2 等差數列　Arithmetic sequences

課本內容 p.13

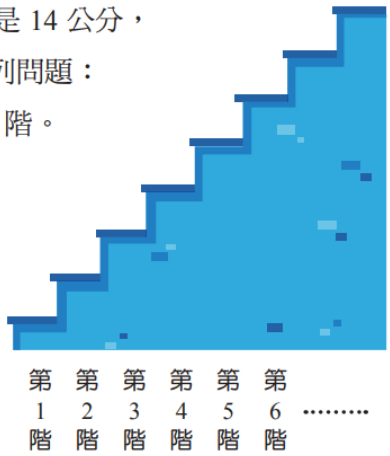
探索活動 等差數列

自強樓樓梯從一樓上到二樓，第 1 階離地面的高度是 14 公分，相鄰兩階的高度差（階梯高）都是 16 公分，回答下列問題：

⑴從站在第 1 階往上走到第 6 階，共走了 _____ 階。

⑵第 1 階是　14　公分，
第 2 階是　30　公分，
第 3 階是　46　公分，
第 4 階是 _____ 公分，
第 5 階是 _____ 公分，
第 6 階是 _____ 公分。

⑶從第 1 階走到第 13 階，共走了 _____ 個階梯，
第 13 階離地面 _____ 公分。



翻譯示例：

探索活動Exploration Activity

*探索 exploration /ˌek.splə'reɪ.ʃən/

From the first floor to the second floor, the height of the first step from the ground is 14 cm, and the difference of the height (step height) between the two adjacent steps is 16 cm. Answer the following questions:

(1) How many steps have you taken from step 1 to step 6?

(2) The first step is 14 cm, the second step is 30 cm, the third step is 46 cm, the fourth step is 62 cm, the fifth step is 78 cm, and the sixth step is 94 cm.

(3) There are 12 steps from the first step to the 13th step. The 13th step is 206 cm from the ground.

雙語使用參考範例：

✧ 開場白 This is an exploration activity about the arithmetic sequence.

✧ 問總共走了幾個階梯?
T: How many steps have you taken from step 1 to step 6?

✧ 問第幾階有多高? T: What is the height of the fourth step?

由探索活動可知，每一階離地面的高度形成一個數列，其中：

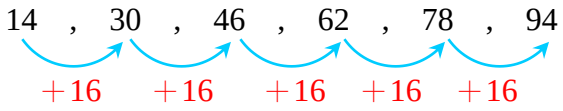
第 1 項 $a_1=14$

第 2 項 $a_2=30$

第 3 項 $a_3=46$

第 4 項 $a_4=62$

⋮



翻譯示例：

It can be seen from the exploration activity that the height of each step from the ground forms a sequence. The first term $a_1=14$ …

這個數列的每一項都是前一項加 16，就另一個觀點，此數列的後項減去前項都等於 16，即 $a_2-a_1=a_3-a_2=a_4-a_3=a_5-a_4=\cdots\cdots=a_n-a_{n-1}=16$。

像這樣，**在一個數列中，任意相鄰兩項「後項減去前項所得的差」都相同，稱為等差數列，這個差稱為公差，通常用 *d* 表示。**

翻譯示例：

Each term of this sequence is the previous term plus 16.

From another point of view, the subsequent term minus the previous term of this sequence is equal to 16.

That is, $a_2-a_1=a_3-a_2=a_4-a_3=a_5-a_4=\cdots\cdots=a_n-a_{n-1}=16$.

In this way, in a sequence, any two adjacent items have the same "difference obtained by subtracting the previous term from the latter term," which is called arithmetic sequences. This difference is called common difference and is usually represented by ***d***.

英文版定義：

Definition: In an arithmetic sequence, the terms increase or decrease by a constant value. This value is called the common difference, or *d*. The common difference can be a positive or a negative value.

課本內容 p.14

【等差數列前後項關係】

1. 一個數列 $a_1, a_2, a_3, a_4, a_5, \ldots\ldots, a_{n-1}, a_n$，
   若 $a_2-a_1=a_3-a_2=a_4-a_3=a_5-a_4=\ \ldots\ldots\ =a_n-a_{n-1}=d$，
   則此數列是公差為 d 的等差數列。
2. 等差數列中，公差＝後項－前項，即 $d=a_n-a_{n-1}$。

翻譯示例：

Relationship between previous term and subsequent term of an arithmetic sequence

1. A sequence $a_1, a_2, a_3, a_4, a_5, \ldots\ldots, a_{n-1}, a_n$.
   If $a_2-a_1=a_3-a_2=a_4-a_3=a_5-a_4=\ \ldots\ldots\ =a_n-a_{n-1}=d$.
   Then this sequence is an arithmetic sequence with common difference d.
2. In the arithmetic sequence,
   common difference＝subsequent term－ previous term,
   that is, $d=a_n-a_{n-1}$.

雙語使用參考範例：

✧ 等差數列介紹

T: An Arithmetic Sequence is made by adding the same value each time.

✧ 觀察數列的公差

T: This sequence 14, 30, 46, 62, 78, 94 has a difference of 16 between each number. Because 30－14＝46－30＝…＝16

The pattern is continued by adding 16 to the last number each time.

✧ 公差介紹

T: The value-added each time is called the "common difference."

✧ 問公差為多少

T: What is the common difference in this example? 19, 27, 35, 43

S: The common difference is 8

✧ 公差也可以為負

T: What is the common difference in this example? 10, 8, 6, 4, 2

S: The common difference is －2

T: Why?

S: Because 8－10＝6－8＝…＝－2

T: So, we know that the pattern is continued by subtracting two each time.

T: Therefore, the common difference could also be negative.

✧ 請學生給幾個等差數列的例子

T: Please give me some examples of an arithmetic sequence.

✧ 引入符號

A sequence $a_1, a_2, a_3, a_4, a_5, \ldots\ldots, a_{n-1}, a_n$，

If $a_2-a_1=a_3-a_2=a_4-a_3=a_5-a_4= \ \ldots\ldots \ =a_n-a_{n-1}=d$,

Then this sequence is an arithmetic sequence with common difference d.

In the arithmetic sequence,

common difference ＝ subsequent term－ previous term,

that is, $d=a_n-a_{n-1}$.

✧ 請學生判斷何謂/是否為等差數列

T: How can you tell that a/this sequence is arithmetic?

等差數列的各項及公差並不限於整數，也可能是分數、小數、含有根號的數或文字符號。

翻譯示例：

Each term and the common difference of the arithmetic sequence is not limited to integers. They may also be fractions, decimals, numbers with roots, or text symbols.

例題：

Fill in the appropriate numbers in the following spaces to make each sequence an arithmetic sequence.

(1) _____, 16 , 19, 22, _____

The common difference *is* $d=19-16=3$ and $a_2=16$，$a_4=22$.

So, $a_1=a_2-d=16-3=13$ and $a_5=a_4+d=22+3=25$.

Therefore, the arithmetic sequence is 13, 16, 19, 22, and 25.

參考資料來源

1. 110 國中數學 2 下翰林版課本
2. IB Maths SL Book Oxford

   Chapter 6 Patterns, sequences, and series
3. Holt McDougal Larson Algebra 2

   Chapter 7 Sequences and Series
4. Number Sequences - Square, Cube, and Fibonacci (mathsisfun.com)

   https://www.mathsisfun.com/numberpatterns.html

☆老師們可以自己從中選擇以做出適合自己學生程度的學習單或是在課堂中適時補充這些英文。

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