三角形的邊角關係
Relationships between Sides and Angles of a Triangle

Class:__________ Name: __________

1. Triangle inequality theorem

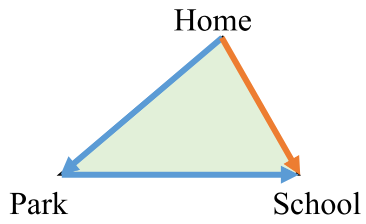
There are two different ways from home to school.

Path 1 : Home → Park → School (Blue Path)

Path 2 : Home → School (Orange Path)

Which one is shorter?

Path 2 is shorter because a straight line is the shortest distance between two points.

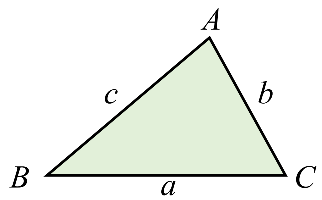


We can use the same way to see $\triangle ABC$ shown on the right. a, b, c are the lengths of the sides of $\triangle ABC$, where $\overline{AB}=c$, $\overline{AC}=b$, $\overline{BC}=a$. Then, we can get the following three results:

(1) $a+b>c$

(2) $b+c>a$

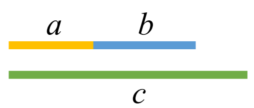
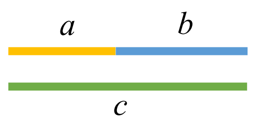
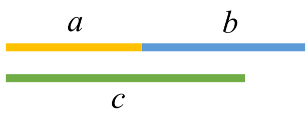
(3) $a+c>b$



★ Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

On the other hand, can every group of three segments be used to form a triangle? Complete the following table to find the conditions which the lengths of segments must fit.

Relationship between segments	Ruler and compass construction (Draw arcs centered at endpoints of c with the radii a and b respectively, and observe the intersection of the two arcs)	Whether they can form a triangle
$a+b<c$ 		
$a+b=c$ 		
$a+b>c$ 		

Exercise. Check whether each group of the side lengths can form a triangle.

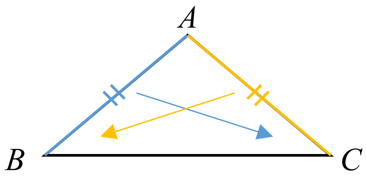
(1) 3, 4, 7

(2) 6, 6, 8

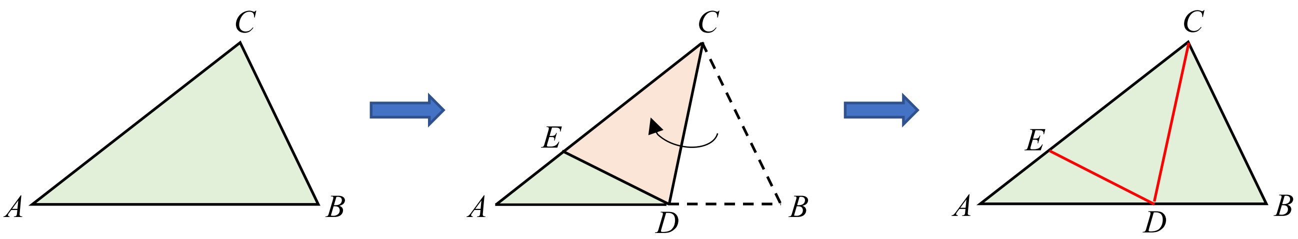
(3) 3, 5, 10

2. The larger angle lies opposite the longer side (大邊對大角)

In $\triangle ABC$, if $\overline{AB}=\overline{AC}$, then this triangle is called an isosceles triangle. The angles opposite the congruent sides are equal because these angles are the base angles of an isosceles triangle. That is, $\angle B=\angle C$.



What if two sides are not equal? If two sides of a triangle are not equal, what is the relationship between the measure of the angles opposite the sides?



We discuss this in the $\triangle ABC$ above. We can see in the figure that $\overline{AC}>\overline{AB}$.

Fold $\triangle ABC$ so that $\overline{CB}$ is folded onto $\overline{AC}$ to point A, where point B is placed at point E on $\overline{DE}$.

Then, $\angle CED=\angle B$.

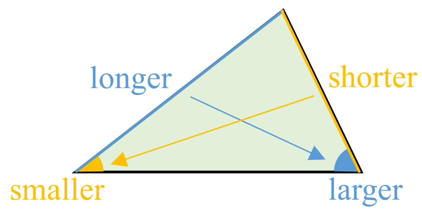
In $\triangle AED$, by exterior angle theorem, we have $\angle CED=\angle A+\angle EDA$.

Then, $\angle B=\angle CED=\angle A+\angle EDA>\angle A$.

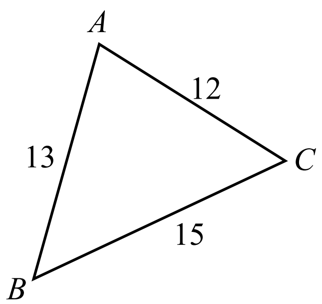
Therefore, in $\triangle ABC$, if $\overline{AC}>\overline{BC}$, then $\angle B>\angle A$.

★ Triangle Longer Side Theorem

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

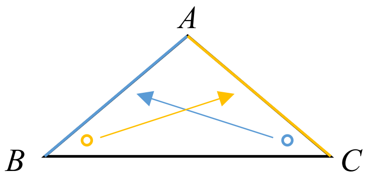


Exercise. List the angles in order from smallest to largest.

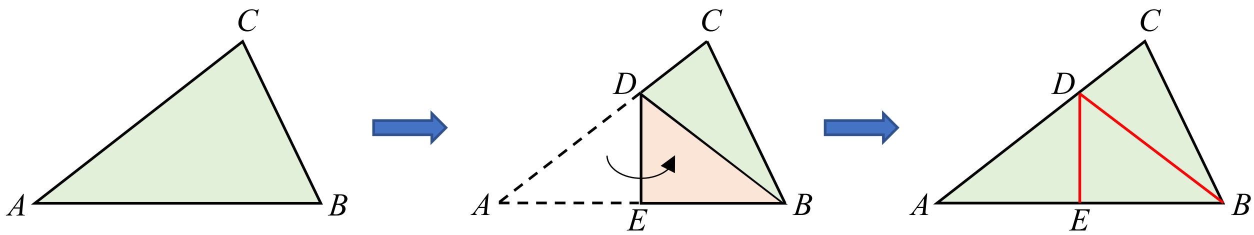


3. The longer side lies opposite the larger angle (大角對大邊)

In $\triangle ABC$, if $\angle B = \angle C$, then this triangle is an isosceles triangle. The sides opposite the congruent angles are equal because these sides are the legs of an isosceles triangle. That is, $\overline{AB} = \overline{AC}$.



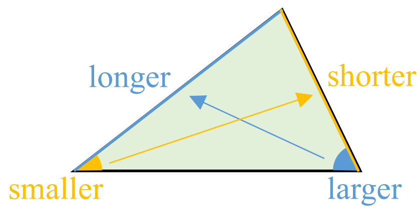
What if two angles are not equal? If two angles of a triangle are not equal, what is the relationship between the length of the sides opposite the angles?



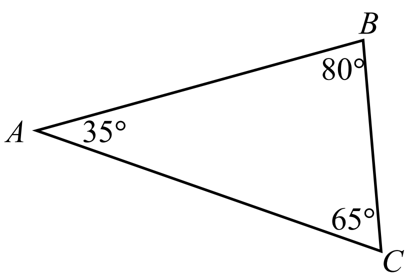
We discuss this in the $\triangle ABC$ above. We can see in the figure that $\angle A < \angle B$.
Fold $\triangle ABC$ so that point A is folded along $\overline{AB}$ to point B, where $\overline{DE}$ is the crease.
Then, $\overline{DA} = \overline{DB}$.
In $\triangle CDB$, by triangle inequality theorem, we have $\overline{CD} + \overline{DA} > \overline{CA}$.
$\overline{CB} = \overline{CD} + \overline{DB} = \overline{CD} + \overline{DA} > \overline{CA}$
Therefore, in $\triangle ABC$, if $\angle A < \angle B$, then $\overline{BC} < \overline{AC}$.

★ Triangle Larger Angle Theorem

If one angle in a triangle is larger than another angle in a triangle, then the side opposite the larger angle will be longer than the side opposite the smaller angle.



Exercise. List the angles in order from shortest to longest.



一、設計理念：

1. 本單元主要談三角形兩邊之和大於第三邊、大邊對大角、大角對大邊等三個概念，其中後兩者均會用到兩邊之和大於第三邊。
2. 配合臺灣課本習慣使用三角形的邊角關係作為標題，本單元英文標題為 Relationship between Sides and Angles of a Triangle，國外課本會使用 Inequalities in a Triangle 作為章節標題。
3. 國外課本中會直接以 Triangle Inequality Theorem(三角不等式)稱三角形兩邊之和大於第三邊的關係，但臺灣的國中課本通常僅寫三角形的三邊長關係，並尚未使用三角不等式這個名詞。
4. 國外課本大邊對大角稱呼為 Triangle Longer Side Theorem，大角對大邊稱呼為 Triangle Larger Angle Theorem。
5. leg 一詞可以指等腰三角形的腰，亦可指直角三角形的股。

二、英文詞彙：

中文	英文
角	angle
邊	side
三角形	triangle
尺規作圖	ruler and compass construction
等腰三角形	isosceles triangle
(等腰三角形的)腰	leg
(等腰三角形的)底角	base angle

三、數學英文用法：

數學表示法	英文
$90°$	90 degrees
$\angle A$	angle A
$\overline{AB}$	line segment
$\triangle ABC$	Triangle ABC
$a>b$	a is greater than b
$a<b$	a is less than b

四、教學參考範例：

<table>
<tr>
<td rowspan="4">L1
【三角形的三邊長關係】
Triangle inequality theorem</td>
<td>1. Triangle inequality theorem
There are two different ways from home to school.
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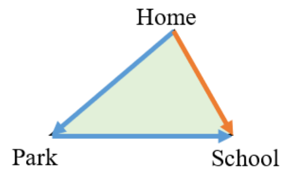
</td>
</tr>
<tr>
<td>Today we are going to learn the relationships between sides and angles of a triangle. In the beginning, we focus on the three sides of a triangle. Let’s look at the situation: There is a home, a park, and a school on the map. We are at home and need to get to the school. We have two options. The first path is to go through the park to get to the school. The second path is to take a direct route from home to the school. Which path is shorter?
Because a straight line is the shortest distance from one point to another. That’s why the second path is the shorter of the two.</td>
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<td>We can use the same way to see $\triangle ABC$ shown on the right. a, b, c are the lengths of the sides of $\triangle ABC$, where $\overline{AB}=c$, $\overline{AC}=b$, $\overline{BC}=a$. Then, we can get the following three results:
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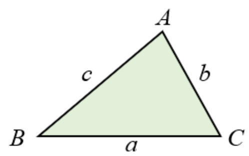

★ Triangle Inequality Theorem
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.</td>
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<td>Let’s use the same way to see the relationship among three sides of a triangle. The length of line segment AB is c, the length of line segment AC is b, and the length of line segment BC is a. Then, if we want to go from A to B, line segment AB is the shortest path. As a result, a plus b is greater than c. Similarly, we have b plus c is greater than a, and a plus c is greater than b.</td>
</tr>
</table>

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【三角形的三邊長關係】

Triangle inequality theorem

On the other hand, can every group of three segments be used to form a triangle? Complete the following table to find the certain relationship which the lengths of segments must fit.

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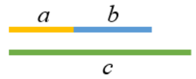
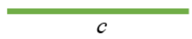
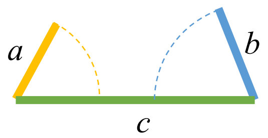
$a+b<c$ 		



We just learn the relationship of three sides in a triangle. Next, we want to know whether every group of three-line segments can be used to form a triangle. We discuss it by ruler and compass construction. Let's complete the following table.

In the first case, the sum of the length of line segments a and b is less than c. Then, we draw an arc with radius a centered at the left endpoint of c. On the other hand, we draw another arc with radius b centered at the right endpoint of c. Then, we find these two arcs do not intersect, so these three sides cannot make a triangle.

<table>
<tr>
<td rowspan="4">ear2
【大邊對大角】
The larger angle lies opposite the longer side</td>
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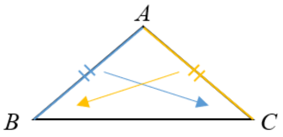
In $\triangle ABC$, if $\overline{AB}=\overline{AC}$, then this triangle is isosceles. The angles opposite the equivalence sides are equal because these angles are the base angles of an isosceles triangle. That is, $\angle B=\angle C$.

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<tr>
<td>Next, we want to find the relationship between the side and the measure of the angle opposite the side. We stat with a special case. In a triangle ABC, if the length of the side AB equals side AC, then what is the relationship between angle B and angle C, which is opposite the side AB and side AC respectively?
If side AB equals side AC, then this is an isosceles triangle. Therefore, angle B equals angle C. It means that in a triangle with sides of equal length, the angles opposite those equal sides are also equal.</td>
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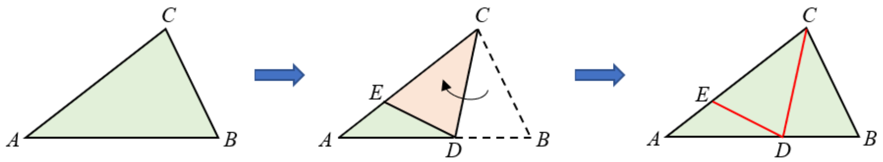

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Then, $\angle B=\angle CED=\angle A+\angle EDA>\angle A$.
Therefore, in $\triangle ABC$, if $\overline{AC}>\overline{BC}$, then $\angle B>\angle A$.</td>
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<td>What happens when two sides of a triangle are not the same length? If two sides of a triangle are not equal, what is the relationship between the measure of the angles opposite the sides? Let's explore this by using a simple folding technique.
In triangle ABC, the side AC is longer than side AB. Fold triangle ABC so that side CB is folded onto side AC to point B. Then, the measure of angle CED equals angle B.
We focus on the triangle AED. By the exterior angle theorem, the measure of the exterior angle of a triangle is equal to the sum of the two remote angles. In this case, the measure of angle CED equals angle A plus angle EDA. This helps us see the connection between the measurement of angle A and angle B. The measure of angle B equals angle CED, and the measure of angle CED equals angle A plus angle EDA. As a result, the measure of angle B is greater than angle A.
To sum it up, in triangle ABC, if the length of side AC is greater than side BC, then the measure of angle B is greater than angle A.</td>
</tr>
</table>

製作者：臺北市雙園國中 劉家宇