畢氏定理

The Pythagorean Theorem

雙語使用參考範例：

T:大家還記得我們上一章節學的根式運算嗎？今天，我們要學習一個很重要數學定理，它被稱為畢氏定理。這個定理將幫助我們解決許多直角三角形相關的問題。
T: Do you remember the square root operations we learned in the previous chapter? Today, we will learn an important mathematical theorem known as the Pythagorean Theorem. This theorem will help us solve various problems related to right angle triangles.

T:首先，我們來看一個生活實例。大家知道電視機的尺寸代表甚麼意思嗎？
First, let's consider a real-life example. Does everyone know what the size of a TV represents?

T:電視機的尺寸通常代表了電視螢幕的對角線長度，以英寸（inches）為單位表示。這是一個用來描述電視螢幕大小的標準測量。對角線長度是從電視螢幕的左下角到右上角的距離，它是電視螢幕對角線的長度，而不是螢幕的實際寬度或高度。
例如，當人們說一台電視是 "50 吋" 時，他們指的是電視螢幕的對角線長度為 50 英寸。這個尺寸通常用來比較不同電視機的大小，因為它提供了一個清晰的指標，使人們能夠快速瞭解電視螢幕的相對大小。
T: TV sizes typically represent the diagonal length of the TV screen, measured in inches. This is a standard measurement used to describe the size of a TV screen. The diagonal length is the distance from the lower-left corner to the upper-right corner of the TV screen, and it represents the diagonal length of the TV screen, not its actual width or height.
For example, when people say a TV is "50 inches," they are referring to the TV screen's diagonal length, which measures 50 inches. This measurement is commonly used to compare the sizes of different TVs because it provides a clear indicator for people to quickly understand the relative size of the TV screen.

T:假設你知道一個電視機的長度、寬度。則該如何不透過測量就能知道對角線的長度呢? (即電視的尺寸)
If you know the length and width of a television, how can you determine the length of the diagonal, without measuring it directly?

T:例如你已經知道電視機的長度（a）為 40 英寸，寬度（b）為 30 英寸，那麼，你如何計算對角線的尺寸？
T: For example, a television has a length of 40 inches and a width of 30 inches. How can you find the length of the diagonal?

讓學生透過實際測量和觀察，自己發現畢氏定理。
給學生多個直角三角形的圖，要求他們觀察並測量每條邊的長度。
學生在測量後，討論他們的觀察結果。問他們是否發現了某種模式或關係。
學生可能會提出他們的觀察，例如直角三角形的兩條邊的平方和是否等於斜邊的平方。

T:我們將測量三個不同的三角形，（分別是邊長 3、4、5 的三角形、邊長 5、12、13 的三角形和邊長 6、8、10 的三角形。）請先測量每個三角形的邊長。
T: We will measure three different triangles, (triangles with side lengths of 3, 4, 5, 5, 12, 13, and 6, 8, 10, respectively). Please measure the side lengths of each triangle.
（學生們分別測量三個三角形的邊長）

T:好，現在我們已經測量了這三個三角形的邊長。接下來，我們要進行一些數學運算，看看是否有什麼有趣的發現。
T: Now we've measured the side lengths of these three triangles. Let’s perform some operations on these numbers to see if we can make any interesting discoveries."

（讓學生自行討論再給予提示）

T:請先計算第一個三角形的邊長的平方：3 的平方、4 的平方和 5 的平方。
T: First, please calculate the squares of the side lengths of the first triangle: the square of 3, the square of 4, and the square of 5.

S: 3 的平方是 9，4 的平方是 16，5 的平方是 25。
S: The square of 3 is 9, the square of 4 is 16, and the square of 5 is 25.

T:現在，讓我們繼續計算第二個三角形（5、12、13）的平方，然後再計算第三個三角形（6、8、10）的平方。我們的目標是找到一些模式或規律。
T: Now, let's proceed to calculate the squares of the side lengths of the second triangle (5, 12, 13), and then calculate the squares of the side lengths of the third triangle (6, 8, 10). Our goal is to identify if there are any patterns.

（學生們分別計算第二和第三個三角形的平方）

S:第二個三角形
5 的平方是 25
12 的平方 144
13 的平方是 169。

S: In the second triangle, the square of 5 is 25, the square of 12 is 144, and the square of 13 is 169.

S:第三個三角形
6 的平方是 36
8 的平方是 64
10 的平方是 100。

S: In the third triangle, the square of 6 is 36, the square of 8 is 64, and the square of 10 is 100.

T:讓我們來比較這些數字的個別平方，看看是否有什麼特別之處。你們有沒有發現什麼有趣的地方？
T: Let's compare these individual squared values and see if there's anything special.
T: Did any of you notice anything interesting?

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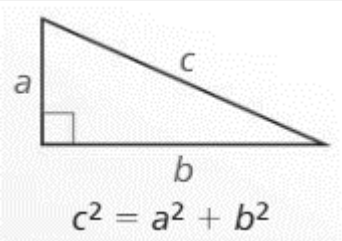
「直角三角形的兩條邊的平方和等於斜邊的平方。

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



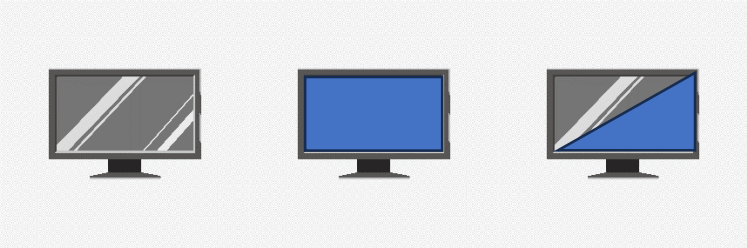
a 和 b 是直角三角形較短的兩條邊的長度。

• a and b are the lengths of the two legs in a right triangle.

c 是直角三角形斜邊的長度。

• c is the length of the hypotenuse in a right triangle.

(A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.)



T:我們可以將電視螢幕視為一個長方形，對角線和其長和寬則可形成一個直角三角形，那麼我們就能應用畢氏定理了

T: We can let the TV screen be a rectangle, and the diagonal, along with its length and width, can form a right-angled triangle, allowing us to apply the Pythagorean theorem.

T:現在，讓我們用畢氏定理解決測量電視機尺寸的問題。假設你已經知道電視機的長度（a）為 40 英寸，寬度（b）為 30 英寸，那麼，你如何計算對角線的尺寸？

T: Now, let's use the Pythagorean Theorem to Solve the problem of measuring the dimension of a TV. Suppose you already know that the length (a) of the television is 40 inches and the width (b) is 30 inches. How would you calculate the diagonal size?

T: 根據畢氏定理， $a^2 + b^2 = c^2$，所以 $40^2 + 30^2 = c^2$。接下來，我們可以計算 c。
T: According to the Pythagorean Theorem, $a^2 + b^2 = c^2$, so $40^2 + 30^2 = c^2$. Now, we can calculate c.

T: 1600 + 900 = 2500。
$c^2 = 2500$ •
$c = \pm\sqrt{2500} = \pm 50$ (Choose positive)

T: 電視機的對角線尺寸是 50 英寸。
T: The diagonal size of the television is 50 inches.

T:現在，讓我們試一些練習題，以確保大家真正理解畢氏定理的應用。
T: Now, let's do some practice to ensure that everyone truly understands the application of the Pythagorean Theorem.

問題 1：一個電視機的長度為 24 英寸，對角線的尺寸為 30 英寸，求寬度。
Question 1: A television has a length of 24 inches, and the diagonal size is 30 inches. Find the width.

問題 2：一個長方形的長度為 10 英尺，寬度為 6 英尺，求對角線的長度。
Question 2: A rectangular piece has a length of 10 feet and a width of 6 feet. Find the length of the diagonal.

Exercise 1: In a right triangle, one of the legs has a length of 3 cm, and the other leg is 4 cm long. Calculate the length of the hypotenuse.

Exercise 2: The length of a rectangular table is 6 feet, and the width is 8 feet. Calculate the length of the diagonal.

Exercise 3: In a right triangle, one of the legs has a length of 7 cm, and the hypotenuse is 25 cm long. Calculate the length of the other leg.

Exercise 4: In a right triangle, one of the legs has a length of 5 m, and the other leg is 12 m long. Calculate the length of the hypotenuse.

Exercise 5: In a right triangle, one of the legs is 9 inches long, and the hypotenuse is 15 inches long. Calculate the length of the other leg.

Exercise 6: In a right triangle, the lengths of the two legs are 10 cm and 24 cm, respectively. Calculate the length of the hypotenuse.

參考資料來源

1. 111 國中數學翰林版課本
2. Into Math Advanced2
3. https://byjus.com/maths/radical/

☆老師們可以自己從中選擇以做出適合自己學生程度的學習單或是在課堂中適時補充這些英文。

製作者:康橋國際學校 陳怡伶