angles of a polygon

5. The sum of the interior angles of a polygon(多邊形的內角和)

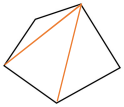
We can divide the quadrilateral into two triangles by connected one diagonal, each of which has interior angles that add up to  $180^\circ$ . Therefore, we can use this method to find the sum of the interior angles of any convex polygons with  $n$  sides.



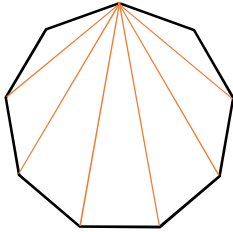
Draw all diagonals from one vertex and divide polygons shown below into triangles. Then, complete the following table.

We already know the sum of the interior angles of a triangle is 180 degrees. Next, we want to find the sum of the interior angles of a polygon. Let's start with an easy polygon – quadrilateral. Draw a diagonal of a quadrilateral, which can divide it into two triangles. A triangle has interior angles that add up to 180 degrees, so these two triangles have interior angles that add up to 360 degrees. Therefore, the sum of the interior angles of a quadrilateral is 360 degrees.

Next, we want to use the same method to find the sum of the interior angles of a polygon. We draw all diagonals from one vertex and divide a polygon into triangles.

polygon	pentagon
shape	
number of sides	5
numbers of triangles	3
sum of the interior angles	$540^\circ$

Let's find the sum of the interior angles of a pentagon. First, a pentagon is a 5-sided polygon, so the number of sides is five. Next, we draw all diagonals from the top vertex. There are 2 diagonals and they divide a pentagon into 3 triangles. The sum of the interior angles of a triangle is 180 degrees, so the sum of the interior angles of a pentagon equals 180 degrees times 3, which equals 540 degrees.

<p>3</p> <p>【正多邊形的內角】</p> <p>The interior angle of a regular polygon</p>	<p>Example 5.</p> <p>Find the measure of an interior angle in a regular nonagon (9-sided polygon).</p>
	
	<p>A regular polygon is a polygon in which all sides and interior angles are equal. Therefore, in a regular nonagon, we can find the sum of all interior angles. Then, because each interior angle is equal, we divide the sum by nine, and we have the measure of an interior angle in nonagon.</p> <p>Let's start our calculation. Drawing all diagonals from one vertex, we divide a nonagon into seven triangles. Each triangle has interior angles that add up to 180 degrees, so the sum of interior angles of a nonagon equals 180 degrees times 7, which equals 1260 degrees.</p> <p>Because the measure of nine interior angles in a nonagon are equal, we divide 1260 by 9 and get an interior angle of a nonagon is 140 degrees.</p>

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