雙語教學主題(國中七年級教材):解一元一次不等式
Topic: solving linear inequality in one variable

這個單元常用到的一些用語
Vocabulary we use in this topic

Integer	fraction
Decimal	reciprocal
Flip	switch
Swap	transpose
Coefficient	distributive property
Denominator	LCD(least common denominator)

From the last lesson, we learned the inequality signs such as: greater than > , less than < , greater than or equal to ≥ and less than or equal to≤. Before we learn how to solve linear inequalities in one variable, I'd like to talk about the properties of inequality here. Most of the properties of inequality are almost the same as the properties of equation, and I'll emphasize the differences down below.

Properties of inequality a,b,c are real numbers a) if a>b, then a+c>b+c Let a=x, b=4 and c=3	Real numbers are like integers, fraction, decimals, negative numbers and positive numbers etc. a)if a is greater than b, then a plus c is greater than b plus c Let a be x, b be 4 and c be 3

if x>4, then x+3>4+3 Check: since x>4, we let x=10 10+3 = 13 and 4+3 = 7 13>7 i.e. 10+3>4+3 The statement is true	If x is greater than 4, then x+3 is greater than 4+3 Since x is any number greater than 4, let x be 10 10 plus 3 is 13 and 4 plus 3 is 7 13 is greater than 7 That is 10 plus 3 is greater than 4 plus 3 The statement is true
b) if a>b, then a-c>b-c Let a=x, b=4 and c=3 if x>4, then x-3>4-3 Check: Since x>4, we let x=10 10-3=7 and 4-3=1 7>1 i.e. 10-3>4-3 The statement is true	b)if a is greater than b, then a minus c is greater than b minus c Let a be x, b be 4 and c be 3 If x is greater than 4, then x-3 is greater than 4-3 Since x is any number greater than 4, let x be 10 10 minus 3 is 7 and 4 minus 3 is 1 7 is greater than 1 That is 10 minus 3 is greater than 4 minus 3 The statement is true

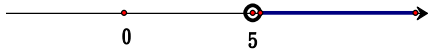
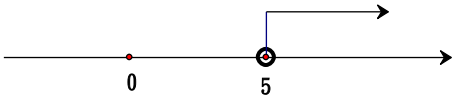
We can see the properties of inequality are about the same as the properties of equation in terms of addition and subtraction.. However, we have to pay extra attention to the properties of inequality when we are dealing with multiplication and division.

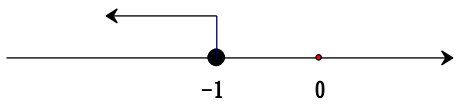
c-1) if a>b, and c>0, then a*c>b*c Let a=x, b=4 and c=3 if x>4, then x*3>4*3 Check: Since x>4, we let x=10 10*3=30 and 4*3=12 30>12 i.e. 10*3>4*3 The statement is true Now let's pay more attention on the following: c-2) if a>b and c<0 then a*c<b*c Let a=x, b=4 and c=3 if x>4, then x*(-3)<4*(-3) Check: Since x>4, we let x=10 10*(-3)=-30 and 4*(-3)=12	c)if a is greater than b, and c is a positive number, then a times c is greater than b times c Let a be x, b be 4 and c be 3 If x is greater than 4, then x*3 is greater than 4*3 Since x is any number greater than 4, let x be 10 10 times 3 is 30 and 4 times 3 is 12 30 is greater than 12 That is 10 times 3 is greater than 4 times 3 The statement is true c-2) if a is greater than b and c is a negative number then a*c is less than b*c Let a be x, b be 4 and c be -3 If x is greater than 4, then x*(-3) is greater than 4*(-3) Since x is any number greater than 4, let x be 10 10 times negative 3 is negative 30 and 4 times negative 3 is negative 12

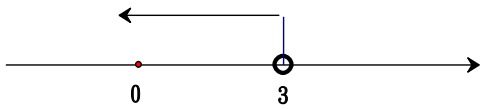
But -30<-12 i.e. 10*(-3)<4*(-3) The statement is true	But negative 30 is less than negative 12 That is 10 times negative 3 is less than 4 times negative 3 The statement is true Whenever we do division, we actually do multiplication of reciprocal. We don't need to do any further explanation for division property.. We have to bear it in mind that when we multiply or divide a negative number on both sides of an inequality, we have to switch the direction of the inequality sign.

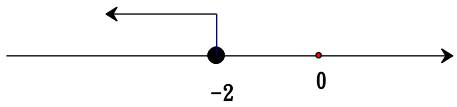
Now let's solve the following inequalities and graph the solutions

Ex1: x-3>2 x-3+3>2+3 x>2+3	X minus 3 is greater than 2 We add the same quantity 3 on both sides (or you can transpose negative 3 to the right side and change its sign to positive)

x>5  It's not easy to draw a thick line with our pens or pencils, here is an alternative way to graph the solution :  	Now we have only x left on the left side , combine constant numbers and get x is greater than 5 See! It's as easy as it is! It's just like solving a linear equation!

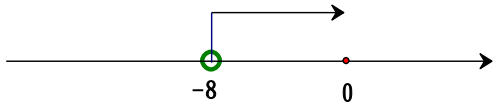
Ex2: $m+2 \leq 1$ $m+2-2 \leq 1-2$ $m \leq -1$  	m plus 2 is less than or equal to 1 subtract 2 on both sides (or transpose positive 2 to the right side and change its sign to negative) We get m is less than or equal to -1
Ex3: $2r<6$	2 times r is less than 6

$\frac{2r}{2} < \frac{6}{2}$ Then r<3  	We divide both sides by 2 (or we multiply the reciprocal of 2 which is $\frac{1}{2}$ on both sides) Now we isolate the variable r on the left side Then we get r is less than 3
Ex4: -3t≥6 $\frac{-3t}{-3} \leq \frac{6}{-3}$ Then t≤-2	Negative 3 times t is greater than or equal to 6 We want to isolate t on the left side, we need to get rid of its coefficient negative 3 So we divide negative 3 on both sides Since we do the division with a negative number We have to swap the inequality sign from greater than or equal to to less than or equal to Then t is less than or equal to negative 2

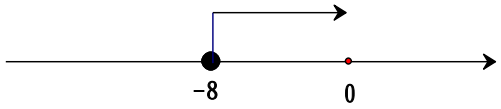
	

Let's try some more multi-step questions

Solve the following inequalities and graph the solutions

Ex5: 2x+3>x-5	2 times x plus 3 is greater than x minus 5
2x+3-x>x-5-x	Subtract x on both sides and combine like terms
x+3>-5	x plus 3 is greater than negative 5
x+3-3>-5-3	Subtract 3 on both sides
Then x>-8	Then x is greater than negative 8
	

Ex6:	
Method 1	Method 1
$3(x-4) \leq 4(x-1)$	3 times the quantity of x minus 4 is less than or equal to 4 times the quantity of x minus 1 (we can also say 3 times x minus 4 in parentheses is ……) We use distributive property and get
$3x-12 \leq 4x-4$	3 times x minus 12 is less than or equal to 4 times x minus 4
$3x-12-4x \leq 4x-4-4x$	If you like to keep x term on the left side, then we subtract 4x from both sides and get
$-x-12 \leq -4$	negative x minus 12 is less than or equal to minus 4
$-x-12+12 \leq -4+12$	We move negative 12 to the right side and becomes positive 12 Then negative x is less than or equal to negative 4 plus 12
$-x \leq 8$	Negative x is less than or equal to 8
$-x(-1) \geq 8(-1)$	We need to isolate x on the left side, , so we multiply negative 1 on both sides
Then we get	Since we multiply a negative number on both sides, we need to switch the inequality sign and get
$x \geq -8$	X is greater than or equal to negative 8



Let me introduce another way to solve this inequality

Method 2

$3(x-4) \leq 4(x-1)$

$3x-12 \leq 4x-4$

$3x-12-3x \leq 4x-4-3x$

$-12 \leq x-4$

$-12+4 \leq x-4+4$

Then $-8 \leq x$,

it's equivalent to

$x \geq -8$

For many beginners, flipping the inequality sign along the solving process is always a threat! Students forget to change the direction of the inequality sign from time to time when they multiply or divide a negative number on both sides.

Therefore, I encourage students to solve the linear inequality as follows:

Method 2

After the distributive property is done, we move 3x towards 4x, because when we subtract 3x from 4x, the result of the coefficient of x is positive

That is negative 12 is less than or equal to 4x minus 4 minus 3x

Combine like terms, we get

Negative 12 is less than or equal to x minus 4

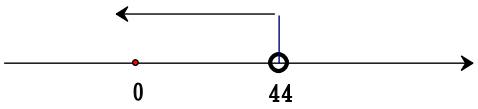
We then move negative 4 to the left side and get positive 4

Negative 12 plus positive 4 is less than or equal to x

Then negative 8 is less than or equal to x

It also means x is greater than or equal to negative 8

We only do the addition and subtraction to solve this inequality. There is no need to switch the inequality sign and students won't get confused or forget to flip the inequality sign.

Ex7: $\frac{x}{2}-7<\frac{1}{3}(x+1)$ $(\frac{x}{2}-7)*6<\frac{1}{3}(x+1)*6$ 3x-42<2(x+1) 3x-42<2x+2 3x-2x<2+42 X<44  	x over 2 minus 7 is less than one third of the sum x+1 We have to simplify fraction from both sides, and 6 is the LCD(least common denominator) of 2 and 3, so we just need to multiply 6 on both sides and we can clear both denominators Distribute 6 to items in parentheses on the left side and cancel the common factor 3 on the right side, we get 3x minus 42 is less than 2 times the sum of x+1 Apply distributive property on the right side 3x-42<2x+2 We move 2x to the left and get negative 2x, also negative 42 to the right and get positive 42 Combine like terms, then x<44

Remember to practice more to get the fluency on doing math.

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