5 等比數列 Geometric sequences

課本內容 p.21

由摺紙認識等比數列：

摺疊 1 次是　2（$=2^1$）	張紙的厚度：$0.1\times 2^1=0.2$（毫米）
摺疊 2 次是　2×2（$=2^2$）	張紙的厚度：$0.1\times 2^2=0.4$（毫米）
摺疊 3 次是　$2\times 2\times 2$（$=2^3$）	張紙的厚度：$0.1\times 2^3=0.8$（毫米）
摺疊 4 次是　$2\times 2\times 2\times 2$（$=2^4$）	張紙的厚度：$0.1\times 2^4=1.6$（毫米）
摺疊 5 次是　$2\times 2\times 2\times 2\times 2$（$=2^5$）	張紙的厚度：$0.1\times 2^5=3.2$（毫米）
⋮	⋮
摺疊 23 次是　$2\times 2\times 2\times\ldots\ldots\times 2$（$=2^{23}$）	張紙的厚度：0.1×2^{23}（毫米）

而 0.2, 0.4, 0.8, 1.6, 3.2,……，
這些數形成一個新的數列，其中 $a_1=0.2$、$a_2=0.4$、$a_3=0.8$、$a_4=1.6$、$a_5=3.2$……，
且這個數列的規律是：每一項都是前一項乘以 2。

a_1 , a_2 , a_3 , a_4 , a_5 , …

0.2　0.4　0.8　1.6　3.2

$\times 2$　$\times 2$　$\times 2$　$\times 2$

翻譯示例：		
Fold a sheet of 0.1mm paper once	the thickness will be equal to the thickness of 2（$=2^1$） sheets of paper:	$0.1\times 2^1=0.2$（mm） thick.
Fold a sheet of 0.1mm paper twice	the thickness will be equal to the thickness of 2×2（$=2^2$） sheets of paper:	$0.1\times 2^2=0.4$（mm） thick.
Fold a sheet of 0.1mm paper 3 times	the thickness will be equal to the thickness of $2\times 2\times 2$($=2^3$) sheets of paper:	$0.1\times 2^3=0.8$（mm） thick.
Fold a sheet of 0.1mm paper 4 times	the thickness will be equal to the thickness of $2\times 2\times 2\times 2$（$=2^4$） sheets of paper:	$0.1\times 2^4=1.6$（mm） thick.
Fold a sheet of 0.1mm paper 5 times	the thickness will be equal to the thickness of $2\times 2\times 2\times 2\times 2$（$=2^5$） sheets of paper:	$0.1\times 2^5=3.2$（mm） thick.

<table>
<tr><td></td><td>⋮</td><td></td></tr>
<tr><td>Fold a sheet of 0.1mm paper 23 times</td><td>the thickness will be equal to the thickness of $2\times2\times2\times\ldots\ldots\times2$（$=2^{23}$） sheets of paper:</td><td>0.1×2^{23}（mm） thick.</td></tr>
<tr><td colspan="3">Then 0.2, 0.4, 0.8, 1.6, 3.2,..., these numbers form a new sequence, where $a_1=0.2$、$a_2=0.4$、$a_3=0.8$、$a_4=1.6$、$a_5=3.2$……
The pattern of this sequence is: each term is obtained by multiplying the previous term by 2. (The sequence is formed)</td></tr>
</table>

就另一個觀點，此數列的每一項除以前一項都等於 2，

即 $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}=\ldots\ldots=\dfrac{a_n}{a_{n-1}}=2$。

翻譯示例： On the other hand, each term of this sequence divided by the previous term is equal to 2. $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}=\ldots\ldots=\dfrac{a_n}{a_{n-1}}=2$

像這樣在一個數列中，**任意相鄰兩項的後項除以前項所得的商（比值）都相同，稱為等比數列，而這個商（或比值）稱為公比，通常用 *r* 表示**。

翻譯示例： In a sequence like this the quotient (ratio) of the subsequent term of any two adjacent terms divided by the previous term is the same, we called this a geometric sequence, and this quotient (or ratio) is called a common ratio, which is usually expressed by ***r***.

英文版定義： Definition: In a geometric sequence, each term can be obtained by multiplying the previous term by a constant value. This value is called the common ratio *r*.

雙語使用參考範例：

- ✧ 開場白

  T: There is a sheet of paper. The thickness is 0.1mm. If you fold this piece of paper in half several times. What do you observe?

- ✧ 引導觀察數列

  T: Let’s think about the numbers 0.2, 0.4, 0.8, 1.6, 3.2,...

  Could these numbers form a special sequence?

- ✧ 發現公比

  T: What is the ratio of 0.4 to 0.2?

  S: 2

  T: What is the ratio of 0.8 to 0.4?

  S: It’s also 2.

  T: The ratio of the subsequent term of any two adjacent terms divided by the previous term is the same.

- ✧ 引導為等比數列

  T: In a geometric sequence, each term can be obtained by multiplying the previous term by a constant value. This value is called the common ratio *r*.

  T: Is this sequence a geometric sequence?

  S: Yes.

  T: What is the common ratio of this geometric sequence?

  S: 2.

- ✧ 小結

  T: Now we know that this is a geometric sequence.

  The first term is 0.2 and the common ratio is 2.

  $$\frac{a_2}{a_1}=\frac{a_3}{a_2}=\ldots\ldots=\frac{a_n}{a_{n-1}}=2$$

課本內容 p.22

等比數列前後項關係

1. 一個數列 a_1, a_2, a_3, a_4, a_5, ……, a_{n-1}, a_n，
   若 $\frac{a_2}{a_1}=\frac{a_3}{a_2}=\cdots\cdots=\frac{a_n}{a_{n-1}}=r$，則此數列是公比為 r 的等比數列。
2. 等比數列中，公比$=\frac{\text{後項}}{\text{前項}}$，即 $r=\frac{a_n}{a_{n-1}}$。

翻譯示例：

Relation between previous term and subsequent term of a geometric sequence

1. A sequence a_1, a_2, a_3, a_4, a_5,……, a_{n-1}, a_n.

   If $\frac{a_2}{a_1}=\frac{a_3}{a_2}=\cdots\cdots=\frac{a_n}{a_{n-1}}=r$.

   Then this sequence is a geometric sequence with common ratio r.
2. In the geometric sequence,

   $\text{common ratio}=\frac{\text{subsequent term}}{\text{previous term}}$,

   that is, $r=\frac{a_n}{a_{n-1}}$.

例題

Identifying geometric sequences.

Determine whether each of the following series forms a geometric sequence.

If yes, find the common ratio of the sequence.

(1) 32, 8, 2, $\frac{1}{2}$

(2) $-1, -1, -1, -1, -1, -1$

(3) 0 , 0 , 0

Use the common ratio to complete the sequence.

Fill in the appropriate numbers in the following spaces, so that each sequence becomes a geometric sequence.

(1) ______, 4, −12, ______, ______

(2) ______, ______, 7, 21 , ______

參考資料來源

1. 110 國中數學 2 下翰林版課本
2. IB Maths SL Book Oxford

   Chapter 6 Patterns, sequences, and series
3. Holt McDougal Larson Algebra 2

   Chapter 7 Sequences and Series
4. Number Sequences - Square, Cube, and Fibonacci (mathsisfun.com)

   https://www.mathsisfun.com/numberpatterns.html

☆老師們可以自己從中選擇以做出適合自己學生程度的學習單或是在課堂中適時補充這些英文。

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