Topic: Find the equation of a parabola in standard form

Warm-up:

1.(a)是從一個頂點在（0,0)且開口向上及數據簡單的例子來帶領學生一步一步從幾何定義寫出式子化簡後得到標準式

1. (b) 一樣頂點在（0,0) 但開口向下的例子讓學生練習並從這個兩個例子裡去觀察給定已知條件跟標準式的係數的關係進而猜測一般式且推導 1(c).

[可參考英文問句/提問/開場/解說]

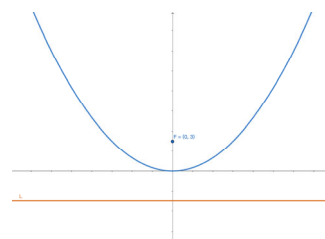 We learned that the graph of a quadratic function is a parabola.

  If we have a parabola, how do we /how can we find the equation of a parabola?

   Let us start with this example.

1.   The vertex is at (0,0) and the directrix is parallel to the y-axis

(a) Find the equation of a parabola in standard form

with focus at the point (0,3) and the directrix $y = -3$

OK, the first step is to recall the definition of a parabola, what is the definition?

P(x,y) is a point on the parabola. Then, by definition.

$\underline{PF} = d(P, L)$	The distance P to F equals the distance P to the directrix.
$\sqrt{(x - 0)^2 + (y - 3)^2} = y - (-3)$	Apply the Distance formula to obtain the square root of x minus zero squared plus y minus three squared equals to y minus negative three.
$[\sqrt{(x - 0)^2 + (y - 3)^2)}]^2 = [y - (-3)]^2$	Square both sides.
$x^2 = 12y$	Expand the brackets and simplify. We will get x squared equals to twelve y. We call this equation the standard form.

Let's rewrite this equation,$y = \frac{1}{12}x^2$ then it becomes the familiar quadratic function.

We'll now look at an example where the parabola opens downward. Now it's your turn to try. We will check the answers after three minutes.

(b)Find the equation of a parabola in standard form with focus F:(0,-3) and the directrix $y = 3$

P(x,y) is a point on the parabola.

$$\sqrt{(x-0)^2 + (y+3)^2} = 3 - y$$

Square both sides and simplify.

We obtain $x^2 = -12y$

From these two examples, what do you notice?

What if we change the focus and the directrix, how will the standard form change?

Let's look at (c)

(c)Find the equation of a parabola in standard form with focus F:(0,c) and the directrix$y = c$

P(x,y) is a point on the parabola.

$$\sqrt{(x-0)^2 + (y-c)^2} = |y-c|$$

Square both sides and simplify.

We obtain $x^2 = 4cy$

Now we can use this conclusion from (c) to quickly sketch a parabola from an equation.

See exercise(d).

(d) Exercise

 Find the vertex, focus, and directrix of a parabola and sketch the parabola.

(1)$x^2 = 32y$

(2)$x^2 = -32y$

(3)$y = 0.5x^2$

How about if we turn a parabola and it opens to the right, how will the standard form change?

Let's look at examples (a) and(b). Have a go. We will check the answers after five minutes.

2. The vertex is at (0,0) and the directrix is parallel to the x-axis

(a) Find the equation of a parabola in standard form with focus F:(3,0) and the directrix $x = -3$

P(x,y) is a point on the parabola.

$$\sqrt{(x-3)^2 + (y-0)^2} = x - (-3)$$

Square both sides and simplify.

We obtain $y^2 = 12x$

(b) Find the equation of a parabola in standard form with focus F:(-3,0) and the directrix $x = 3$

P(x,y) is a point on the parabola.

$$\sqrt{(x+3)^2 + (y-0)^2} = 3 - x$$

Square both sides and simplify.

We obtain $y^2 = -12x$

After these two examples, can you come up with a conclusion for (c)?

(c) Find the equation of a parabola in standard form with focus F:(c,0) and the directrix $x = a$

P(x,y) is a point on the parabola.

$$\sqrt{(x-c)^2 + (y-0)^2} = |x - c|$$

Square both sides and simplify.

We obtain $y^2 = 4cx$

Now we can use this conclusion from (c).

See exercise(d).

(d) Exercise

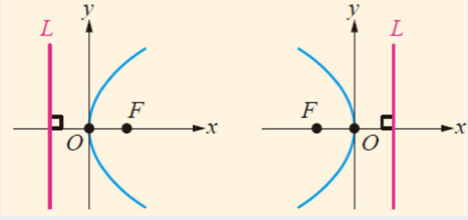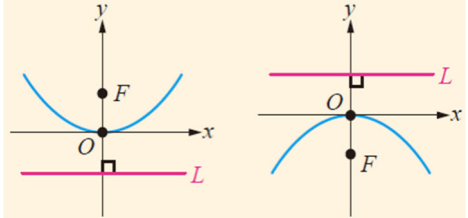Find the vertex, focus, and directrix of a parabola and sketch the parabola.

(1)$y^2 = 32x$

(2)$y^2 = -6x$

(3)$y^2 = 3x$

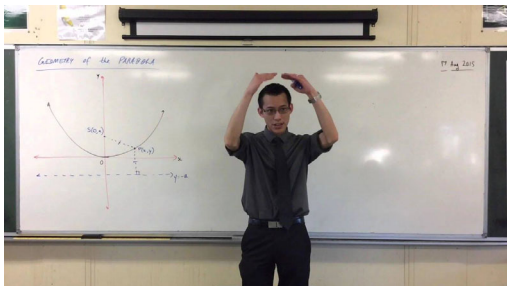Let's sum up what we've learned. Complete the following form.

3. The conclusion

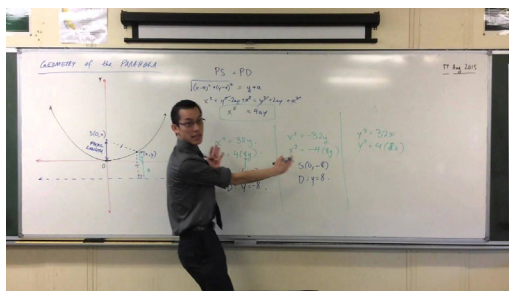| Standard form	Focus	Directrix	Graph
			$c > 0$ $\qquad$ $c < 0$
			$c > 0$ $\qquad$ $c < 0$

4. Challenge

Find the equation of a parabola in standard form with vertex V(1,2) and the directrix $y = -1$ (Hint: use the concept of translation)

[參考資料]

1. Standard Form of a Parabola (1 of 3: From the Locus Definition)

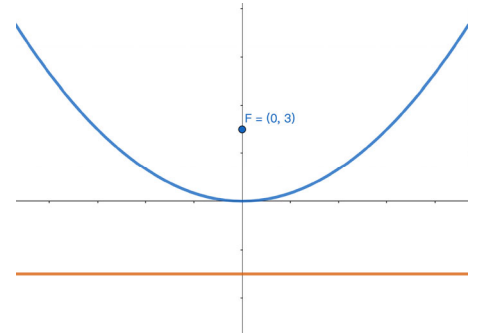2. Standard Form of a Parabola (2 of 3: Determine the focus and directrix)

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Topic: Find the equation of a parabola in standard form

1.   The vertex is at (0,0) and the directrix parallel to the y-axis
(a) Find the equation of a parabola in standard form with focus at
   the point (0,3) and the directrix $y = -3$

F = (0, 3)

(b) Find the equation of a parabola in standard form with focus F:(0,-3) and the directrix $y = 3$

(c) Find the equation of a parabola in standard form with focus F:(0,c) and the directrix $y = a$

(d) Exercise
 Find the vertex, focus, and directrix of a parabola and sketch it.
   $(1) x^2 = 32y$
   $(2) x^2 = -32y$
   $(3) y = 0.5x^2$

2. The vertex is at (0,0) and the directrix is parallel to the x-axis
(a) Find the equation of a parabola in standard form with focus F:(3,0) and the directrix $x = -3$

(b) Find the equation of a parabola in standard form with focus F:(-3,0) and the directrix $x = 3$

(c) Find the equation of a parabola in standard form with focus F:(c,0) and the directrix $x = a$
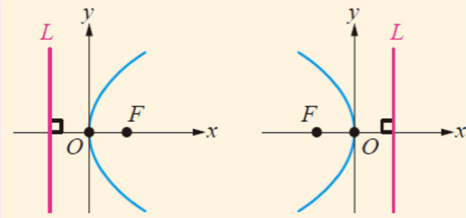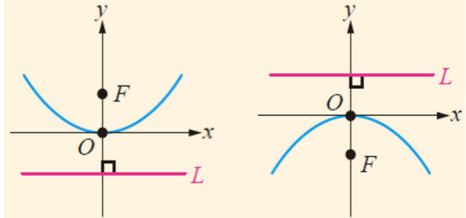
(d) Exercise
 Find the vertex, focus, and directrix of a parabola and sketch it.
 $(1) y^2 = 32x$
 $(2) y^2 = -6x$
 $(3) y^2 = 3x$

3. The conclusion

Standard form	Focus	Directrix	Graph
			$c > 0$ $c < 0$
			$c > 0$ $c < 0$

4. Challenge
Find the equation of a parabola in standard form with vertex V:(1,2) and the directrix $y = -1$
  (Hint: use the concept of translation)

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