雙語教學主題(國中七年級教材):解一元一次不等式應用問題

Topic: solving word problems involving linear inequalities in one variable

這個單元常用到的一些用語

Some words or expressions we generally use in this topic

minimum, maximum, quotient, division, compare, symbol, requirement, determine, integer,

When we solve word problems, the most important thing is to READ THE QUESTIONS. You have to fully understand the questions and answer yourself:

What do we know? And what don't we know?

Just like solving real-world problems, there's a procedure we can follow as we solve and write the inequalities to solve real-world problems.

First step: define your variable (set the unknown)
This is "what don't we know?"

Second step: write an inequality to model the situation
This is "what do we know?" and use the appropriate inequality sign to get the inequality statement

Third step: solve the inequality and give the solution

There are some common used words for inequality signs we see in word problems.

>	greater, over, above, more, exceed, higher, larger, bigger,
<	less, fewer, lower, under, below, smaller, beneath,
≥	at least, minimum, no less than,
≤	at most, maximum, no greater than,

.

Word problems examples

Q1: The quotient of a number and 2 is less than or equal to 5 Sol: let the number be n $\frac{n}{2}$ is less than 5, $\frac{n}{2}$ is a smaller number comparing to 5, so $\frac{n}{2}$ stays on the left side of the symbol < The arrow always points to the smaller value and the larger value stays on the right side of the symbol $\frac{n}{2}<5$ $\frac{n}{2}$ and 5 can be equal too, so we use the symbol $\leq$ instead of < $\frac{n}{2} \leq 5$ $n \leq 10$	When we read the word quotient, it means there's a division. The quotient of a number and 2 means the unknown number n divided by 2, that is: $\frac{n}{2}$ Set the unknown The inequality would be n divided by 2 is less than or equal to 5 we move 2 to the right side and get n is less than or equal to 10

Q2: $\frac{1}{2}$ is greater than the product of $\frac{3}{4}$ and a number Sol: Let the number be x Remember the arrow of the symbol always points to the smaller value Then $\frac{1}{2} > \frac{3}{4}x$ $(4)\frac{1}{2} > \frac{3}{4}x(4)$ $2>3x$ $\frac{2}{3} > x$, the same as $x < \frac{2}{3}$	Product means multiplication The product of $\frac{3}{4}$ and a number means Three-quarters times a number Let the unknown be x The inequality would be one half is greater than three-fourths times x We multiply 4(LCD of denominators) on both sides and get two is greater than 3x divide both sides by 3 and two thirds is greater than x or x is less than two thirds

Q3: Find all pairs of consecutive odd natural numbers, both of which are larger than 15 such that the sum is less than 40 . Sol: Let the smaller odd number be x, then the larger odd number would be x+2 x>15 and x+(x+2)<40 2x+2<40 2x<38 X<19 So 15<x<19 x can be 17 only and x+2 would be19 The solution is: (17,19) Check: Let x=17, then x+2=19 17+19=36, 36<40 X=17 fits the requirement If we take x=19, then x+2=21 19+21=40, 40 is not less than 40 So x=19 is not the solution	Consecutive odd natural numbers are like 3,5 or 7, 9… Both of which are larger than 15, since x is the smaller odd number, if x is greater than 15, x plus 2 will also be greater than 15 X plus x plus 2 is less than 40 Combine like terms, we get two times x is less than thirty-eight divide both sides by 2, then x is less than nineteen with x is greater than 15 we get 15<x<19 x is an odd number and is greater than 15, x cannot be 15, so x can be 17 or more x is less than 19, x cannot be 19, so x can be 17 or less So x can only be 17 and x+2 would be 19 So there's only one solution (17,19) Checking after solving the word problems is essential. Please do it from time to time

Q4: A can of soda costs 35 dollars. Harry has 300 dollars with him. Please determine the number of cans of soda he can buy under 300 dollars? Sol: Let c be the number of cans of soda Then we get $35c<300$ $\frac{35c}{35}<\frac{300}{35}$ $c<8\frac{4}{7}$ $c\leq 8$ So Harry can buy at most 8 cans of soda under 300 dollars Check: If Harry buys 8 cans,$8\cdot 35=280$ $280<300$, he is able to pay for them If Harry wants to buy 9 cans $9\cdot 35=315$, $315>300$, he can't afford it	Set the unknown c The inequality would be thirty-five times c is less than three hundred divide both sides by thirty-five we get c is less than the mixed number eight and four seventh But c is the number of cans of soda, we cannot buy part of a can of drinks, this means c must be a whole number The largest whole number for c is 8

Q5:

A father is 40 years old and his son is 12 years old. in how many years will twice the son's age be more than the father's age?

Sol:

Let y be the number of years

person / time	Now	After y years
Father's age	40	40+y
Son's age	12	12+y

according to the question

This inequality should be

$2(y+12) > y+40$

Then $2y+24>y+40$

$y>16$

So $y=17$

In 17 years or more, twice of the son's age will be more than father's age

Check:

Take y=17,

Son's age will be 29, father's age will be 57

$2 \cdot 29=58$, $58>57$, it is a solution

Let's take y=18

Son's age will be 30, father's age will be 58

$2 \cdot 30=60$, $60>58$, it is a solution

If y=16

Son's age will be 28, father's age will be

Key phrases

twice the son's age 2(y+12)

more than >

father's age y+40

Set the unknown y

Please do not forget, after some years, <u>both father and son are growing old together</u>

2 times in parentheses y plus 12 is greater than y plus 40

Distribute 2 to the parentheses, we get

2 times y plus 24 is greater than y plus 40

Combine like terms

y is greater than 16

Since y is a whole number, y has to be 17 or more

So we choose the minimum

y is equal to 17

56 2·28=56, 56 is not more than 56 It doesn't meet the requirement	
	We know that An average is the sum of all scores/number of tests

Q6: There are three major exams in one semester. Amy has scores 73 and 81 from the last two exams in math. What score does she need in math in the final exam to get an average of no less than 85 in this semester? (full marks would be 100) Sol: Let the last score Amy needs be s We have $\frac{73+81+s}{3} \geq 85$ $73+81+s \geq 85 \cdot 3$ $154+s \geq 255$ $s \geq 101$ But the full mark is 100, so Amy **cannot** get the average of her scores in math no less than 85 in this semester	Set the unknown score to be s We have The quantity of 73 plus 81 plus s over 3 is greater than or equal to 85 Multiply 3 on both sides 73 plus 81 plus s is greater than or equal to 85 times 3 Combine like terms and transpose 154 to the right side We get s is greater than or equal to 101 But Amy can never make it due to the full mark is 100

Q7: Joseph wants to join a game social club. The club offers two kinds of plans for people to choose from. Plan A: 100 dollars to sign up, then charges 30 dollars each month Plan B: free to sign up, charges 50 dollars each month For what number of months will plan B charge at least as much as plan A? Sol" First we let n be the number of months Then $50n \geq 100+30n$ $20n \geq 100$ $n \geq 5$ So at least 5 months, plan B will charge equal to or more than plan A Check: Let's take n=5 Plan A charges $100+30 \cdot 5=250$ Plan B charges $0+50 \cdot 5=250$ Plan B charges as much as plan A Let's take n=6 Plan A charges $100+30 \cdot 6=280$ Plan B charges $0+50 \cdot 6=300$ $300>280$ Plan B charges more than plan A If n=4 Plan A charges $100+30 \cdot 4=220$ Plan B charges $0+50 \cdot 4=200$ $200<220$ Plan B charges less than plan A This doesn't meet the requirement	Set the unknown n The total Joseph needs to pay on each choice after n months Plan A: 100 plus 30n Plan B: 50n As much as means equal At least as much as means at least equal In other words, greater than or equal to

Q8: A YouTuber wants to get a sponsorship from a big brand by collecting more than fifty thousand viewers. The record shows he can get 3000 new viewers each month. Now he has forty thousand viewers already. At least how many months does this YouTuber need to get enough viewers to reach his goal?	
Sol: Let the YouTuber need at least x months to reach his goal	Set the unknown
$40000+3000x\geq 50000$	Forty thousand is a fixed number, the number of his viewers changes as the number of months increases Move forty thousand to the right side tree thousand times x is greater than or equal to fifty thousand
$3000x\geq 10000$	Divide both sides by one thousand 3 times x is greater than or equal to 10
$3x\geq 10$	Divide 3 on both sides x is greater than or equal to 3 point 333…
$x\geq 3.333\ldots$	
Since the number of months is a whole number So $x\geq 4$ This means the YouTuber needs at least 4 months to get this sponsorship	So x is greater than or equal to 4
Check: When x=4 This youtuber gets 40000+3000·4=52000 viewers which is more than 50 thousand	

If x=3 then 40000+3000·3=49000 49 thousand is less than 50 thousand He doesn't get enough viewers to get the sponsorship	

Q9: Three integers a, b, and c are in the ratio a:b:c=3:4:5, and the sum of the numbers are at most 100. Find the maximum values of these three numbers. Sol: Since a:b:c=3:4:5 Let a=3x, then b=4x, c=5x The sum of the number is a+b+c=3x+4x+5x According to the question, we have the inequality $3x+4x+5x\leq 100$ $12x\leq 100$ $x\leq \frac{25}{3}=8\frac{1}{3}$ a, b and c are integers, x must be an integer, so $x\leq 8$ We choose x=8 Then a=3x=3·8=24, b=4x=4·8=32, c=5x=5·8=40 The maximum values of a, b and c are 24, 32 and 40 Check: Let's try x=9 Then a=3x=27, b=4x=36, c=5x=45 a+b+c=27+36+45=108 108 is more than 100 So x=9 is not a solution	The ratio of a, b and c is a to b to c is 3 to 4 to 5 Set the unknown x a equals 3 times x, b equals 4 times x, and c equals 5 times x a plus b plus c is equal to 3 times x plus 4 times x plus 5 times x 3 times x plus 4 times x plus 5 times x is less than or equal to 100 Combine like terms Divide both sides by 12, we get x is less than or equal to twenty-five over 3, x is also equal to eight and one third Since a, b and c are the multiples of 3, 4, and 5 respectively, and under the condition that a, b and c are integers X has to be an integers too X can only be less than 8 or equal to 8 The maximum of x would be 8 Then a equals 3 times 8, a is 24 b equals 4 times 8, b is 32 c equals 5 times 8, c is 40

Q10: A surf shop has a weekly expense of 50 thousand dollars. Due to the pandemic situation, the shop has slow business and wants to boost their business this week by having promoting sales as "BUY ONE GET ONE FREE" That is: whenever you buy longboard then you can get a skimboard for free. You can also pay $1000 for a piece of skimboard only. A longboard costs $3000. According to the inventory: There are 50 longboards and 20 skim boards in the shop	
At least how many longboards does the surf shop need to sell to make a profit this week? Please describe the situation of the sales for this week.	Make profit means the income is greater than the expense
Sol: Let n be the number of longboards which will be sold this week Since the shop has only 20 skim boards, means the shop can only sell at most 20 longboards for the skimboards have to be give-away We have $n \leq 20$ Assume there're no skim boards left Then we have an inequality	
$3000n+1000(20-n)>50000$	3 thousand times n plus one thousand times the quantity of 20 minus n is greater than 50 thousand Divide both sides by 1000
$3n+(20-n)>50$	3 times n + parentheses twenty minus n is greater than fifty Combine like terms

<table>
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2n>30
n>15 n is a whole number, so
n≥16
We have
<table>
<tr><th>Long</th><th>Skim</th><th>Income</th></tr>
<tr><td>16·3000</td><td>0</td><td>48000</td></tr>
<tr><td>16·3000</td><td>1·1000</td><td>49000</td></tr>
<tr><td>16·3000</td><td>2·1000</td><td>50000</td></tr>
<tr><td>16·3000</td><td>3·1000</td><td>51000</td></tr>
<tr><td>17·3000</td><td>Any sales of skim boards is ok</td><td>51000</td></tr>
<tr><td>...</td><td>...</td><td></td></tr>
<tr><td>20·3000</td><td>No skim boards left</td><td>60000</td></tr>
</table>
The shop sells
(1) at least 16 longboards and 3 skimboards
Or
(2) 17 longboards or more, up to 20 pieces
The surf shop can then make their profit
</td>
<td>Divide both sides by 2
n is greater than 15</td>
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Q11: Sally has $5000and she spends $100 every day. Kevin has no money for now and he saves $200 every day. How many days does evin need to have at least as much money as Sally has? Sol: Let Kevin needs at least d days Then 200d≥5000-100d 300d≥5000 $d \geq \frac{50}{3} = 16\frac{2}{3}$, d is a whole number d≥17 The minimum of d would be 17 Kevin needs 17 days or more to have at least as much money as Sally has Check: When d=17 Kevin has 17·200=3400 Sally has 5000-100·17=3300 3400>3300 So it's a solution If d=16 Kevin has 16·200=3200 Sally has 5000-100·16=3400 3200<3400 Kevin has less money than Sally does	Let the unknown days Kevin needs be d 2 hundred times d is greater than or equal to 5 thousand minus one hundred times d Combine like terms and simplify 3 hundred times d is greater than or equal to 5 thousand d is greater than or equal to 50 over 3 or 16 and 2 thirds Since d is a whole number d has to be greater than or equal to 17

Q12: Rectangle ABCD is 6 cm long and 4cm wide. Equal quantities are to be added to each side, so that the perimeter becomes greater than 100. What possible values must be added? Sol: Let the equal quantity to be added be x	
Then the new length is (6+2x)cm the new width is (4+2x)cm The perimeter of the new rectangle has to be greater than 100 The inequality should be	6 plus 2 times x centimeter 4 plus 2 times x centimeter The perimeter of a rectangle is the sum of the length of all sides
2(6+2x)+2(4+2x)>100	2 times the quantity of 6 plus 2 times x plus 2 times the quantity of 4 plus 2 times x is greater than one hundred Distribute 2 to the parentheses
12+4x+8+4x>100	twelve plus 4 times x plus 8 plus 4 times x is greater than one hundred
8x>80	Combine like terms 8 times x is greater than 80
x>10 The equal quantity to be added is any number greater than 10 Check: Let's take x=10 2(6+2x)+2(4+2x)=100 100 is not greater than 100 It doesn't meet the requirement	x is greater than 10

If x=11 2(6+2x)+2(4+2x)=108 108>100 x=11 would be one of the solutions	

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