正弦、餘弦定理

Law of Sines and Cosines

<table>
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1st Period</th></tr>
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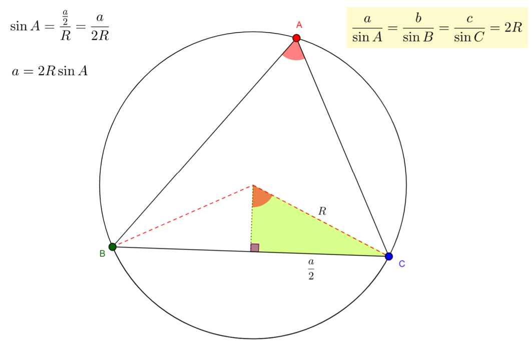

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Vocabulary: Circumcircle (外接圓), Radius (半徑), Inscribed angle (圓周角), Central angle (圓心角).

Sentences:

1. The measure of an inscribed angle is half of the measure of the central angel with the same intercepted arc. (對應同弧的圓心角是圓周角的兩倍。)
2. The same can be proved that … (同理可證…)

GeoGebra Resource:

羅驥韡 (Pegasus Roe) – *正弦定理*

https://www.geogebra.org/m/pP86kbJA

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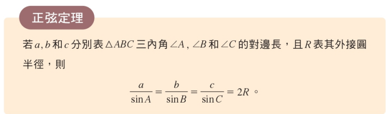

正弦定理

若 a, b 和 c 分別表 $\triangle ABC$ 三內角 $\angle A$, $\angle B$ 和 $\angle C$ 的對邊長，且 R 表其外接圓半徑，則

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \text{。}$$

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Translations:

In a triangle, side “a” divided by the sine of angle A is equal to the side “b” divided by the sine of angle B is equal to the side “c” divided by the sine of angle C is equal to 2 times radius of circumcircle.

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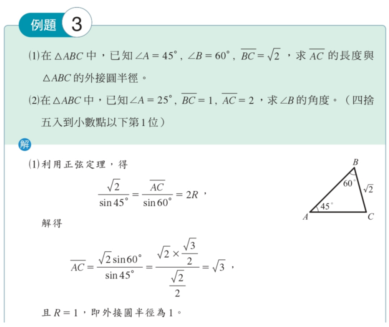

例題 3

(1)在 $\triangle ABC$ 中，已知 $\angle A = 45^\circ$, $\angle B = 60^\circ$, $\overline{BC} = \sqrt{2}$，求 $\overline{AC}$ 的長度與 $\triangle ABC$ 的外接圓半徑。

(2)在 $\triangle ABC$ 中，已知 $\angle A = 25^\circ$, $\overline{BC} = 1$, $\overline{AC} = 2$，求 $\angle B$ 的角度。（四捨五入到小數點以下第 1 位）

解

(1)利用正弦定理，得

$$\frac{\sqrt{2}}{\sin 45^\circ} = \frac{\overline{AC}}{\sin 60^\circ} = 2R，$$

解得

$$\overline{AC} = \frac{\sqrt{2}\sin 60^\circ}{\sin 45^\circ} = \frac{\sqrt{2} \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{3}，$$

且 $R = 1$，即外接圓半徑為 1。

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Vocabulary: Segment (線段).

Translations:

By the law of sines, we have square root of 2 over sine 45 degrees is equal to segment AC over sine

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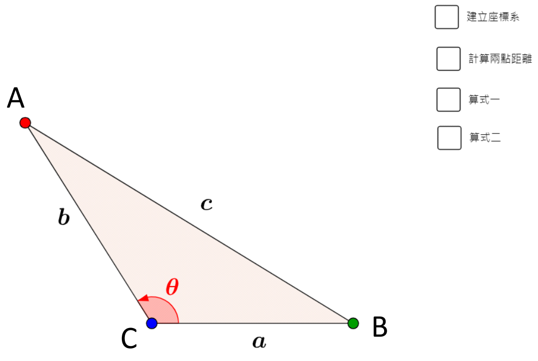
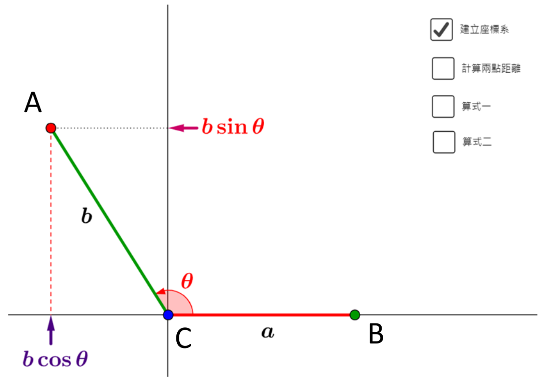
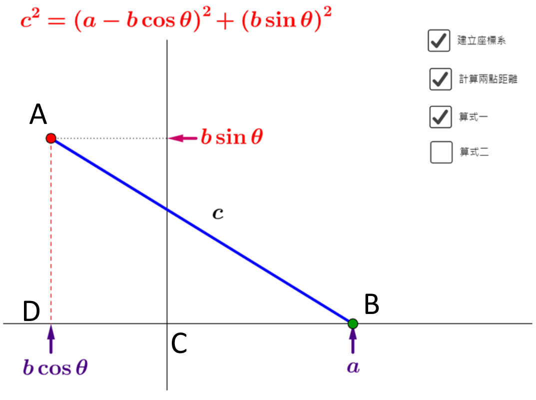
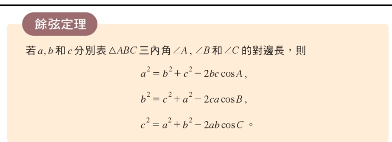
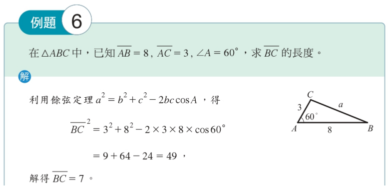
	60 degrees is equal to 2R. Solving this equation, we will have segment AC is equal to square roots of 3. Furthermore, the radius of circumcircle is 1.
	**Vocabulary:** Acute Angle (銳角), Right Angle (直角), Obtuse Angle (鈍角), Drag (拉). **Sentences:** 1. Let a, b, and c be the lengths of the three sides of a triangle. A, B, and C be the three corresponding vertices of the triangle. (三角形 ABC，其對應邊為 a、b、c。) 2. By dragging point A, the θ can be an acute angle, a right angle or an obtuse angle. (藉由拖拉 A 點，θ 可以是銳角、直角、鈍角。) **GeoGebra Resources:** 羅驥韡 (Pegasus Roe) – *餘弦定理* https://www.geogebra.org/m/YyXAydgg
	**Vocabulary:** Coordinate (坐標), Start Edge (始邊), End Edge (終邊). **Sentences:** 1. Let's discuss this triangle on coordinate plane. (建立坐標系。) 2. Let segment BC be the starting edge, and segment AC be the ending edge. (讓線段 BC 為始邊，線段 AC 為終邊。)

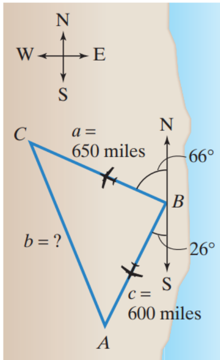
	3. The coordinate of point A will be $(b\cos\theta, b\sin\theta)$. (則 A 點座標即為 $(b\cos\theta, b\sin\theta)$。)
	**Vocabulary:** Projection (投影), Pythagorean Theorem (畢氏定理), Simplify (簡化). **Sentences:** 1. Set the projection of a point A on X-axis is a point D. (設點 A 投影在 X 軸上為點 D。) 2. The X-coordinate of point D is ... (點 D 的 x 坐標為⋯) 3. As triangle ADB is a right triangle, we'll have Pythagorean Theorem which is $(a-b\cos\theta)^2+(b\sin\theta)^2=c^2$. (因為三角形 ADB 是直角三角形，所以我們可以使用畢氏定理，因此得到式子 $(a-b\cos\theta)^2+b\sin^2\theta=c^2$。) 4. We simplify this formula ... (我們將式子簡化⋯)
 餘弦定理 若 a, b 和 c 分別表 $\triangle ABC$ 三內角 $\angle A$, $\angle B$ 和 $\angle C$ 的對邊長，則 $a^2=b^2+c^2-2bc\cos A$, $b^2=c^2+a^2-2ca\cos B$, $c^2=a^2+b^2-2ab\cos C$。	**Translations:** Let a, b, and c be the lengths of the three sides of a triangle. A, B, and C be the three corresponding vertices of the triangle. Then, the law of cosine states that: a square is equal to b square plus c square minus 2 times b times c times cosine A.
 例題 6 在 $\triangle ABC$ 中，已知 $\overline{AB}=8$, $\overline{AC}=3$, $\angle A=60^\circ$，求 $\overline{BC}$ 的長度。 解 利用餘弦定理 $a^2=b^2+c^2-2bc\cos A$，得 $\overline{BC}^2=3^2+8^2-2\times3\times8\times\cos60^\circ$ $=9+64-24=49$， 解得 $\overline{BC}=7$。 	**Translation:** By using law of cosines we have line segment BC square is equal to 3 square plus 8 square minus 2

	times 3 times 8 times cosine 60 degrees. And it is equal to 49. Thus, we have segment BC is 7.

補充題

Materials

Two airplanes leave an airport at the same time on different runways. One flies at a bearing of N66°W at 325 miles per hour. The other airplane flies at a bearing of S26°W at 300 miles per hour. How far apart will the airplanes be after two hours?



Solutions :

After two hours, the plane flying at 325 miles per hour travels 325×2 miles, or 650 miles. Similarly, the plane flying at 300 miles per hour travels 600 miles. The situation is illustrated in Figure.

Let b be the distance between the planes after two hours. We can use a north-south line to find angle B in triangle ABC. Thus, $B = 180° - 66° - 26° = 88°$

We now have $a = 650, c = 600$ and $B = 88°$. We use the Law of Cosines to find b in this SAS situation.

$b^2 = a^2 + c^2 - 2ac\cos B$ (Apply the Law of Cosines.)

$b^2 = 650^2 + 600^2 - 2(650)(600)\cos 88°$ (Substitute: $a = 650, c = 600$ *and* $B = 88°$.)

$\approx 775{,}278$ (Use a calculator.)

$b \approx \sqrt{775{,}278} \approx 869$ (Take the square root and solve for b.)

After two hours, the planes are approximately 869 miles apart.

Note

Vocabulary: Runway (飛機跑道), Bearing (方位), direction(方向), Illustrate (說明), Approximately (大約).

Sentences:

1. A plane flies at a bearing of N66°W at 325 miles per hour. (一架飛機以每小時 325 英里往北

66 度西飛行。)

2. A plane flies in the direction of N66°W at 325 miles per hour. (一架飛機以每小時 325 英里往北 66 度西飛行。)
3. The plane flying at 300 miles per hour travels 600 miles. (一架飛機以每小時 300 英里飛行，飛行里程為 600 英里。)

參考資料

References

1. 許志農、黃森山、陳清風、廖森游、董涵冬（2019）。*數學 2：單元 12 三角比的性質*。龍騰文化。
2. Openstax. (2022, May 24). *AlgebraAndTrigonometry.* https://reurl.cc/x9pKbe.
3. Ron Larson. (2018). *Precalculus 10th Edition.* Cengage Learning.
4. Miami Beach Senior High School. (2022, July 4). *Chapter 6 Additional Topics in Trigonometry.* https://reurl.cc/6Z6jK6.

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