雙語教學主題(國中九年級上學期教材): 相似三角形的應用問題

Topic: Applications of triangle similarity theorems and word problems solving

Vocabulary

similar, similarity, polygon, corresponding sides, corresponding angles, proportional sides, congruent angles, corresponding heights, equilateral triangle, pentagon, shadow, cast, hockey, bouncing, the angle of incidence, the angle of reflection, approximate, estimate, visually, align, angle of depression, angle of elevation, simplifying, cross multiplying, isosceles triangle, bisect, bisector, intersect,

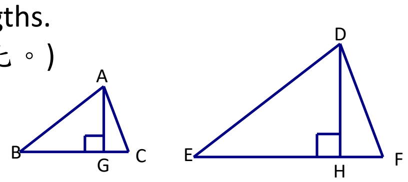
CPCTC: corresponding parts of congruent triangles are congruent

After we have learned so much about triangle similarity, we are going to explore some of the very useful extended similarity properties.

Property 1:

The ratio of corresponding heights among similar triangles is equal to the ratio of their corresponding side lengths.

(兩相似三角形中，對應高的比等於其對應邊的比。)



As shown, $\triangle ABC \sim \triangle DEF$,

$\overline{AG} \perp \overline{BC}$ and $\overline{AG}$ intersects $\overline{BC}$ at point G,

$\overline{DH} \perp \overline{EF}$ and $\overline{DH}$ intersects $\overline{EF}$ at point H.

Then

$$\frac{\overline{AG}}{\overline{DH}} = \frac{\overline{AB}}{\overline{DE}} = \frac{\overline{AC}}{\overline{DF}} = \frac{\overline{BC}}{\overline{EF}}$$

Pf:

Before we do further discussion, please think for a second if you can do the reasoning yourself.

(for teachers)

We want to draw the connection between $\overline{AG}$ and $\overline{DH}$, we need to focus on the triangles related to these two segments. So we look at $\triangle ABG$ and $\triangle DEH$(Of course, you can also choose the right hand side of corresponding triangles.).

In $\triangle ABG$ and $\triangle DEH$

$\angle AGB = 90° = \angle DHE$ ($\overline{AG} \perp \overline{BC}$ and $\overline{DH} \perp \overline{EF}$)

$\angle B = \angle E$ (CPCTC

$\triangle ABC \sim \triangle DEF$, corresponding angles are congruent.)

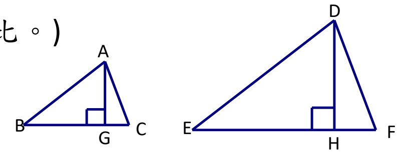
$\Rightarrow \triangle ABG \sim \triangle DEH$ (AA similarity theorem)

$\Rightarrow \frac{\overline{AG}}{\overline{DH}} = \frac{\overline{AB}}{\overline{DE}}$ (triangle similarity property)

$\Rightarrow \frac{\overline{AG}}{\overline{DH}} = \frac{\overline{AB}}{\overline{DE}} = \frac{\overline{AC}}{\overline{DF}} = \frac{\overline{BC}}{\overline{EF}}$ #

We extend the above to another useful property.

Property 2:
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding side lengths.
(兩相似三角形中，面積比等於其對應邊的平方比。)



As shown, $\triangle ABC \sim \triangle DEF$,
$\overline{AG} \perp \overline{BC}$ and $\overline{AG}$ intersects $\overline{BC}$ at point G,
$\overline{DH} \perp \overline{EF}$ and $\overline{DH}$ intersects $\overline{EF}$ at point H.
Then

$$\frac{\text{the area of } \Delta ABC}{\text{the area of } \Delta DEF} = \left(\frac{\overline{AB}}{\overline{DE}}\right)^2 = \left(\frac{\overline{AC}}{\overline{DF}}\right)^2 = \left(\frac{\overline{BC}}{\overline{EF}}\right)^2$$

Pf:

The area of $\triangle ABC = \frac{1}{2}\overline{AG} \cdot \overline{BC}$

The area of $\triangle DEF = \frac{1}{2}\overline{DH} \cdot \overline{EF}$

Then

$$\frac{\text{the area of } \Delta ABC}{\text{the area of } \Delta DEF} = \frac{\frac{1}{2}\overline{AG}\cdot\overline{BC}}{\frac{1}{2}\overline{DH}\cdot\overline{EF}}$$

$$= \frac{\overline{AG}}{\overline{DH}} \cdot \frac{\overline{BC}}{\overline{EF}}$$

$$= \frac{\overline{BC}}{\overline{EF}} \cdot \frac{\overline{BC}}{\overline{EF}} \qquad \text{(Property 1 above)}$$

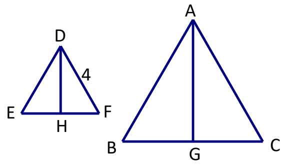
$$= \left(\frac{\overline{BC}}{\overline{EF}}\right)^2$$

So $\frac{\text{the area of } \Delta ABC}{\text{the area of } \Delta DEF} = \left(\frac{\overline{BC}}{\overline{EF}}\right)^2 = \left(\frac{\overline{AB}}{\overline{DE}}\right)^2 = \left(\frac{\overline{AC}}{\overline{DF}}\right)^2$ # $\quad \left(\frac{\overline{BC}}{\overline{EF}} = \frac{\overline{AB}}{\overline{DE}} = \frac{\overline{AC}}{\overline{DF}}\right)$

We will illustrate these properties through some examples. We learned the formula of the height and the area of an equilateral triangle in ~~the~~ eighth grade. We will explore how to get the heights and areas between similar triangles without looking for a lot of information.

Ex 1:

Two equilateral triangles △ABC and △DEF as shown.



$\overline{DF}=4$. $\overline{AG}$ and $\overline{DH}$ are the heights of equilateral triangles ABC and DEF respectively.

$\frac{\overline{BC}}{\overline{EF}}=\frac{3}{2}$.

Find

(1) the measure of $\overline{AG}$

(2) the area of △ABC

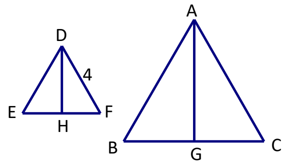
Sol:

First, we want to ask: do you think △ABC and △DEF are similar?

Share your answers and reasons with classmates.

After our discussion, we all know that all equilateral triangles are similar.

(all the regular geometric shapes are similar, like a square is similar to another square, a regular pentagon is similar to another regular pentagon, etc....., and circles are similar to circles, too.)



(在我們討論之後，我們就知道，所有的正三角形都相似，就像所有正方形也都相似，所有正五邊形也都相似等等。圓也互相相似喔。)

So △ABC and △DEF are similar

(1)

In equilateral triangle DEF, the height

$\overline{DH}=\frac{\sqrt{3}}{2}\cdot 4=2\sqrt{3}$ … ♥

△ABC～△DEF,

and $\frac{\overline{AG}}{\overline{DH}}=\frac{\overline{BC}}{\overline{EF}}=\frac{3}{2}$ (property 1)

$\Rightarrow \overline{AG}=\frac{3}{2}\cdot\overline{DH}=\frac{3}{2}\cdot 2\sqrt{3}=3\sqrt{3}$ # (from ♥)

(2)

the area of △DEF$=\frac{\sqrt{3}}{4}\cdot 4^2=4\sqrt{3}$ …✦

and

the area of △ABC=(the area of △DEF) $\cdot(\frac{3}{2})^2$

$=4\sqrt{3}\cdot\frac{9}{4}$ (from ✦ and property 2)

$=9\sqrt{3}$ #

Attention:
If you don't understand what I did above or don't remember all the formulas I use, please thoroughly review the formulas concerning the height and the area of equilateral triangles we learned in the eighth grade. Please!
(如果你不了解我上面用了麼性質，請你一定要複習八年級所學。)

There is an advanced example here. Let's do it together.

Ex 2:
There are four congruent equilateral triangles in a row in Figure 1.
Connect point A and point F

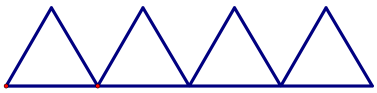


Figure 1

We get the shape in Figure 2.
If the area of a single equilateral triangle is 12, what is the area of the region shaded in brown?

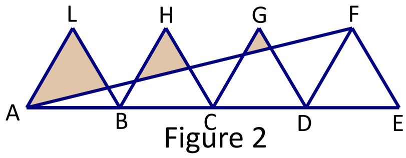


Figure 2

It's not a difficult question, but you need to make some attempts and thinking along the way.
Thinking and trying are the most attractive parts of solving math questions.
Don't be afraid, grab a good pen and some paper to start trying.
(思考和嘗試是在解決數學問題時最有趣的時候了。不要害怕，拿一隻好筆和一些計算紙一起來試試看吧!)
Sol:

Connect $\overline{LF}$ and label the side lengths we need of the brown triangles as a, b, c, d, e, and f respectively as shown in Figure 3.
We can see the side lengths of $\overline{LH}$, $\overline{HG}$, and $\overline{GF}$ all equal $\overline{AL}$=a.
Then we create the relationship among those brown triangles.
Can you see what we have here?
Please discuss it with others and show us your conclusions.

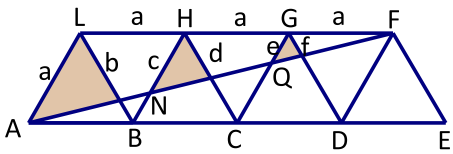


Figure 3

(連接$\overline{LF}$，目的是要在棕色三角形之間建立起關係。你們(學生們)有看到一些什麼關係嗎?討論並分享你們的結果。)
Yes! The most important conclusion is that all corresponding sides of equilateral triangles are parallel(They have congruent corresponding angles.), and that implies the two adjacent sides of the vertex angles in brown triangles are parallel correspondingly. That is:
$\overline{AL}//\overline{BH}//\overline{CG}$ and $\overline{BL}//\overline{CH}//\overline{DG}$
In △AFL,
$\overline{LH}:\overline{HG}:\overline{GF}$=a:a:a=1:1:1
In △QFG and △AFL,
∠QFG = ∠AFL (reflexive property)
∠GQF = ∠LAF (corresponding angles of parallel lines are congruent)
⇒△QFG～△AFL (AA similarity theorem)

Similarly,

$\triangle QFG \sim \triangle NFH \sim \triangle AFL$

We get

$$\frac{e}{a}=\frac{1}{3} \text{ and } \frac{c}{a}=\frac{2}{3} \Rightarrow e=\frac{1}{3}a \text{ and } c=\frac{2}{3}a \quad \cdots(1) \quad \text{(property)}$$

To relate b, d, and f, we construct another congruent equilateral triangle on the left side as shown in Figure 4.

Since it's a congruent equilateral triangle,

$\overline{KA}=\overline{KL}$=a. And $\overline{KL}:\overline{LH}:\overline{HG}:\overline{GF}$=a:a:a:a=1:1:1:1

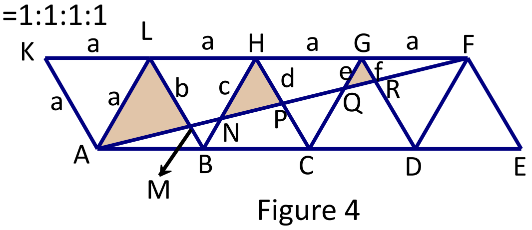


Figure 4

In $\triangle RFG$ and $\triangle AFK$,

$\angle RFG = \angle AFK$ (reflexive property)

$\angle RGF = \angle AKF = 60°$ (an interior angle of an equilateral triangle is 60°)

$\Rightarrow \triangle RFG \sim \triangle AFK$ (AA similarity theorem)

Similarly,

$\triangle RFG \sim \triangle PFH \sim \triangle MFL \sim \triangle AFK$

Then

$$\frac{f}{a}=\frac{1}{4},\ \frac{d}{a}=\frac{2}{4}=\frac{1}{2}, \text{ and } \frac{b}{a}=\frac{3}{4}$$

$$\Rightarrow f=\frac{1}{4}a,\ d=\frac{1}{2}a, \text{ and } b=\frac{3}{4}a \qquad \cdots(2)$$

We head back to the question: the area of the region shaded in brown.

Let's enlarge $\triangle AML$ and take a deep look.

To find the area of one triangle, we need the base and its corresponding height.

In $\triangle AML$, we see $\angle L=60°$

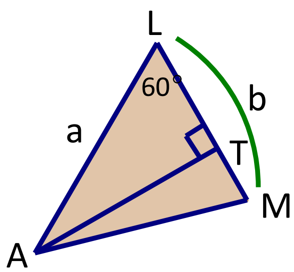
(It's an interior angle of an equilateral triangle.)

This reminds us of what we learned in t~~he~~ eighth grade.

Which is:

In a right triangle with interior angles 30°-60°-90°,

The ratio of three corresponding side lengths is 1:$\sqrt{3}$:2.



So we construct a height $\overline{AT} \perp \overline{LM}$.

$$\frac{\overline{AT}}{\overline{AL}}=\frac{\sqrt{3}}{2} \Rightarrow \overline{AT}=\frac{\sqrt{3}}{2}\overline{AL}=\frac{\sqrt{3}}{2}a$$

And the area of $\triangle AML=\frac{1}{2}\overline{AT}\cdot\overline{ML}=\frac{1}{2}\cdot\frac{\sqrt{3}}{2}a\cdot b=\frac{\sqrt{3}}{4}ab$

Similarly,

the area of $\triangle NPH=\frac{1}{2}\cdot\frac{\sqrt{3}}{2}c\cdot d=\frac{\sqrt{3}}{4}cd$

the area of △QRG $= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} e \cdot f = \frac{\sqrt{3}}{4} ef$

The area of the region shaded in brown is:

the area of △AML+ the area of △NPH+ the area of △QRG

$$= \frac{\sqrt{3}}{4} ab + \frac{\sqrt{3}}{4} cd + \frac{\sqrt{3}}{4} ef$$

$$= \frac{\sqrt{3}}{4} (ab + cd + ef) \qquad \text{(from (1) and (2))}$$

$$= \frac{\sqrt{3}}{4} (a \cdot \frac{3}{4} a + \frac{2}{3} a \cdot \frac{1}{2} a + \frac{1}{3} a \cdot \frac{1}{4} a)$$

$$= \frac{\sqrt{3}}{4} a^2 (\frac{3}{4} + \frac{1}{3} + \frac{1}{12})$$

$$= \frac{\sqrt{3}}{4} a^2 \cdot \frac{14}{12}$$

$$= \frac{\sqrt{3}}{4} a^2 \cdot \frac{7}{6}$$

Let's stop here for a while. When a side length of an equilateral triangle is a, the area of it is $\frac{\sqrt{3}}{4} a^2$. So the area of the region shaded in brown:

$$\frac{\sqrt{3}}{4} a^2 \cdot \frac{7}{6}$$

$$= 12 \cdot \frac{7}{6} \qquad \text{(given the area of the equilateral triangle is 12)}$$

$$= 14_{\#}$$

This example sure looks a little complicated. When we learn trigonometry in ~~the~~ tenth grade, we can solve this problem a little faster.

Similar triangles are very powerful in solving the problems we encounter in real life. Especially for those big quantities that are not easy for us to get the measurements. (對於我們不容易測量到的長、寬和高的距離時，相似三角形的應用就非常有幫助。)

Let's take a short break and dive into some basic word problems.

Ex 3:

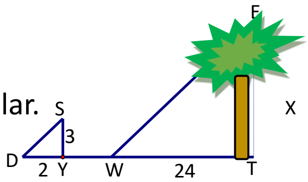
If a tree casts a 24-foot shadow, at the same time, a 3-foot yardstick casts a 2-foot shadow. Find the height of the tree

Sol:

Set the height of the tree $\overline{TE}$ equal to x

The shapes of the two triangles in the figure look similar.

We want to make sure if they are similar triangles.



Consider sunbeams are parallel to us on Earth.

$\Rightarrow \overline{DS} // \overline{WE}$

$\Rightarrow \angle SDY = \angle EWT$ (corresponding angles)

In $\triangle DYS$ and $\triangle WTE$,

$\angle SDY = \angle EWT$

$\angle DYS = \angle WTE = 90°$ (The tree and the yardstick are both perpendicular to the ground.)

$\Rightarrow \triangle DYS \sim \triangle WTE$ (AA similarity theorem)

Now we can estimate the height of the tree $\overline{TE}$ by applying similarity properties.

We have

$$\frac{\overline{TE}}{\overline{YS}} = \frac{\overline{WT}}{\overline{DY}}$$

$$\Rightarrow \frac{x}{3} = \frac{24}{2} \quad \text{(replace with the given)}$$

$$\Rightarrow x = 36 \quad \text{(cross multiplying)}$$

So the height of the tree is 36 feet. #

Always check after solving real world problems to see if the answers we get are reasonable and acceptable.

Ex 4:

Hockey player Yuli standing on point K passes the puck to his teammate by bouncing the puck off the wall of the rink as shown in Figure 1.

Yuli is 3 feet far from the wall, $\overline{HB}$ =4 ft, and $\overline{BP}$ =9 ft.

How far from the wall will the pass be picked up by his teammate?

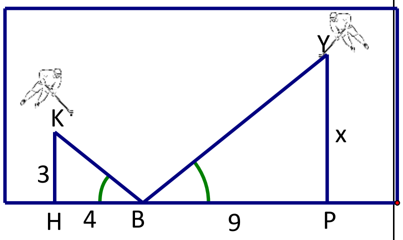


Figure 1

Sol:

Set $\overline{PY}$ equal to x

From physics, we know the angle of incidence is equal to the angle of reflection.

Construct $\overline{BC} \perp \overline{HP}$, then $\alpha = \beta$ shown in Figure 2.

$\angle HBK = 90° - \alpha$

$= 90° - \beta$

$= \angle PBY$

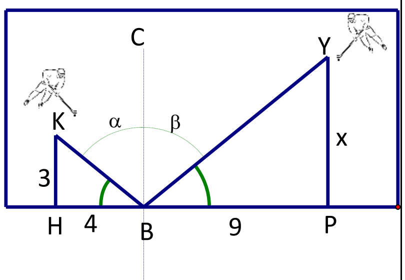


Figure 2

In $\triangle BHK$ and $\triangle BPY$,

$\angle HBK = \angle PBY$ (Complementary angles of congruent Incidence angle and reflection angle are congruent..)

$\angle BHK = \angle BPY = 90°$ (The measure of distance from a wall is always measured perpendicularly to the wall.)

$\Rightarrow \triangle BHK \sim \triangle BPY$ (AA similarity theorem)

$\Rightarrow \dfrac{\overline{PY}}{\overline{HK}} = \dfrac{\overline{BP}}{\overline{BH}}$

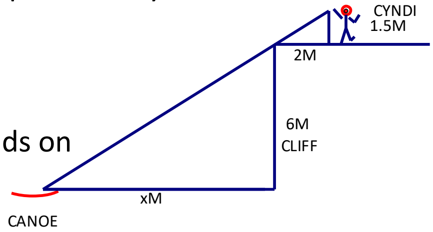
$\Rightarrow \dfrac{x}{3} = \dfrac{9}{4}$ (replace with the given)

$\Rightarrow x = \dfrac{27}{4}$ (cross multiplying)

So Yuli's teammate can pick up the puck if he is $\dfrac{27}{4}$ or 6.75 feet from the wall. #

When solving real life problems, I highly recommend translating the words into drawings.
(當解決應用問題時，我非常建議大家把條件畫出來輔助思考。)

Ex 5:
Cyndi's canoe has come untied and floated away on the lake. She is standing on top of a cliff 6M from the water in a lake. If she stands 2M from the edge of the cliff, she can visually align the top of the cliff with the water at the back of her canoe. Her eye level is 1.5M above the ground. Approximately how far out from the cliff is Cyndi's canoe?



Sol:
Set Cyndi's canoe xM far from the cliff.
We see two triangles here. The ground Cyndi stands on and the water surface is parallel, there is a pair of congruent corresponding angles. And assume Cyndi and the cliff both stand perpendicularly to the ground, there is a pair of congruent right angles. So the two triangles we see are similar (AA similarity theorem)
Then

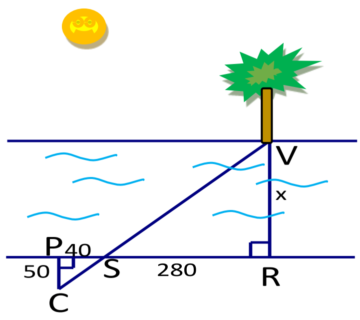
$$\frac{x}{2}=\frac{6}{1.5} \quad \text{(similar property)}$$

$\Rightarrow x=8$

So Cyndi's canoe is 8M far from the cliff. #

Ex 6:

Judo went camping with his classmates. He saw a river nearby and wanted to measure the distance across the river as shown. Judo and his classmates wanted to estimate the distance across the lake to see if they could swim across the river...
They decided to measure the distance across the river by applying triangle similarity that they learned from math class recently. They saw a tree on the other side of the river and drew a simple map as shown in the figure.



The measures they got are as follows:

$\overline{PC}$ =50M, $\overline{PS}$ =40M, and $\overline{SR}$ =280M.

Please answer the following questions:

(1) What is the approximate distance across the river?
That is the length of $\overline{RV}$.

(2) If the maximum swimming distance for Judo at one time is 300M, can Judo safely swim across the river without any swimming kits?
Please explain why you get the conclusion.

Sol:

Set the river's width $\overline{SR}$ equal to x.

In $\triangle$CPS and $\triangle$VRS,

$\angle$CPS = $\angle$VRS=90° (as shown in the figure.)

$\angle$CSP= $\angle$VSR (congruent vertical angles)

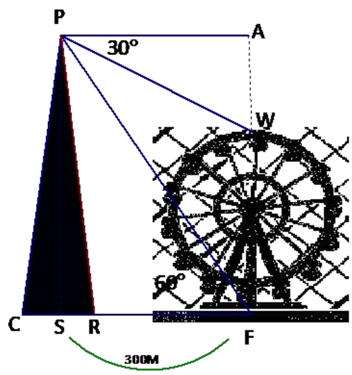
$\Rightarrow \triangle CPS \sim \triangle VRS$ (AA similarity theorem)

$\Rightarrow \frac{x}{50} = \frac{280}{40}$ (similar property)

$\Rightarrow$x=350

The river's width is about 350M, so Judo canNOT swim across the river himself. #

Ex 7:



Diane stands on the top of a skyscraper and wants to know how tall the Ferris wheel would be in the neighborhood. She measures the angle of depression between the horizontal line and. the top of the Ferris wheel to be 30 degrees. Diane then returns to the bottom of the Ferris wheel and measures the angle of elevation between the ground and the top of the skyscraper to be 60 degrees, as shown in the figure.

The distance between the Ferris wheel and the skyscraper is 300M.

(1) How tall is the skyscraper?

(2) How tall is the Ferris wheel?

Sol:

(1) First of all, we have two right triangles here. Both are 30°-60°-90° triangles.

In $\triangle PSF$ and $\triangle PAW$,

$\angle PSF = \angle PAW = 90°$ ($\overline{PS} \perp \overline{SF}$ and $\overline{PA} \perp \overline{AF}$)

$\angle AWP = 90° - \angle APW$ (Two acute angles are complementary in a right triangle.)

$= 90° - 30°$

$= 60°$ -

$= \angle PSF$ (given)

$\Rightarrow \triangle PSF \sim \triangle PAW$ (AA similarity theorem)

$$\Rightarrow \frac{\overline{PS}}{\overline{SF}} = \frac{\overline{PA}}{\overline{AW}}$$

$$\Rightarrow \frac{\overline{PS}}{\overline{SF}} = \frac{\overline{SF}}{\overline{AW}}$$ (Quadrilateral PSFA is a rectangle. $\overline{PA} = \overline{SF}$)

$$\Rightarrow \frac{\overline{PS}}{300} = \frac{300}{\overline{AW}} = \frac{\sqrt{3}}{1}$$ …✲

$\Rightarrow \overline{PS} = 300\sqrt{3} \fallingdotseq 519.6$ (calculator)

So the skyscraper is about 519.6M.

(2) from ✲

$$\frac{300}{\overline{AW}} = \frac{\sqrt{3}}{1}$$ (cross multiplying)

$$\Rightarrow \overline{AW} = \frac{300}{\sqrt{3}}$$

$= 100\sqrt{3}$

$\Rightarrow \overline{FW} = \overline{FA} - \overline{AW}$

$= \overline{PS} - \overline{AW}$ ($\overline{FA} = \overline{PS}$)

$= 300\sqrt{3} - 100\sqrt{3} = 200\sqrt{3}$

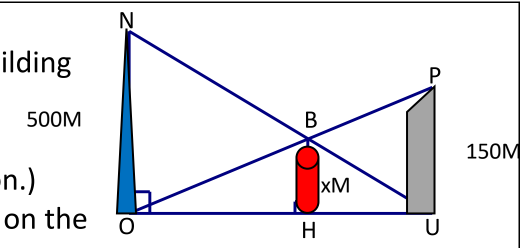
$\fallingdotseq 346.4$

So the height of the Ferris wheel is about 346.4M#

Ex 8:

Shania is living in a building between 101 Building and Uni-president Department Store (I use UPD stands for Uni-president Department Store for the rest of the question.) The top of the building she's living in is right on the point B as shown in the figure. Shania checks out that the height of 101 Building is 500M and the height of UPD is 150M. Please calculate how tall the building is where she lives.

(All buildings are perpendicularly standing on the ground.)



Sol:

We have some triangles here. See if we can find out whether there are similar triangles.

In $\triangle OHB$ and $\triangle OUP$,

$\angle OHB = \angle OUP = 90°$ (given)

$\angle UBH = \angle UNO$ (Corresponding angles are congruent.)

$\Rightarrow \triangle OHB \sim \triangle OUP$ (AA similarity theorem)

$\Rightarrow \dfrac{\overline{HB}}{\overline{UP}} = \dfrac{\overline{OH}}{\overline{OU}}$ …(a)

On the left hand side, in $\triangle UHB$ and $\triangle UON$,

$\angle UHB = \angle UON = 90°$ (given)

$\angle UBH = \angle UNO$ (Corresponding angles are congruent.)

$\Rightarrow \triangle UHB \sim \triangle UON$ (AA similarity theorem)

$\Rightarrow \dfrac{\overline{HB}}{\overline{ON}} = \dfrac{\overline{UH}}{\overline{OU}}$ …(b)

Since $\dfrac{\overline{OH}}{\overline{OU}} + \dfrac{\overline{UH}}{\overline{OU}} = \dfrac{\overline{OU}}{\overline{OU}} = 1$,

$\Rightarrow \dfrac{\overline{HB}}{\overline{UP}} + \dfrac{\overline{HB}}{\overline{ON}} = 1$ (from (a) and (b))

$\Rightarrow \dfrac{x}{150} + \dfrac{x}{500} = 1$ (simplifying)

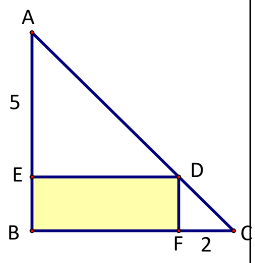
$\Rightarrow \dfrac{13x}{1500} = 1$

$\Rightarrow x \fallingdotseq 115.4$

So the height of the building where Shania lives is about 115.4M#

Ex 9:

In the figure on the right, quadrilateral EBFD is an inscribed rectangle in triangle ABC. $\overline{AE}=5$ and $\overline{CF}=2$. Find the area of rectangle EBFD.



Sol:

The area of rectangle EBFD is the product of two adjacent sides, but it seems that we don't have much information. We see two right triangles $\triangle AED$ and $\triangle DFC$. So let's find out if these two right triangles are similar.

In $\triangle AED$ and $\triangle DFC$,

$\angle AED = \angle DFC = 90°$ (exterior angles of a rectangle)

$\angle EAD = \angle FDC$ (Both pairs of opposite sides of a rectangle are parallel. And corresponding angles are congruent.)

$\Rightarrow \triangle AED \sim \triangle DFC$ (AA similarity theorem)

$$\Rightarrow \frac{\overline{AE}}{\overline{DF}} = \frac{\overline{ED}}{\overline{FC}}$$

$$\Rightarrow \frac{5}{\overline{DF}} = \frac{\overline{ED}}{2}$$

$$\Rightarrow \overline{DF} \cdot \overline{ED} = 10$$

Wala! $\overline{DF} \cdot \overline{ED}$ is the area of rectangle EBFD.(We don't need to find out the lengths of $\overline{DF}$ and $\overline{ED}$ respectively.)

So the area of rectangle EBFD is 10. #

Isn't it fun doing math! Let's keep going on.

Ex 10:

A pool table is 1M by 2M. There are 6 pockets in total, 4 in the corners and 2 at the midpoints of each 2M side. Tommy places a cue ball 0.25M from the north wall and 0.25M away from the west wall as shown in Figure 1.

The angles formed as the ball approaches and deflects form a mirror image of each other.

At what distance from the southeast corner should the cue ball hit the east wall, so the cue ball sinks into the pocket at the midpoint of the south wall?

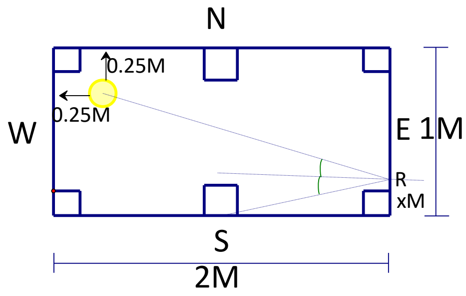


Figure 1

Sol:

Set the distance to be xM.

From physics again, the angle of incidence equals the angle of reflection. Do you see two right triangles? Let me label some points and you'll see them.

In Figure 2, $\overline{AB}$ and $\overline{CD}$ are parallel to the west side of the pool, and $\overline{BR}$ is parallel to the south side of the pool.

They form right angles as they intersect.

So $\angle ABR$ and $\angle DCR$ are right angles.

Set the cue ball to hit the east wall xM away from the southeast corner.

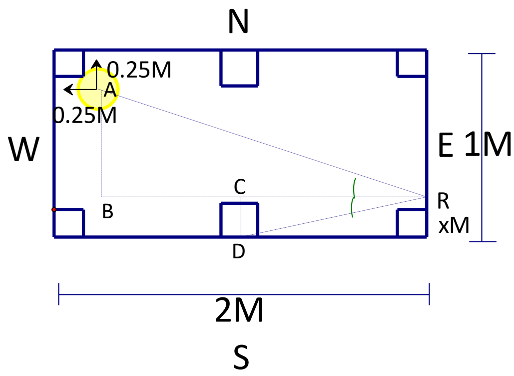


Figure 2

Now we see in $\triangle ABR$ and $\triangle DCR$,

$\angle ABR = \angle DCR = 90°$ ($\angle ABR$ AND $\angle DCR$ are right angles.)

$\angle ARB = \angle DRC$ (The incidence angle equals the reflection angle.)

$\Rightarrow \triangle ABR \sim \triangle DCR$ (AA similarity theorem)

$\Rightarrow \dfrac{\overline{AB}}{\overline{CD}} = \dfrac{\overline{BR}}{\overline{CR}}$ (triangle similar property)

$\Rightarrow \dfrac{1-x-0.25}{x} = \dfrac{2-0.25}{1}$ (The distance from point B to the south wall equals x and $\overline{CD}$ bisects the south wall.)

$\Rightarrow \dfrac{0.75-x}{x} = \dfrac{1.75}{1}$

$\Rightarrow 1.75x = 0.75 - x$ (cross multiplying)

$\Rightarrow 2.75x = 0.75$

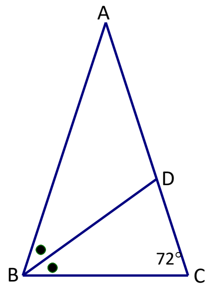
$\Rightarrow x = \dfrac{3}{11}$

$\Rightarrow x \fallingdotseq 0.27$

So the cue ball will sink into the pocket at the midpoint of the south wall when it hits the east wall 0.27M away from the southeast corner. #

Cheer up! Finally, here comes the last example.

Ex 11:



Given isosceles triangle ABC, $\overline{AB}=\overline{AC}=1$.

$\angle C=72°$, $\overline{BD}$ is the angle bisector of $\angle ABC$ and intersects $\overline{AC}$ at point D.

Find the length of $\overline{CD}$.

Sol:

Do we see some isosceles triangles in the figure?

Yes! There are three isosceles triangles, please show us what they are.

In isosceles $\triangle ABC$,

$\angle ABC=\angle C=72°$ (The base angles are congruent in isosceles triangles.)

$\angle ABD=\angle CBD=36°$ …(1) ($\overline{BD}$ is the angle bisector of $\angle ABC$.)

$\angle A=180°-\angle ABC-\angle ACB=180°-72°-72°=36°$ …(2)

(The sum of the interior angles of a triangle equals 180°.)

In $\triangle BCD$,

$\angle BDC=180°-\angle C-\angle CBD=180°-72°-36°=72°$ …(3)

(The sum of the interior angles of a triangle equals 180°.)

So $\triangle BCD$ is an isosceles triangle($\angle BDC=\angle C=72°$) and $\overline{BC}=\overline{BD}$ …(4)

In $\triangle ABD$, $\angle A=\angle ABD=36°$ (from (1)and (2)), so $\triangle ABD$ is also an isosceles triangle, $\overline{AD}=\overline{BD}$ …(5)

Now let's wrap up all the information up there.

In $\triangle ABC$ and $\triangle BDC$,

$\angle A=\angle CBD=36°$ (from (1)and (2))

$\angle C=\angle C$ (self reflexive property)

$\Rightarrow \triangle ABC \sim \triangle BDC$ (AA similarity theorem)

$\Rightarrow \dfrac{\overline{AB}}{\overline{BC}}=\dfrac{\overline{BC}}{\overline{CD}}$ (triangle similar property and (4))

Set $\overline{CD}$ equal to x, then

$\overline{BC}=\overline{BD}=\overline{AD}=1-x$ ((4) and (5))

$\Rightarrow \dfrac{\overline{AB}}{\overline{BC}}=\dfrac{\overline{BC}}{\overline{CD}}$

$\Rightarrow \dfrac{1}{1-x}=\dfrac{1-x}{x}$

$\Rightarrow (1-x)^2=x$ (cross multiplying)

$\Rightarrow x=\dfrac{3\pm\sqrt{5}}{2}$ (the quadratic formula)

$\Rightarrow x=\dfrac{3-\sqrt{5}}{2}$ # (x<1)

Hope you all get a better idea of how to apply triangle similarity to solve real life problems. Math is fun and very useful, right?

Reference:
MIND YOUR DECISIONS
https://www.youtube.com/watch?v=mOxnEn9qWwo

製作者 北市金華國中 郝曉青