Topic: the locus of an ellipse

1. Watch this clip to 2:16.   Share with your partner what you see in this clip.

2. Check these words

English	中文	圖示
Ellipse		
Foci		

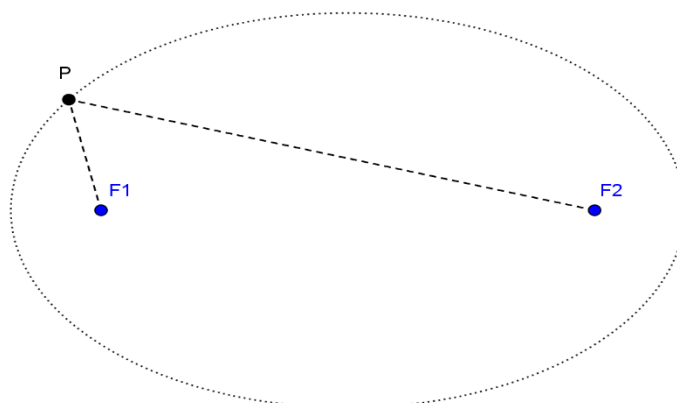3. Watch this clip from 2:16. Then finish the following geometric definition of the ellipse.

條件：_____

規則： P is a point …..

定義：

An ellipse is the set of all points (x,y)…..

4. 橢圓的名詞要素：

   (1) Focus/Foci：

   (2) Major axis：

   (3) Minor axis：

   (4) Center：

   (5) Vertex/vertices：

   (6) Co-vertices :

   (7) Length of the major axis:

   (8) Length of the minor axis:

   (9) The distance between the foci :

   (10) The distance from center to either focus:

3. Property of an ellipse： If an ellipse with major and minor axes of lengths 2a and 2b, respectively, where 0<b<a. The distance between the foci is 2c, then _____.

4. The diagram below shows several concentric circles centered at points F_1 and F_2. The radius of each circle is one unit away from the adjacent circle. Use the definition of an ellipse to sketch it.
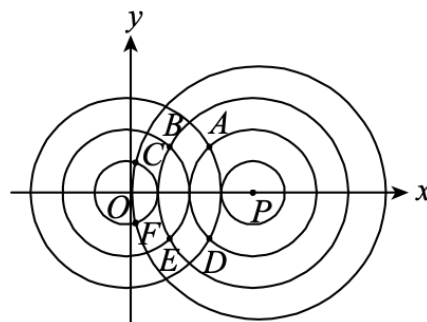
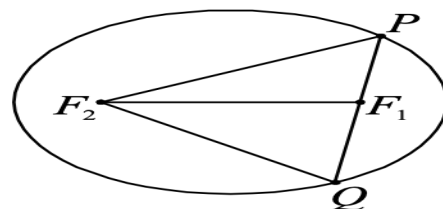   (1) The moving point P on the ellipse satisfy $\overline{PF_1} + \overline{PF_2} = 16$
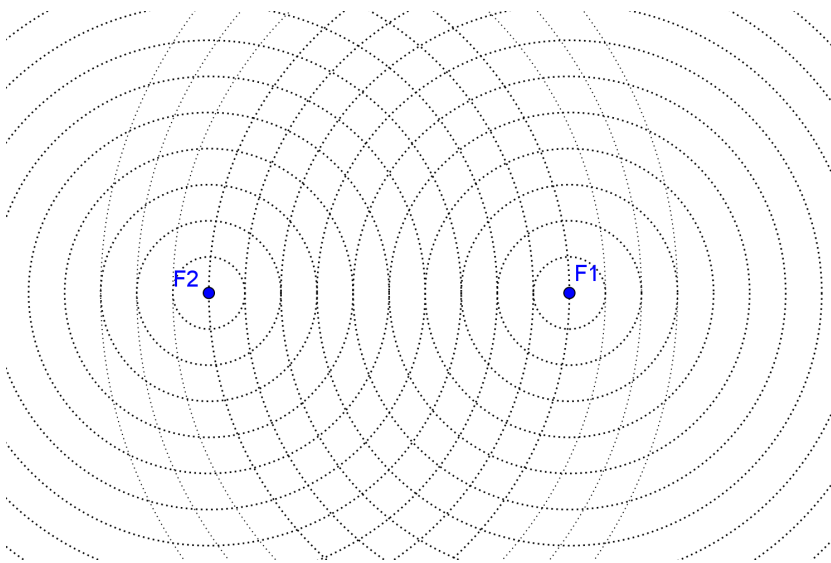
(2) Find the length of the major axis、 the distance between the foci and the length of the minor axis

(3) Find a、b、c

5.如圖，以$O(0,0)$為圓心，半徑為1，2，3畫三個同心圓，以$P(4,0)$為圓心，半徑為1，2，3，4畫四個同心圓。若A，B，C，D，E，F在某一個橢圓上，則關於該橢圓，下列哪些選項是正確的？

(1)中心為$(2,0)$　　　(2)長軸長為4　　　(3)短軸長為3　　　(4)$(4,0)$為其焦點

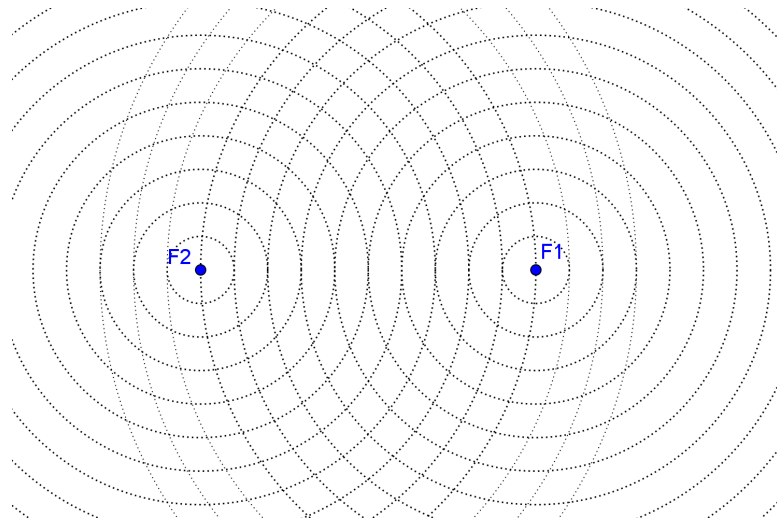6.一橢圓形撞球台，其長軸長為10，且其兩焦點為F_1，F_2，今有一人從F_2將一球依直線方向打至邊上一點P，反彈過F_1，撞至邊上另一點Q，再回到原焦點處F_2，試求ΔPQF_2的周長。

7. Challenge：

(1)($2a$=10，$2c$=10)， The set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 10$. Determine what kind of graph will we get from using the concentric circle centered at points F1 and F2.

(2) ($2a$=4，$2c$=10)， The set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 4$. Determine what kind of graph will we get from using the concentric circle centered at points F1 and F2.
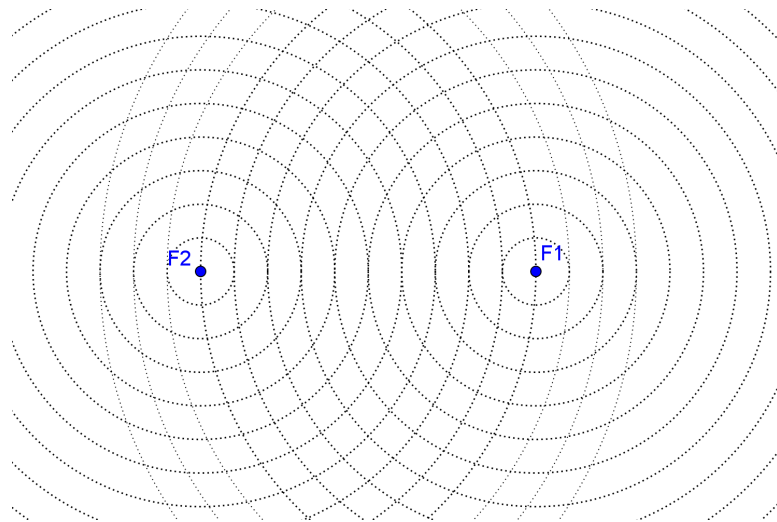
Conclusion：(1) If **$2a > 2c$**， the set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 2a$ is an ellipse.

(2) If **$2a = 2c$**， the set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 2a$ is a line segment $\overline{F_1F_2}$.

(3) If **$2a < 2c$**， the set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 2a$ is an empty set.

Topic: the locus of an ellipse使用建議

1. Watch this clip to 2:16. Share with your partner what you see in this clip.

[教學活動安排]

讓學生透過觀看影片引發學習橢圓的動機

並可利用此機會練習英聽及記筆記擷取重點

[可參考的英文問句/提問/開場]

In this class, we are going to learn about the ellipse. What is an ellipse and why should we learn about it?

Let's watch this clip to 2:16. Then, share with your partner what you see in this clip.

2. Check these words

[教學活動安排]

-教師可讓學生兩兩一組,用手機上網查這些將會使用到的單字、關鍵字 及發音並完成表格

| English	中文	圖示
Ellipse		
Foci		

-也可以請學生將在這堂課中遇到的不熟的單字記錄下來

3. Watch this clip from 2:16.
    Then finish the following geometric definition of the ellipse.

條件： Two points on the planeF_1、 F_2. The Sum of distance from$F_1$and F_2 is constant(α).

規則： P is a point that $\overline{PF_1} + \overline{PF_2} = \alpha$

定義：

An ellipse is the set of all points (x,y) in a plane, the sum of whose distances from two distinct fixed points(F_1、 F_2) is constant.
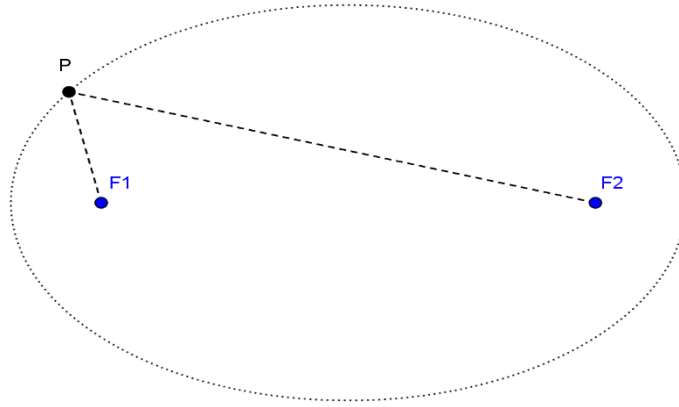
[教學活動安排]

 Think-pair-share

利用全英文影片中的定義說明，讓學生練習在全英的語境下理解橢圓的幾何定義

有需要時可以讓學生多聽幾次．當學生完成時,讓學生兩兩一組,互相跟對方說自己從影片中聽到

並寫下來對於橢圓的幾何定義．

註：紅字部分為參考解答

4. 橢圓的名詞要素：

   (1) Focus/Foci：

   (2) Major axis：

   (3) Minor axis：

   (4) Center：

   (5) Vertex/vertices：

   (6) Co-vertices：

   (7) Length of the major axis:

   (8) Length of the minor axis:

   (9) The distance between the foci：

   (10) The distance from center to either focus:

   [教學活動安排]

   教師介紹並說明

   可參考這影片 https://youtu.be/Uy-Ig5zgNsE

   (1) Focus/Foci： The two fixed points are called the foci (plural of focus).

(2) Major axis：The line segment through the foci is called the major axis.

(3) Minor axis：The line segment through the center and perpendicular to the major axis is called the minor axis.

(4) Center：The mid-point of the major axis.

(5) Vertex/vertices：The endpoints of the major axis. We denote them as A and B

(6) Co-vertices: The endpoints of the minor axis. We denote them as C and D
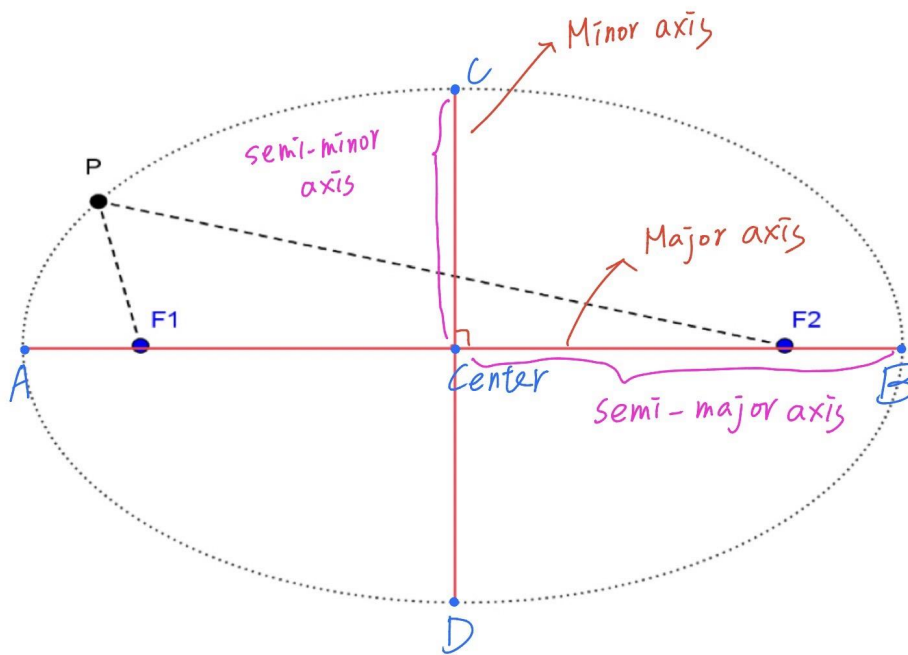
(7) Length of the major axis: $\overline{AB} = 2a$

By the definition, we know that $\overline{AF_1} + \overline{AF_2} = 2a$ and $\overline{BF_2} = \overline{AF_1}$(symmetry)

Therefore,$2a = \overline{AF_1} + \overline{AF_2} = \overline{BF_2} + \overline{AF_2} = \overline{AB}$

(8) Length of the minor axis:$\overline{CD} = 2b$ (We denote it as 2b, we will explain this later when introducing the property)

(9) The distance between the foci:$\overline{F_1F_2} = 2c$ (We denote it as 2c)

(10) The distance from center to either focus: c

3. The property of the ellipse： If an ellipse with major and minor axes of lengths 2a and 2b, respectively, where 0<b<a. The distance between the foci is 2c, then _____.

[教學活動安排]

強調橢圓中a、b、c的關係, $a^2 = b^2 + c^2$

4. The diagram below shows several concentric circles centered at points F_1 and F_2. The radius of each circle is one unit away from the adjacent circle. Use the definition of an ellipse to sketch it.

(1) The moving point P on the ellipse satisfy $\overline{PF_1} + \overline{PF_2} = 16$
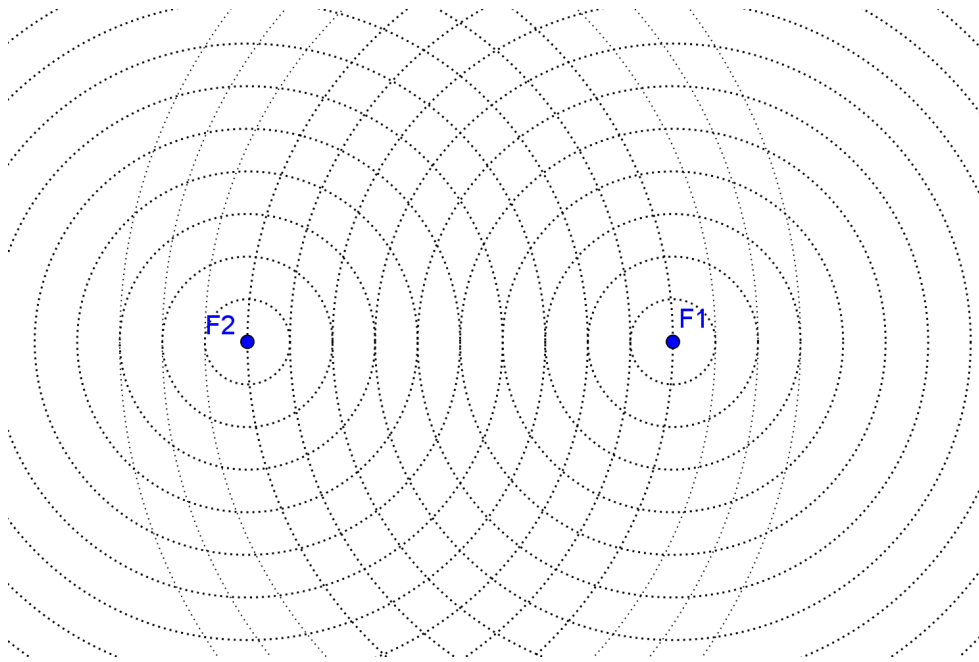
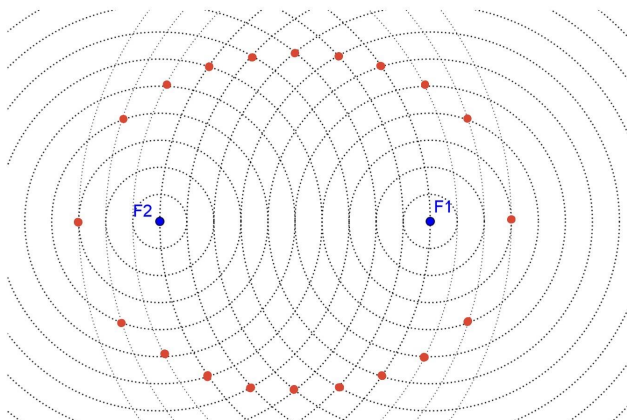(2) Find the length of the major axis、 the distance between the foci and the length of the minor axis

(3) Find a、b、c

[教學活動安排]

在此之前都是介紹橢圓的幾何定義、性質,因此接下來利用同心圓及例子讓學生實際練習用幾何定義去畫出橢圓,同時透過此活動驗收學生對橢圓幾何定義是否掌握

Answers:

(1)

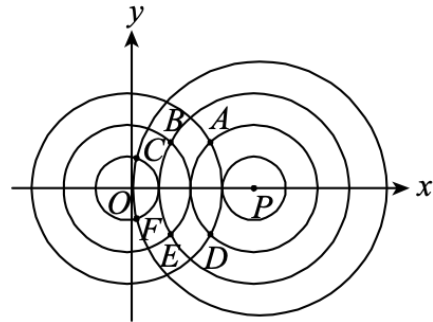(2) $16 \cdot 10 \cdot 2\sqrt{39}$

(3) $8 \cdot 5 \cdot \sqrt{39}$

5.如圖，以$O(0,0)$為圓心，半徑為1，2，3畫三個同心圓，以$P(4,0)$為圓心，半徑為1，2，3，4畫四個同心圓。若A，B，C，D，E，F在某一個橢圓上，則關於該橢圓，下列哪些選項是正確的？
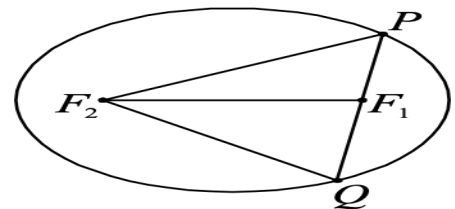
(1)中心為(2,0)　　　(2)長軸長為4　　　(3)短軸長為3　　　(4)(4,0)為其焦點

[教學活動安排]

練習並驗收學生對橢圓的幾何定義及重要性質是否掌握

Answer:(1)(3)(4)

6.一橢圓形撞球台，其長軸長為10，且其兩焦點為F_1，F_2，今有一人從F_2將一球依直線方向打至邊上一點P，反彈過F_1，撞至邊上另一點Q，再回到原焦點處F_2，試求 ΔPQF_2 的周長。
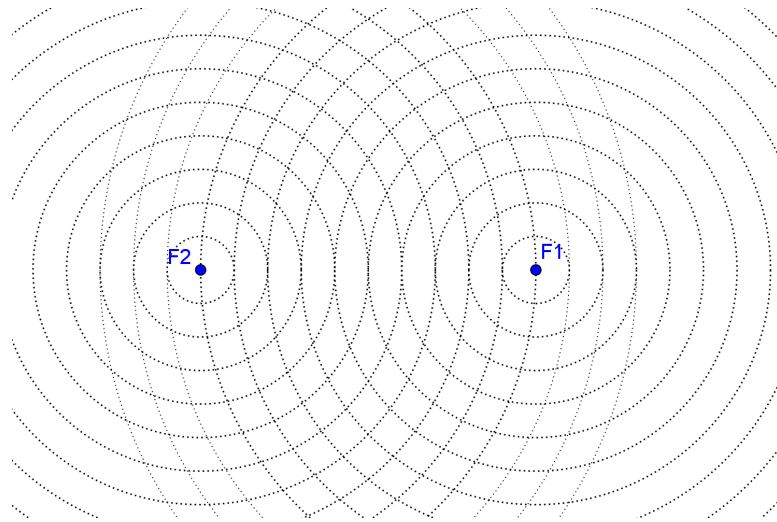

[教學活動安排]

練習並驗收學生對橢圓的幾何定義及重要性質是否掌握

Answer:20

7. Challenge：

(1)($2a$=10，$2c$=10)， The set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 10$. Determine what kind of graph will we get from using the concentric circle centered at points F_1 and F_2.

(2) (2a=4，2 c=10)， The set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 4$. Determine what kind

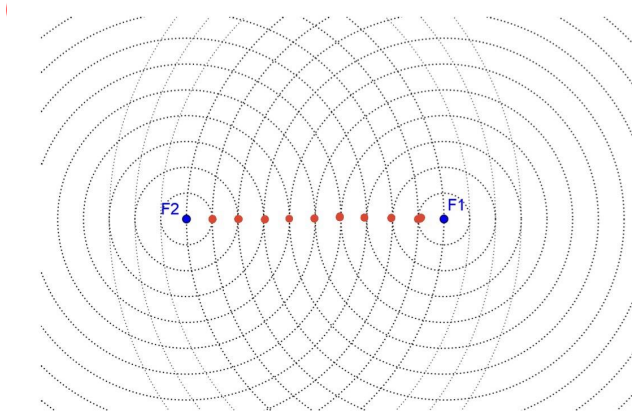of graph will we get from using the concentric diagram centered at points F_1 and F_2
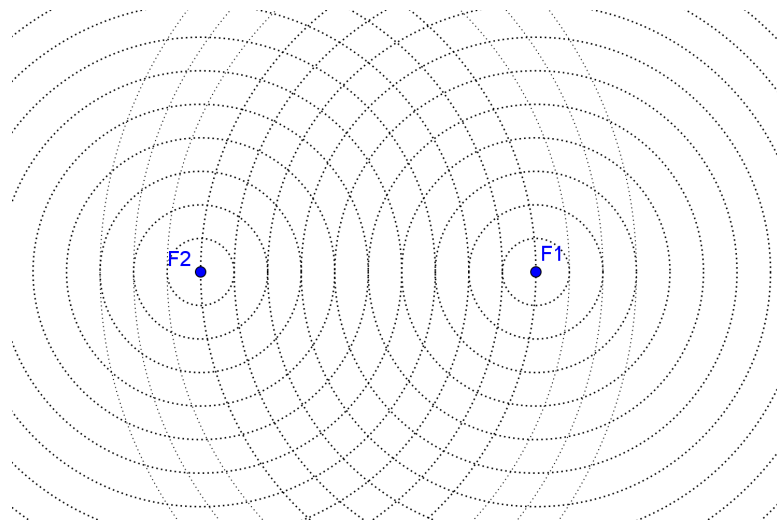
Conclusion：(1) If **2a＞2c**， the set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 2a$ is an ellipse.

(2) If **2a＝2c**， the set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 2a$ is a line segment $\overline{F_1F_2}$.

(3) If **2a＜2c**， the set of all points P(x,y) satisfies $\overline{PF_1} + \overline{PF_2} = 2a$ is an empty set, there is
no graph.

[教學活動安排]

此處教師可自行依學生程度安排是否要加入教學活動中

Answer:

(2)We can't find any points. So there is no graph.

參考資料：

1. https://youtu.be/nzwCInIMlU4
2. https://youtu.be/Uy-Ig5zgNsE
3. https://www.slideshare.net/itutor/ellipse-26682425

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