Topic: Find the equation of an ellipse in standard form from the locus definition

1. The center is at (0,0)

(a) Find the equation of an ellipse in standard form with foci (4,0) and (−4,0), P is any point on the ellipse satisfied $\overline{PF_1} + \overline{PF_2} = 10.$

(b) Find the equation of an ellipse in standard form with foci (c,0) and (−c,0), P is any point on the ellipse satisfied $\overline{PF_1} + \overline{PF_2} = 2a.$

(c) Exercise

Find a、b、c of the ellipse, then find the center, vertices, foci, the length of the major axis,
and the length of the minor axis. Then sketch the ellipse.

(1) $\dfrac{x^2}{100} + \dfrac{y^2}{64} = 1$

(2) $9x^2 + 16y^2 = 144$

(3) $\sqrt{(x-12)^2 + y^2} + \sqrt{(x+12)^2 + y^2} = 26$

2. The center is at (0,0)

(a) Find the equation of an ellipse in standard form with foci (0,4) and (0,−4), P is any point on the ellipse satisfied $\overline{PF_1} + \overline{PF_2} = 10.$

(b) Find the equation of an ellipse in standard form with foci (0,c) and (0,−c), P is any point on the ellipse satisfied $\overline{PF_1} + \overline{PF_2} = 2a.$

(c) Exercise

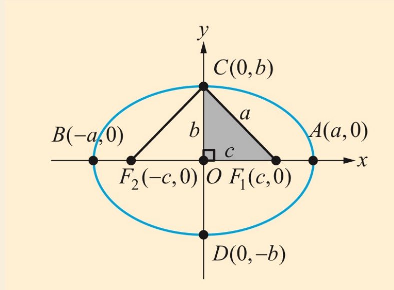
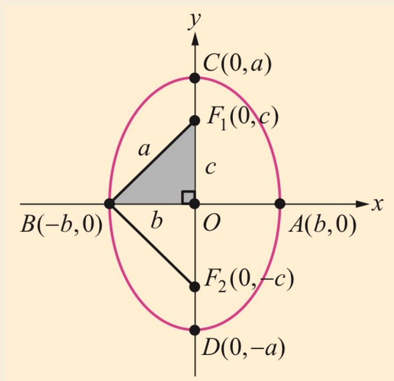
Find a、b、c of the ellipse, then find the center, vertices, foci, the length of the major axis,
and the length of the minor axis. Then sketch the ellipse.

(1) $\dfrac{x^2}{144} + \dfrac{y^2}{169} = 1$

(2) $9x^2 + 4y^2 = 36$

(3) $\sqrt{x^2 + (y-2)^2} + \sqrt{x^2 + (y+2)^2} = 10$

3. The conclusion

Standard form	Foci	Graph
	(c,0) and (-c,0)	The major axis is horizontal. 
	(0,c) and (0,-c)	The major axis is vertical. 

4. Challenge

Find the equation of an ellipse in standard form with foci (0,1) and (4,1) and the major axis of length 6.

(Hint: translation)

Topic: Find the equation of an ellipse in standard form

Warm-up:

[教學活動安排]

1.(a)從中心在(0,0)且數據簡單的左右型橢圓來帶領學生一步步從幾何定義寫出式子化簡後得到標準式,1(b)為推導一般式,1(c)是練習

[可參考英文問句/提問/開場/解說]

If we have an ellipse, how do we find the equation?

Just like what we did with a parabola.

Let's start with this example.

1. The center is at (0,0)

(a) Find the equation of an ellipse in standard form with foci (4,0) and (−4,0). P is any point on the ellipse satisfied $\overline{PF_1} + \overline{PF_2} = 10$.

The first step is to recall the definition of an ellipse, what is the definition?

P(x,y) is a point on the ellipse. Then, by the definition.

$\overline{PF_1} + \overline{PF_2} = 10$	The sum of the distance P to F_1 and F_2 is ten.
$\sqrt{(x-4)^2+y^2} + \sqrt{(x+4)^2+y^2} = 10$	Apply the distance formula to obtain the square root of x minus four squared plus y squared plus the square root of x plus four squared plus y squared equals
$(\sqrt{(x-4)^2+y^2})^2 = (10 - \sqrt{(x+4)^2+y^2})^2$	Rewrite the equation and then square both sides.
$-16x - 100 = -20\sqrt{(x+4)^2+y^2}$	Expand and simplify. We will get negative sixteen x minus one hundred equals minus twenty times the square root of x plus four squared plus y squared.

$144x^2 + 400y^2 = 3600$	Square both sides and simplify. We will get one hundred forty four x squared plus four hundred y squared equals three thousand and six hundred.
$\frac{x^2}{25} + \frac{y^2}{9} = 1$	Divide both sides by three thousand six hundred. We will get x squared over twenty five plus y squared over nine equals one. We say this equation the standard form.

Take a closer look at the coefficients of this standard form, what have you noticed? What will the standard form become if we change the focus and the length of the major axis?

Let's look at (b)

(b) Find the equation of an ellipse in standard form with foci (c,0) and (−c,0), P is any point on the ellipse satisfied $\overline{PF_1} + \overline{PF_2} = 2a$.

P(x,y) is a point on the ellipse.

$$\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$$

Rewrite, square both sides, simplify, square both sides again, expand and simplify, and reduces to

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

We know that $b^2 = a^2 - c^2$

So the equation becomes $b^2x^2 + a^2y^2 = a^2b^2$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Now we can use this conclusion from (b) to quickly sketch an ellipse from an equation.

See exercise(c)

(c) Exercise

Find a、b、c of the ellipse, then find the center, vertices, foci, the length of the major axis, and the length of the minor axis. Then sketch the ellipse.

(1) $\frac{x^2}{100}+\frac{y^2}{64}=1$

(2) $9x^2+16y^2=144$

(3) $\sqrt{(x-12)^2+y^2}+\sqrt{(x+12)^2+y^2}=26$

What if the major axis of an ellipse is vertical? How will the standard form change?

Let's look at this example. Have a go!

2. The center is at (0,0)

(a) Find the equation of an ellipse in standard form with foci (0,4) and (0,−4), P is any point on the ellipse satisfied $\overline{PF_1}+\overline{PF_2}=10$.

P(x,y) is a point on the ellipse. Then, by the definition.

$\overline{PF_1}+\overline{PF_2}=10$	The sum of the distance P to F_1 and F_2 is ten.

$\sqrt{x^2+(y-4)^2}+\sqrt{x^2+(y+4)^2}=10$	Apply the distance formula to obtain the square root of x squared plus y minus four squared plus the square root of x squared plus y plus four squared equals ten.
$(\sqrt{x^2+(y-4)^2})^2=(10-\sqrt{x^2+(y+4)^2})^2$	Rewrite the equation and then square both sides.
$-16y-100=-20\sqrt{x^2+(y+4)^2}$	Expand and simplify. We will get negative sixteen y minus one hundred equals minus twenty times the square root of x squared plus y plus four squared.
$400x^2+144y^2=3600$	Square both sides and simplify. We will get four hundred x squared plus one hundred forty four y squared plus four equals three
$\dfrac{x^2}{9}+\dfrac{y^2}{25}=1$	Divide both sides by three thousand six hundred. We will get x squared over nine plus y squared over twenty five equals one. We say this equation the standard form.

Look at the coefficients of this standard form, what have you noticed? What will the standard form become if we change the focus and the length of the major axis? Let's look at (b)

(b) Find the equation of an ellipse in standard form with foci (0,c) and (0,−c), P is any point on the ellipse satisfied $\overline{PF_1} + \overline{PF_2} = 2a$.

P(x,y) is a point on the ellipse.

$$\sqrt{x^2 + (y-c)^2} + \sqrt{x^2 + (y+c)^2} = 2a$$

Revise, square both sides, simplify, square both sides again, expand and simplify, and reduces to

$$a^2x^2 + (a^2 - c^2)y^2 = a^2(a^2 - c^2)$$

We know that $b^2 = a^2 - c^2$

So the equation becomes $a^2x^2 + b^2y^2 = a^2b^2$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Now we can use this conclusion from (b) to quickly sketch an ellipse from an equation.

See exercise(c)

(c) Exercise

Find a、b、c of the ellipse, then find the center, vertices, foci, the length of the major axis,

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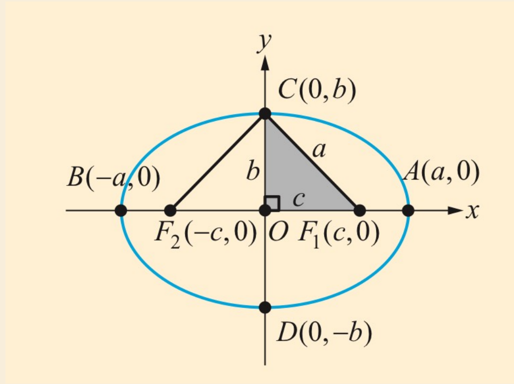
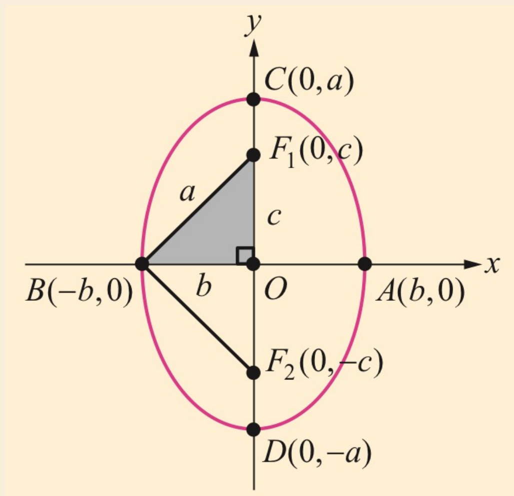
Let’s sum up what we’ve learned. Complete the following form.

3. The conclusion

Standard form	Foci	Graph
	(c,0) and (-c,0)	The major axis is horizontal. 
	(0,c) and (0,-c)	The major axis is vertical. 

4. Challenge

Find the equation of an ellipse in standard form with foci (0,1) and (4,1) and the major axis of length 6.

(Hint: use the concept of translation)

製作者：台北市立育成高中 林玉惇老師