

Topic: the locus of a hyperbola



1. Watch this clip to 1:49. Share with your partner what you see in this clip.

2. Check these words

English	中文	圖示
Hyperbola		
Hyperboloid		
Hyperbolic		

3. Watch this clip from 1:49. Then finish the following geometric definition of the hyperbola.

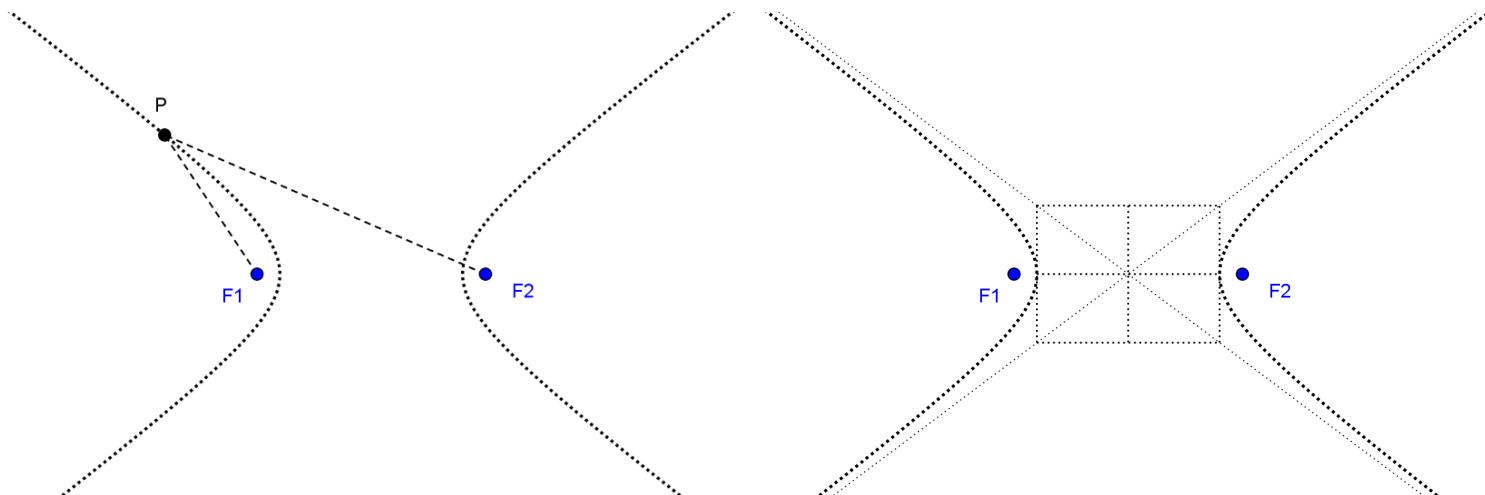


條件：_____

規則：P is a point

定義：

A hyperbola is the set of all points (x,y)



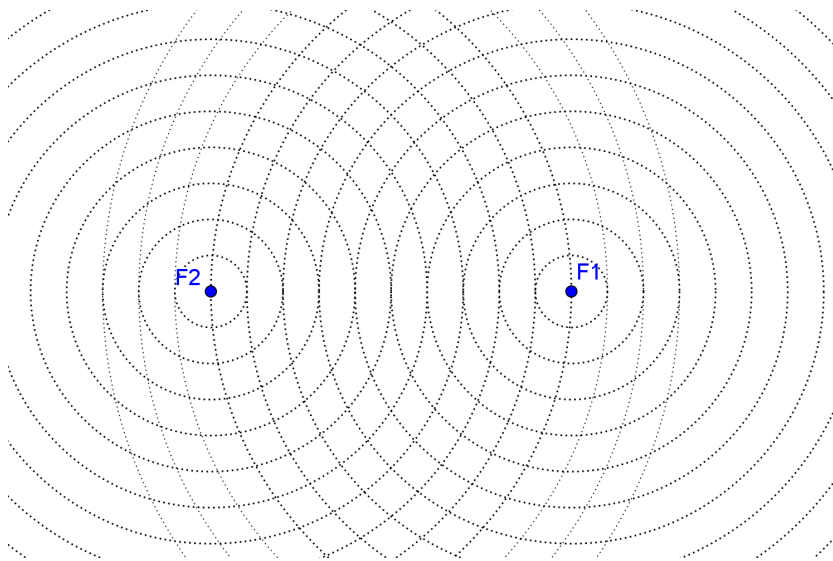
4. 雙曲線的名詞要素：

- (1) Focus/Foci :
- (2) Transverse axis :
- (3) Center :
- (4) Vertex/vertices :
- (5) Length of the transverse axis:
- (6) The distance between the foci:
- (7) The distance from center to either focus:
- (8) Conjugate axis:
- (9) Co-vertices:
- (10) Length of the conjugate axis:
- (11) Asymptote/asymptotes:

5. Property of a hyperbola : If a hyperbola with transverse and conjugate axes of lengths $2a$ and $2b$ respectively. The distance between the foci is $2c$, then _____.

6. The diagram below shows several concentric circles centered at points F_1 and F_2 . The radius of each circle is one unit away from the adjacent circle. Use the definition of a hyperbola to sketch it.

(1) The moving point P on the hyperbola satisfy $|\overline{PF_1} - \overline{PF_2}| = 6$

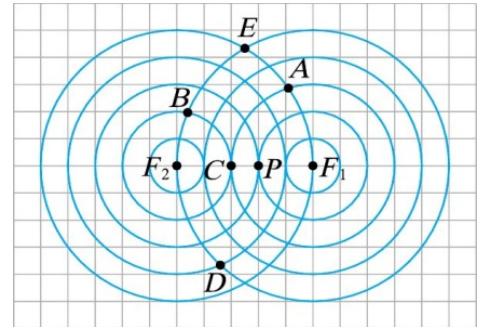


(2) Find the length of the transverse axis 、 the distance between the foci and the length of the conjugate axis.

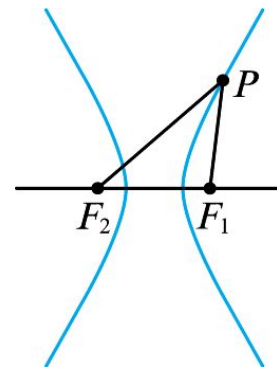
(3) Find a 、 b 、 c .

7. 如圖，在方格紙中有兩組同心圓，圓心分別為 F_1 與 F_2 ，若 P 點在以 F_1, F_2 為焦點的雙曲線上，判斷下列各點是否在此雙曲線上。

() A () B () C () D () E

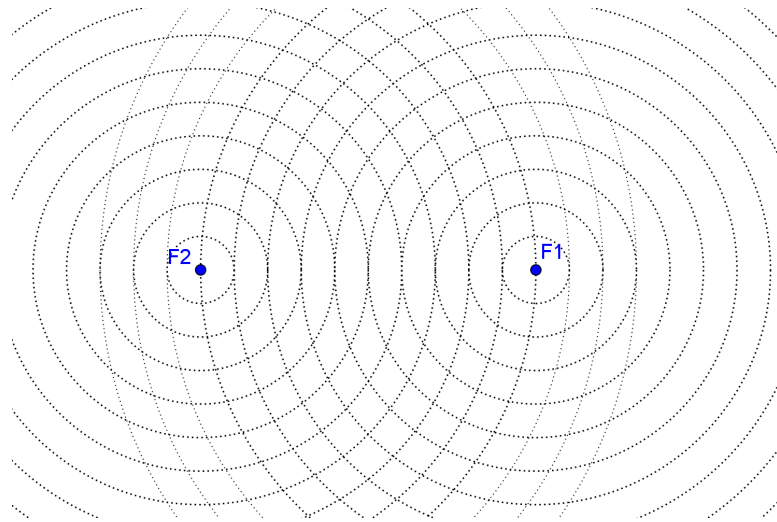


8. 如圖，設 P 為雙曲線的一點且 $|\overline{PF_1} - \overline{PF_2}| = 6$ ，若 F_1, F_2 為此雙曲線的兩焦點且 $\overline{F_1F_2} = 10$ ，已知 $\overline{PF_1} : \overline{PF_2} = 1 : 3$ ，則 $\triangle F_1PF_2$ 的周長為多少？

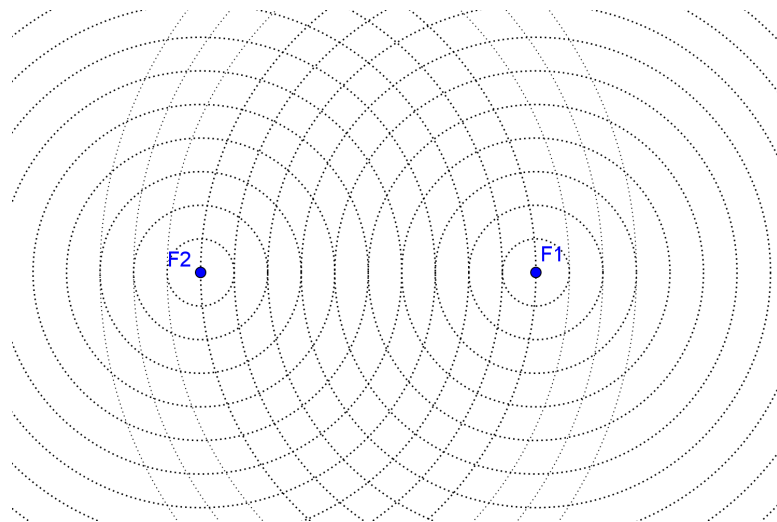


9. Challenge :

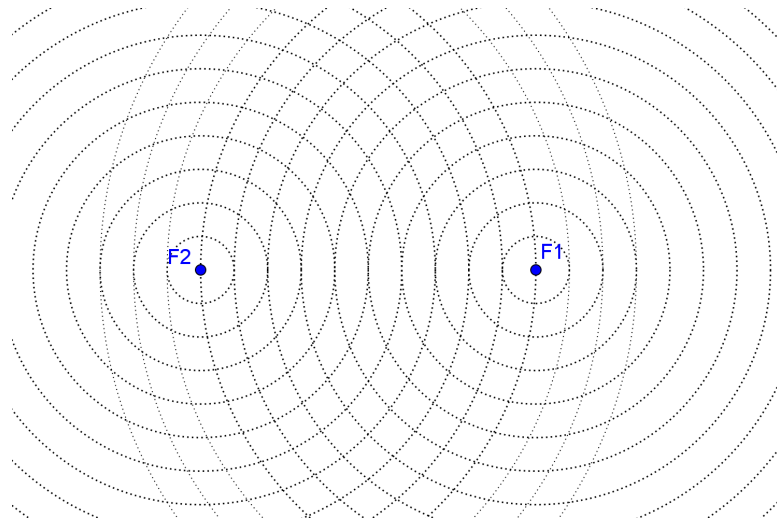
(1) ($2a=10, 2c=10$) The set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 10$. Determine what kind of graph will we get from using the concentric circle centered at points F_1 and F_2 .



(2) ($2a=20, 2c=10$) The set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 20$. Determine what kind of graph will we get from using the concentric circle centered at points F_1 and F_2 .



(3) ($2a=0, 2c=10$) The set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 0$. Determine what kind of graph will we get from using the concentric circle centered at points F_1 and F_2 .



Conclusion :

- (1) If $2a < 2c$, the set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 2a$ is a hyperbola.
- (2) If $2a = 2c$, the set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 2a$ is two rays (two half-lines).
- (3) If $2a > 2c$, the set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 2a$ is an empty set.
- (4) If $2a = 0$, the set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 2a$ is the perpendicular bisector of $\overline{F_1F_2}$.

Topic: the locus of a hyperbola 使用建議



1. Watch this clip to 1:49. Share with your partner what you see in this clip.

[教學活動安排]

讓學生透過觀看影片引發學習雙曲線的動機,並可利用此機會練習英聽及筆記擷取重點

[可參考的英文問句/提問/開場]

In this class, we are going to learn about hyperbola. What is a hyperbola and why should we learn about it?

Let's watch this clip to 1:49. Then, share with your partner what you see in this clip.

2. Check these words

[教學活動安排]

教師可讓學生兩兩一組,用手機上網查這些將會使用到的單字、關鍵字及發音並完成表格

也可以請學生將在這堂課中遇到不熟的單字記錄下來

English	中文	圖示
Hyperbola		
Hyperboloid		
Hyperbolic		

3. Watch this clip from 1:49. Then finish the following geometric definition of the hyperbola.

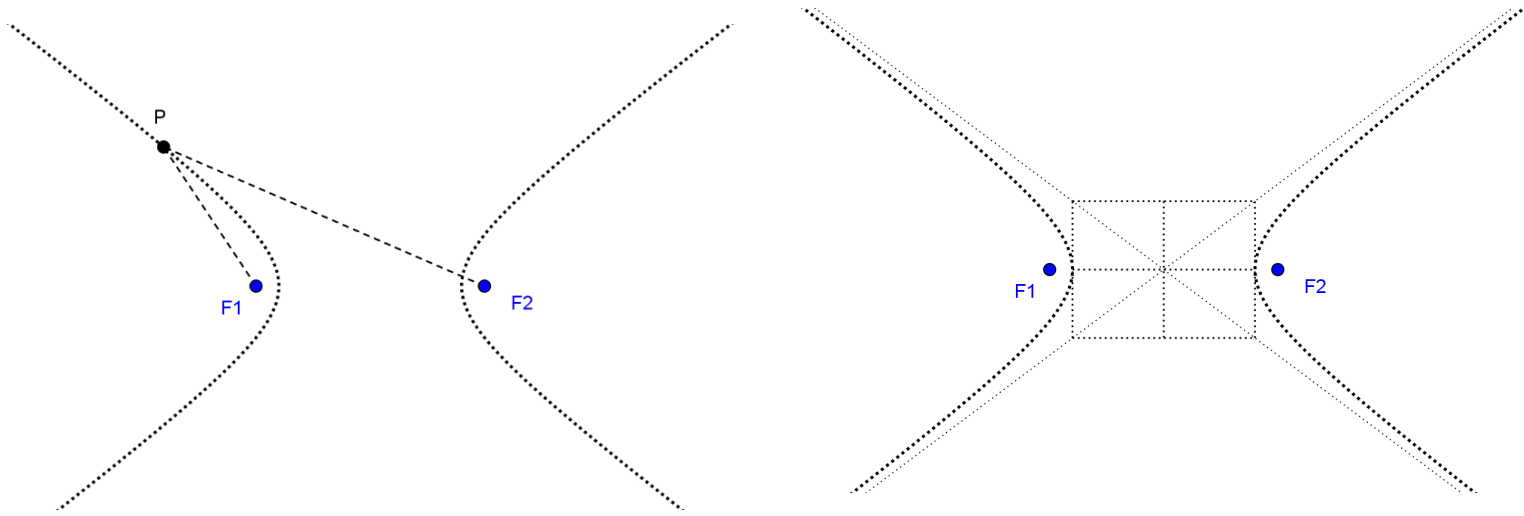


條件：Two points on the plane F_1 and F_2 . The difference of distance from F_1 and F_2 is constant(α)

規則：P is a point satisfied $|\overline{PF_1} - \overline{PF_2}| = \alpha$

定義：

A hyperbola is the set of all points (x,y) in a plane, the difference of whose distances from two distinct fixed points(F_1 and F_2) are constant



[教學活動安排]

Think-pair-share

利用全英文影片中的定義說明，讓學生練習在全英的語境下理解雙曲線的幾何定義，有需要時可以讓學生多聽幾次。當學生完成時，讓學生兩兩一組，互相跟對方說自己從影片中聽到並寫下來對於雙曲線的幾何定義。

註：紅字部分為參考解答

4. 雙曲線的名詞要素：

- (1) Focus/Foci :
- (2) Vertex/vertices :
- (3) Transverse axis :
- (4) Center :
- (5) Length of the transverse axis:
- (6) The distance between the foci:
- (7) The distance from center to either focus:
- (8) Conjugate axis:
- (9) Co-vertices:
- (10) Length of the conjugate axis:
- (11) Asymptote/asymptotes:

[教學活動安排]
教師介紹並說明

[可參考的英文問句/提問/說明]

The graph of a hyperbola has two disconnected parts(branches). How many axes of symmetry does the hyperbola have? Can you draw it?

(1) Focus/Foci : The two fixed points F_1 、 F_2 are called foci (focus是單數，foci是複數).

(2) Vertex/vertices : The line through the foci intersects the hyperbola at two points are called vertices. We denote them as A and B.

(3) Transverse axis (貫軸) : The line segment connecting vertices A and B is the transverse axis.

(4) Center : The midpoint of the transverse axis.

(5) Length of the transverse axis: $\overline{AB} = 2a$

By the definition, we know that $\overline{AF_2} - \overline{AF_1} = 2a$ and $\overline{AF_1} = \overline{BF_2}$ (symmetry)

Therefore, $2a = \overline{AF_2} - \overline{AF_1} = \overline{AF_2} - \overline{BF_2} = \overline{AB}$

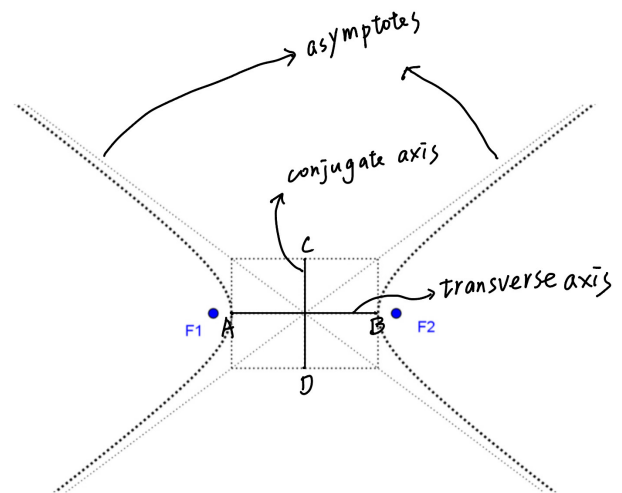
(6) The distance between the foci: $\overline{F_1F_2} = 2c$. We denote it as $2c$

(7) The distance from center to either focus: c

(8) Conjugate axis (共軛軸) : The line segment with length $2b$ through the center and perpendicular to the transverse axis is called the conjugate axis. 與貫軸垂直於中心且長度為 $2b$ 的線段

(9) Co-vertices(共軛軸頂點): The endpoints of the conjugate axis. We denote them as C and D.

(10) Asymptote/asymptotes(漸近線): Every hyperbola has two asymptotes that intersect at the center of the hyperbola, as shown in Figure. The asymptotes pass through the vertices of a rectangle of the dimensions $2a$ by $2b$.



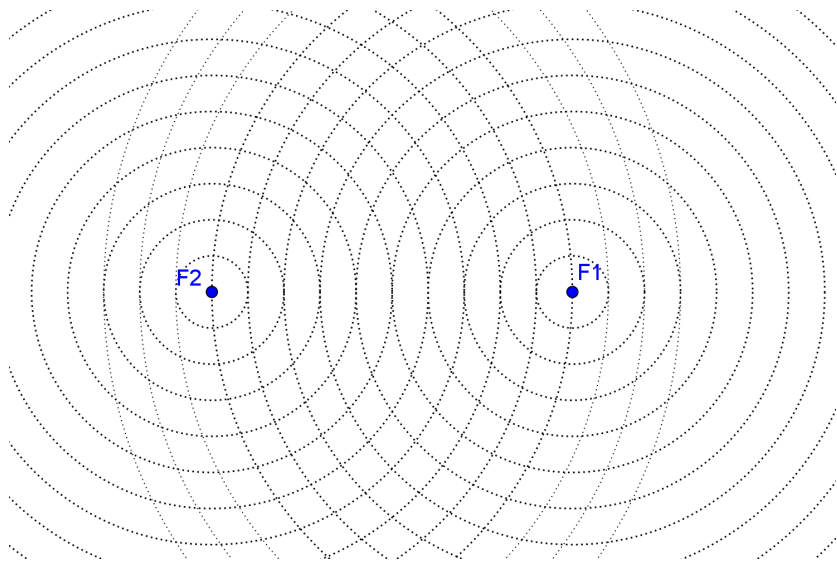
5. Property of a hyperbola : If a hyperbola with transverse and conjugate axes of lengths $2a$ and $2b$ respectively. The distance between the foci is $2c$, then _____.

[教學活動安排]

強調雙曲線中 a 、 b 、 c 的關係, $c^2 = a^2 + b^2$

6. The diagram below shows several concentric circles centered at points F_1 and F_2 . The radius of each circle is one unit away from the adjacent circle. Use the definition of a hyperbola to sketch it.

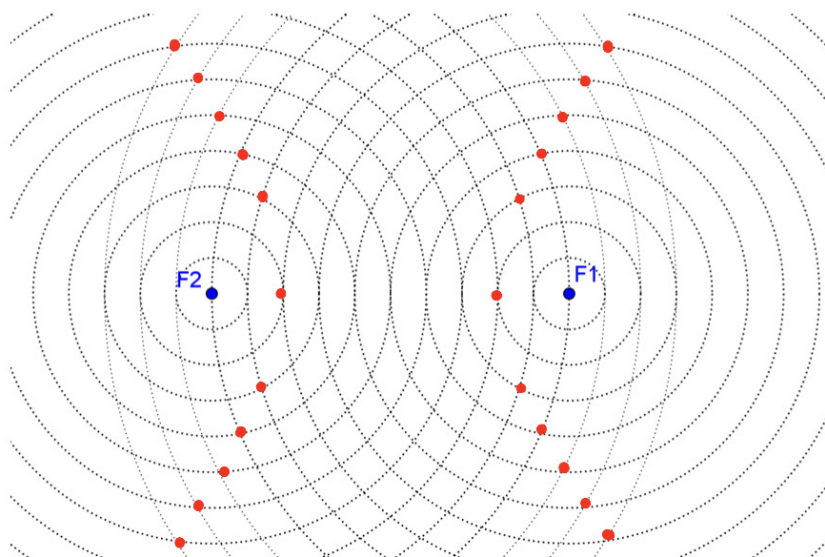
(1) The moving point P on the hyperbola satisfy $|\overline{PF_1} - \overline{PF_2}| = 6$



(2) Find the length of the transverse axis 、 the distance between the foci and the length of the conjugate axis.

(3) Find a 、 b 、 c.

Answer: (1)

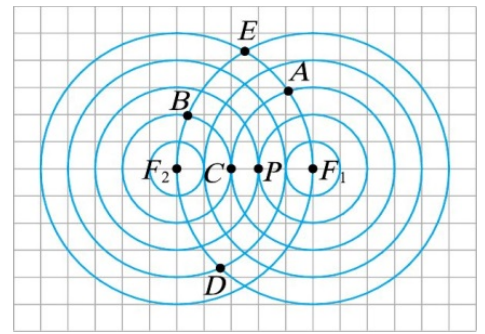


(2) 6 、 10 、 8 (3) 3 、 5 、 4

7.如圖，在方格紙中有兩組同心圓，圓心分別為 F_1 與 F_2 ，若 P 點在以 F_1, F_2 為焦點的雙曲線

上，判斷下列各點是否在此雙曲線上。

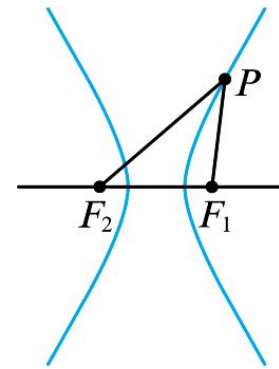
()A ()B ()C ()D ()E



Answer: (X)A (X)B (O)C (O)D (X)E

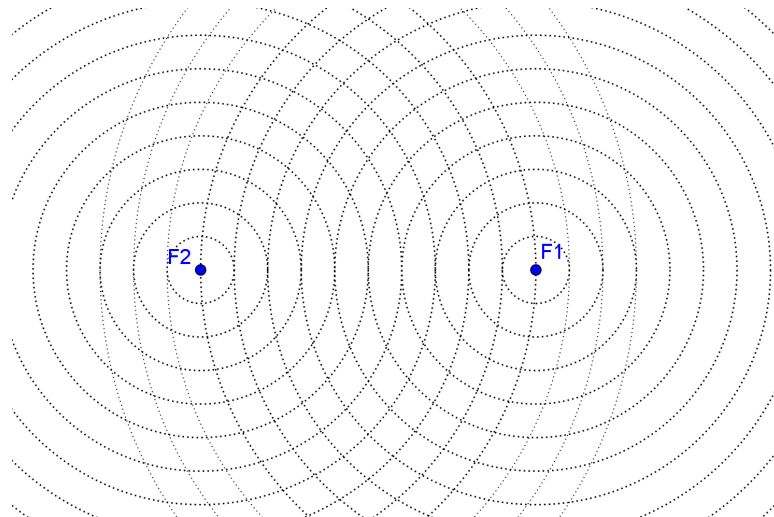
8. 如圖，設 P 為雙曲線的一點且 $|\overline{PF_1} - \overline{PF_2}| = 6$ ，若 F_1, F_2 為此雙曲線的兩焦點且 $\overline{F_1F_2} = 10$ ，已知 $\overline{PF_1} : \overline{PF_2} = 1 : 3$ ，則 $\triangle F_1PF_2$ 的周長為多少？

Answer: 22

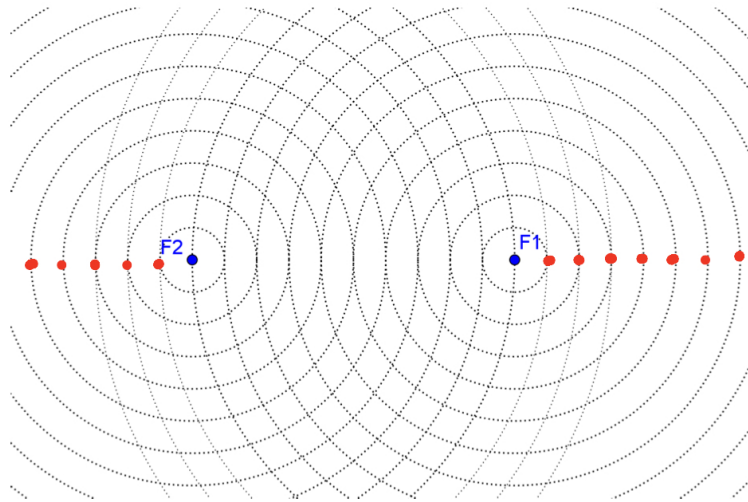


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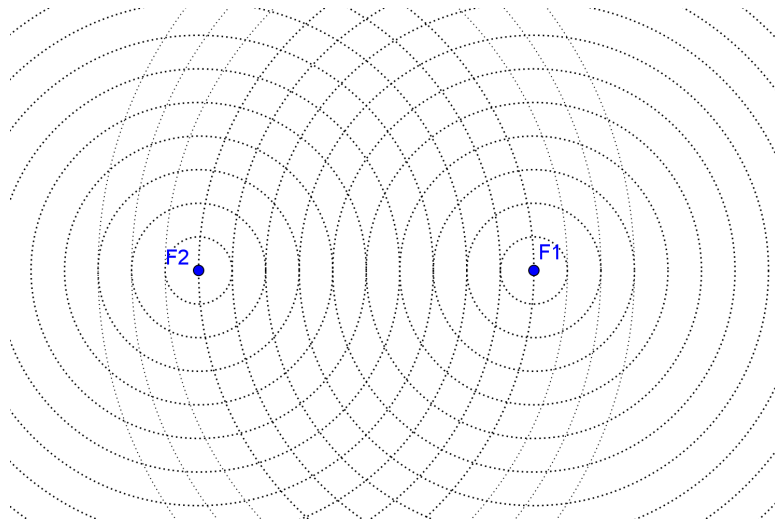
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Answer:

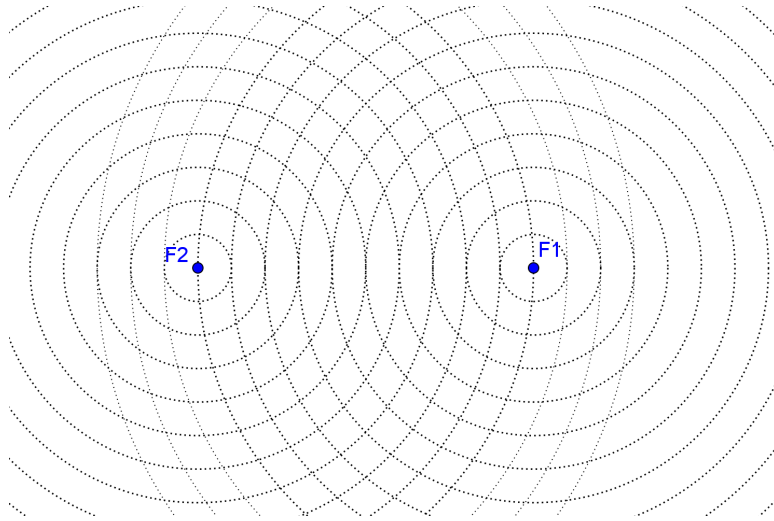


(2) ($2a=20, 2c=10$) The set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 20$. Determine what kind of graph will we get from using the concentric circle centered at points F_1 and F_2 .

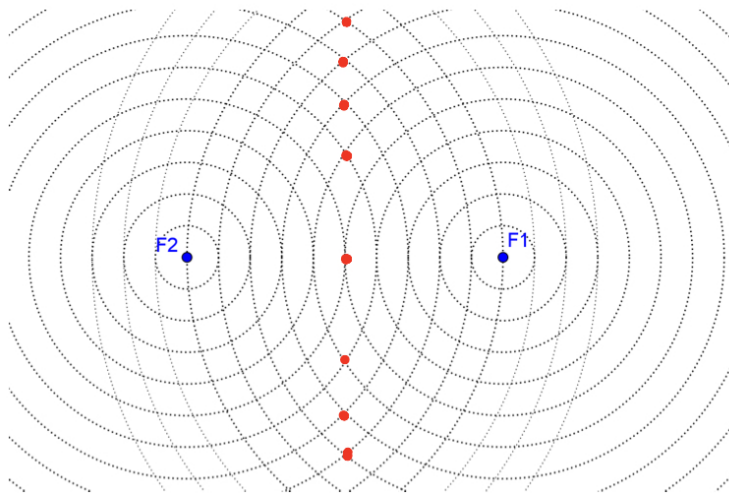


Answer: no graph (無圖形) .

(3) ($2a=0, 2c=10$) The set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 0$. Determine what kind of graph will we get from using the concentric circle centered at points F_1 and F_2 .



Answer:



Conclusion :

- (1) If $2a < 2c$, the set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 2a$ is a hyperbola.
- (2) If $2a = 2c$, the set of all points $P(x,y)$ that satisfies $|\overline{PF_1} - \overline{PF_2}| = 2a$ is two rays (two half-lines).
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製作者：台北市立育成高中 林玉惇