

3 等差數列一般項

General formula for the n th term of an arithmetic sequence

課本內容 p.16

從自強樓樓梯的一樓走到二樓時，每一階樓梯離地面的高度是 14，30，46，62，78，……，這些數形成一個數列，我們先觀察數列中各項與首項及公差的關係：

$$30 = a_2 = 14 + 1 \times 16 \text{ (首項加 1 個公差)}$$

$$46 = a_3 = 14 + 2 \times 16 \text{ (首項加 2 個公差)}$$

$$62 = a_4 = 14 + 3 \times 16 \text{ (首項加 3 個公差)}$$

$$78 = a_5 = 14 + 4 \times 16 \text{ (首項加 4 個公差)}$$

$$\begin{array}{ccccccccc} a_1 & , & a_2 & , & a_3 & , & a_4 & , & a_5 & \dots\dots \\ 14 & & 30 & & 46 & & 62 & & 78 & \\ \frown & & \frown & & \frown & & \frown & & & \\ & & +16 & & +16 & & +16 & & +16 & \end{array}$$

公差 $d = 30 - 14 = 46 - 30 = 62 - 46 = 78 - 62 = \dots\dots = 16$ 。

Thinking

第 12 階樓梯離地面的高度等於第 1 階的高度再加多少個階梯的高度？

翻譯示例：

When walking from the first floor to the second floor of the Ziqiang building stairs, the height of each step from the ground is 14, 30, 46, 62, and 78,.... These numbers form a sequence. First, we observe the relationship between each term, the first term, and the common difference:

$$30 = a_2 = 14 + 1 \times 16 \text{ (the first term adds 1 common difference)}$$

$$46 = a_3 = 14 + 2 \times 16 \text{ (the first term adds 2 common difference)}$$

$$62 = a_4 = 14 + 3 \times 16 \text{ (the first term adds 3 common difference)}$$

$$78 = a_5 = 14 + 4 \times 16 \text{ (the first term adds 4 common difference)}$$

$$\text{Common difference } d = 30 - 14 = 46 - 30 = 62 - 46 = 78 - 62 = \dots\dots = 16.$$

【Thinking】

How many steps are needed to go from the 1st step to the 12th step?

11.

雙語使用開場範例：

◇ 開場白

T: There is a stairway with several steps. We found that the height of each step from the ground is 14, 30, 46, 62, and 78. What can you observe between the steps and the height of each step?

◇ 引導觀察數列

T: Let's think about the numbers 14, 30, 46, 62, and 78.

Do you agree these numbers could form a particular sequence?

◇ 發現公差

T: What is the difference between 14 and 30?

S: 16.

T: What is the difference between 30 and 46?

S: It is also 16.

◇ 引導為等差數列

T: What did we learn in the last class?

S: Arithmetic sequence.

T: What is the definition of an arithmetic sequence?

S/T: In an arithmetic sequence, the terms increase or decrease by a constant value. This value is called the common difference, or d . The common difference can be a positive or a negative value.

T: Yes. Is this sequence an arithmetic sequence?

S: Yes.

T: What is the common difference between this arithmetic sequence?

S: 16.

◇ 小結

T: Now we know that this is an arithmetic sequence.

The first term is 14, and the common difference is 16.

◇ 如何利用第一項與公差找其他項

T: If we observe the relationship between each term, the first term, and the common difference, what can we find?

S/T: We can find that

$30 = a_2 = 14 + 1 \times 16$ (the first term adds 1 common difference)

$46 = a_3 = 14 + 2 \times 16$ (the first term adds 2 common difference)

$62 = a_4 = 14 + 3 \times 16$ (the first term adds 3 common difference)

$78 = a_5 = 14 + 4 \times 16$ (the first term adds 4 common difference)

設首項為 a_1 ，公差為 d 的等差數列，各項與首項及公差的關係如下：

$$a_2 = a_1 + 1 \times d \text{ (首項加 1 個公差)}$$

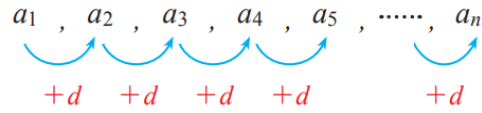
$$a_3 = a_1 + 2 \times d \text{ (首項加 2 個公差)}$$

$$a_4 = a_1 + 3 \times d \text{ (首項加 3 個公差)}$$

$$a_5 = a_1 + 4 \times d \text{ (首項加 4 個公差)}$$

⋮

$$a_n = a_1 + (n-1) \times d \text{ (首項加 } n-1 \text{ 個公差)}$$



翻譯示例：

Let the first term be a_1 , and the common difference be d . The relationship between each term, the first term, and the common difference are as follows:

$$a_2 = a_1 + 1 \times d \text{ (the first term adds 1 common difference)}$$

$$a_3 = a_1 + 2 \times d \text{ (the first term adds 2 common difference)}$$

$$a_4 = a_1 + 3 \times d \text{ (the first term adds 3 common difference)}$$

$$a_5 = a_1 + 4 \times d \text{ (the first term adds 4 common difference)}$$

⋮

$$a_n = a_1 + (n-1) \times d \text{ (the first term adds } n-1 \text{ common difference)}$$

【General formula for the n th term of an arithmetic sequence】

If the first term of an arithmetic sequence is a_1 and the common difference is d , the n th term is $a_n = a_1 + (n-1)d$, also called the general term of an arithmetic sequence.

雙語使用參考範例：

◇ 如何利用第一項與公差找其他項

T: If I want to find the 10th term (a_{10}), could you use a formula that can quickly help me find the 10th term?

S/T: Because $a_2 = a_1 + 1 \times d$ (a_2 is the first term added 1 common difference)

$a_3 = a_1 + 2 \times d$ (a_3 is the first term added 2 common difference)

⋮

$$a_{10} = a_1 + (10 - 1) \times d \text{ (} a_{10} \text{ is the first term added } 10 - 1 \text{ common difference)}$$

So, for this stair, the height of the 10th step

$$a_{10} = a_1 + 9 \times d = 14 + 9 \times 16 = 158$$

◇ 如何利用第一項與公差找第 n 項

T: We can find any term of the sequence by adding the common difference, d , to the previous term. However, it will waste time. If I want to find the n^{th} term (a_n), could you use a formula that can quickly help me find the 10th term?

S: Sure.

$$\text{S/T: } a_n = a_1 + (n - 1) \times d \text{ (} a_n \text{ is the first term add } n - 1 \text{ common difference)}$$

◇ 總結

You can find the n^{th} term of an arithmetic sequence using the formula:

$$a_n = a_1 + (n - 1) \times d$$

◇ 常見錯誤 common error

In the general rule for an arithmetic sequence,

note that the common difference d is multiplied by $n - 1$, not n .

◇ **【General formula for the n^{th} term of an arithmetic sequence】**

If the first term of an arithmetic sequence is a_1

and the common difference is d ,

the n^{th} term is $a_n = a_1 + (n - 1) d$,

which is also called the general term of an arithmetic sequence.

參考資料來源

1. 110 國中數學 2 下翰林版課本
2. IB Maths SL Book Oxford
Chapter 6 Patterns, sequences, and series
3. Holt McDougal Larson Algebra 2
Chapter 7 Sequences and Series
4. [Number Sequences - Square, Cube, and Fibonacci \(mathsisfun.com\)](https://www.mathsisfun.com/numberpatterns.html)
<https://www.mathsisfun.com/numberpatterns.html>

☆老師們可以自己從中選擇以做出适合自己學生程度的學習單或是在課堂中適時補充這些英文。

製作者：臺北市立誠正國民中學 陳怡伶