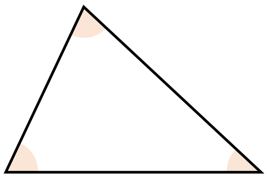
三角形的外角
The exterior angles in a triangle

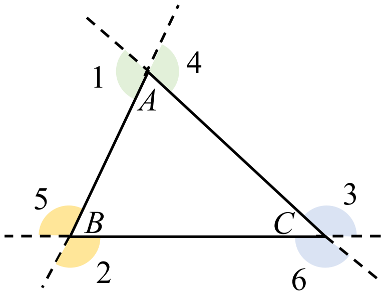
Class:__________ Name: __________

1. Interior angle and exterior angle(內角與外角)

The interior angles of a triangle are three original angles.



The exterior angle is the angle between any side of a triangle and an extended adjacent side.



$\angle 1$ and $\angle 4$ are the exterior angles of $\angle BAC$.

$\angle 2$ and $\angle 5$ are the exterior angles of $\angle ABC$.

$\angle 3$ and $\angle 6$ are the exterior angles of $\angle ACB$.

2. Exterior angle theorem(外角定理)

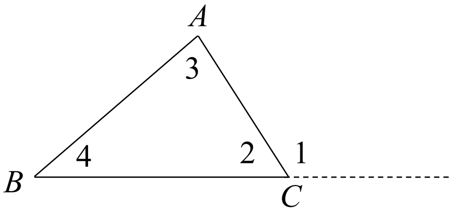
Let's find the relationship between interior angles and exterior angles.

$\angle 1 + \angle 2 = 180°$($\angle 1$ and $\angle 2$ are supplementary.)

$\angle 2 + \angle 3 + \angle 4 = 180°$(The sum of the interior angles of a triangle is 180°)

⇨ $\angle 1 + \angle 2 = \angle 2 + \angle 3 + \angle 4$

⇨ $\angle 1 = \angle 3 + \angle 4$

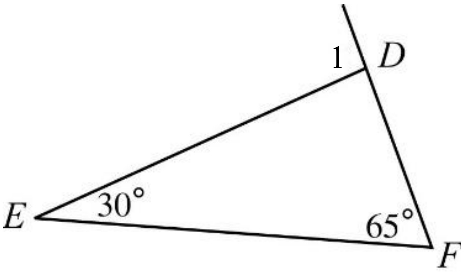


★ Exterior angle theorem:

The measure of the exterior angle of a triangle is equal to the sum of the two remote angles, which are nonadjacent interior angles.

Example 1

Find the measure of $\angle 1$ in the figure.



[Solution]

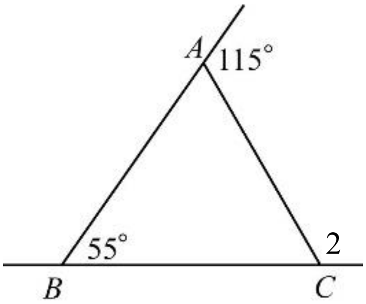
$\angle 1$ is the exterior angle of $\triangle DEF$.

We can apply the exterior angle theorem.

$\angle 1 = 30° + 65° = 95°$

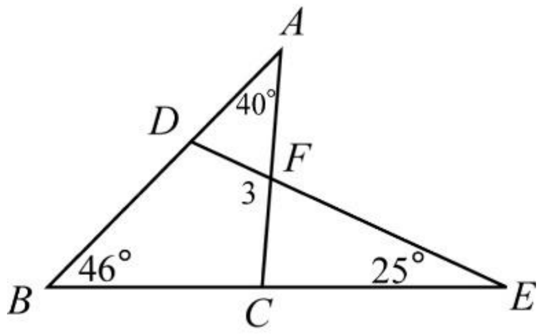
Exercise 1

Find the measure of $\angle 2$ in the figure.



Example 2

Find the measure of $\angle 3$ in the figure.

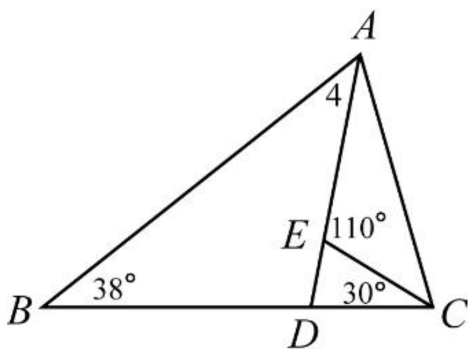


[Solution]

> $\angle ACE$ is the exterior angle of $\triangle ABC$.
> We can apply the exterior angle theorem.
> $\angle ACE = 40° + 46° = 86°$
> Similarly, $\angle 3$ is the exterior angle of $\triangle CEF$.
> Apply the exterior angle theorem again.
> $\angle 3 = 86° + 25° = 111°$

Exercise 2

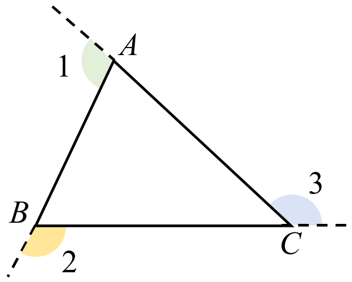
Find the measure of $\angle 4$ in the figure.



3. The sum of exterior angles of a triangle (三角形的外角和)

We know the sum of the interior angles of a triangle is 180°. Next, we are going to find the sum of the exterior angles of a triangle.

[Method 1]



In $\triangle ABC$,

$\angle BAC + \angle 1 = 180°$……(1)

$\angle ABC + \angle 2 = 180°$……(2)

$\angle ACB + \angle 3 = 180°$……(3)

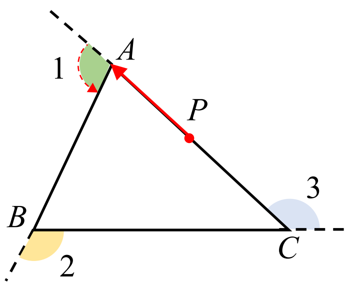
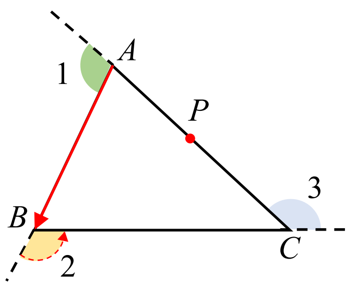
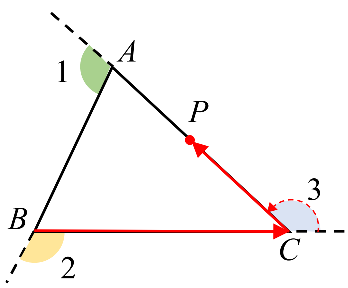
From (1)+(2)+(3), we get $(\angle BAC + \angle ABC + \angle ACB) + (\angle 1 + \angle 2 + \angle 3) = 540°$

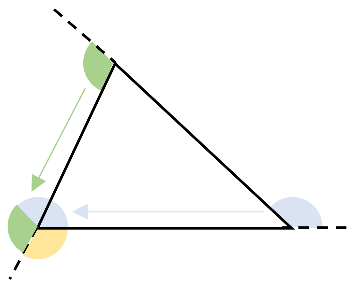
The sum of the interior angle of a triangle is 180°. That is, $\angle BAC + \angle ABC + \angle ACB = 180°$.

Therefore, $\angle 1 + \angle 2 + \angle 3 = 360°$.

[Method 2]star

Tom walks on the sides of a triangular park. He walks around the park stating from point P, goes counterclockwise, and comes back to P.

Tom walks from P to A and turns to face point B. The turning angle is $\angle 1$.	Tom walks from A to B and turns to face point C. The turning angle is $\angle 2$.	Tom walks from B to C and turns to face A. The turning angle is $\angle 3$. He goes back to P in the end.
		



Switch the green angle ($\angle 1$) and blue angle ($\angle 3$) to the lower left corner. We can find these three angles form a full rotation angle. Therefore, we have $\angle 1 + \angle 2 + \angle 3 = 360°$.

一、設計理念：

1. 延續前一份學習單教多邊形的內角，本份學習單主要介紹外角定理及三角形的外角和。
2. 多邊形的外角和因為在 108 課綱中屬於補充教材，故未列於本份學習單中。

二、英文詞彙：

中文	英文
角	angle
補角	supplementary angle
內角	interior angle
外角	exterior angle
多邊形	polygon
三角形	triangle

三、數學英文用法：

數學表示法	英文
$90°$	90 degrees
$\angle A$	angle A
$\angle A$ 和 $\angle B$ 互補	Angle A and angle B are supplementary.

四、教學參考範例：

<table>
<tr>
<td rowspan="2">1
【外角定理】
Exterior angle theorem</td>
<td>2. Exterior angle theorem(外角定理)
Let’s find the relationship between interior angles and exterior angles.

$\angle 1+\angle 2=180°$($\angle 1$ and $\angle 2$ are supplementary.)
$\angle 2+\angle 3+\angle 4=180°$(The sum of the interior angles of a triangle is 180°)
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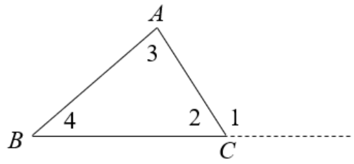

★ Exterior angle theorem:
The measure of the exterior angle of a triangle is equal to the sum of the two remote angles, which are nonadjacent interior angles.</td>
</tr>
<tr>
<td>We just learned the interior angles and exterior angles of a triangle. Next, we want to find the relationship between interior angles and exterior angles.
Here is a triangle ABC, and we draw the exterior angle of angle C. Angle 1 and angle 2 form a straight angle, so angle 1 plus angle 2 equals 180 degrees.
On the other hand, we know the sum of the interior angles of a triangle is 180 degrees, so angle 2 plus angle 3 plus angle 4 equals 180 degrees.
Both these two sums equal 180 degrees. Therefore, angle 1 plus angle 2 equals angle 2 plus angle 3 plus angle 4. Subtract angle 2 on both sides. We have angle 1 equals angle 3 plus angle 4.
Angle 3 and angle 4 are the interior angles of triangle ABC, which are not adjacent to angle 1. These two angles are called “remote angles” of angle 1. From the process above, we get the measure of the exterior angle of a triangle is equal to the sum of the two remote angles. This conclusion is called the exterior angle theorem.</td>
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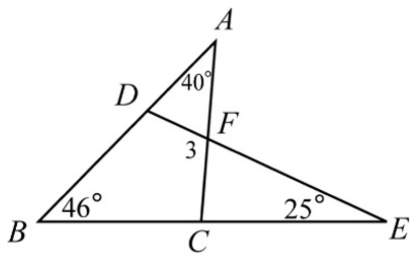
【Example 2】</td>
<td>Example 2
Find the measure of $\angle 3$ in the figure.


[Solution]
$\angle ACE$ is the exterior angle of $\triangle ABC$.
We can apply the exterior angle theorem.
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Similarly, $\angle 3$ is the exterior angle of $\triangle CEF$.
Apply the exterior angle theorem again.
$\angle 3 = 86° + 25° = 111°$</td>
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<td>Let’s see another example. In this figure, we want to find the measure of angle 3.
There are many triangles in the figure, such as triangle ADF, triangle ABC, triangle BDE, and triangle FCE.
Our target is to find the measure of angle 3, which is the exterior angle of triangle ADF and triangle CFE. From the exterior angle theorem, angle 3 equals angle E plus angle FCE, and we have angle E equals 25 degrees, so we can get the answer if we find the measure of angle FCE.
On the other hand, because we know the two interior angles of triangles ABC and BDE, it is easier to start with these two triangles. In triangle ABC, Angle ACE is the exterior angle, and according to the exterior angle theorem, angle ACE equals angle A plus angle B, which equals 40 degrees plus 46 degrees. Therefore, the angle ACE equals 86 degrees.
We know angle ACE, or we can say angle an~~gle~~ FCE equals 86 degrees. Apply the exterior angle theorem again, and we get angle 3 equals 25 degrees plus 86 degrees, which equals 111 degrees.</td>
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