

## 【The general formula for the nth term of a geometric sequence】

問題：一個球在每次彈跳時，都會彈回原來高度的一半。如果這個球一開始從 1 米高的位置下落，而每次彈跳 10 次，請計算它在第 10 次彈跳後到達的高度是多少？我們先觀察下列的規律：

$$a_1 = 1$$

$$a_2 = \frac{1}{2} \quad (\text{首項乘以公比的 } 1 \text{ 次方})$$

$$a_3 = \frac{1}{4} = 1 \times \frac{1}{2} \times \frac{1}{2} = 1 \times \frac{1}{2}^2 \quad (\text{首項乘以公比的 } 2 \text{ 次方})$$

$$a_4 = \frac{1}{8} = 1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1 \times \frac{1}{2}^3 \quad (\text{首項乘以公比的 } 3 \text{ 次方})$$

⋮

$$a_{10} = 1 \times \frac{1}{2}^9 \quad (\text{首項乘以公比的 } 9 \text{ 次方})$$

翻譯示例：

Question: A ball bounces back to half of its original height with each bounce. If the ball starts by falling from a height of 1 meter and bounces 10 times, please calculate the height it reaches after the 10th bounce. Let's first observe the following pattern:

$$a_1 = 1$$

$$a_2 = \frac{1}{2}$$

(The first term is multiplied by the common ratio to the power of 1.)

$$a_3 = \frac{1}{4} = 1 \times \frac{1}{2} \times \frac{1}{2} = 1 \times \frac{1}{2}^2$$

(The first term is multiplied by the common ratio to the power of 2.)

$$a_4 = \frac{1}{8} = 1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1 \times \frac{1}{2}^3$$

(The first term is multiplied by the common ratio to the power of 3.)

⋮

$$a_{10} = 1 \times \frac{1}{2}^9$$

(The first term is multiplied by the common ratio to the power of 9.)

同理，首項為  $a_1$ ，公比為  $r$  的等比數列，首項及公比的關係如下：

$$a_2 = a_1 \times r = a_1 \times r^1 \text{ (首項乘以公比的 1 次方)}$$

$$a_3 = a_1 \times r \times r = a_1 \times r^2 \text{ (首項乘以公比的 2 次方)}$$

$$a_4 = a_1 \times r \times r \times r = a_1 \times r^3 \text{ (首項乘以公比的 3 次方)}$$

⋮

$$a_n = a_1 \times r \times r \times \dots \times r = a_1 \times r^{n-1} \text{ (首項乘以公比的 } n-1 \text{ 次方)}$$

翻譯示例：

Similarly, given a geometric sequence with the first term  $a_1$  and the common ratio  $r$ .

The pattern is as follows:

$$a_2 = a_1 \times r = a_1 \times r^1$$

(The first term is multiplied by the common ratio to the power of 1.)

$$a_3 = a_1 \times r \times r = a_1 \times r^2$$

(The first term is multiplied by the common ratio to the power of 2.)

$$a_4 = a_1 \times r \times r \times r = a_1 \times r^3$$

(The first term is multiplied by the common ratio to the power of 3.)

⋮

$$a_n = a_1 \times r \times r \times \dots \times r = a_1 \times r^{n-1}$$

(The first term is multiplied by the common ratio to the power of  $n-1$ .)

### 【等比數列第 $n$ 項公式】

如果一個等比數列的首項為  $a_1$ ，公比為  $r$ ，則第  $n$  項  $a_n = a_1 \times r^{n-1}$ 。

翻譯示例：

【The general formula for the  $n$ th term of a geometric sequence】

Given a geometric sequence with the first term  $a_1$  and the common ratio  $r$ , the  $n$ th (or general) term is given by  $a_n = a_1 \times r^{n-1}$ .

計算機使用舉例：

◇ 相加

Add numbers with the "+" key. Hit the "+" button between any 2 numbers to add them.

For example, to add 5 to 10, press "5," "+," and then "10."

Add additional numbers to a series.

For example, press "+" and "5" to add to the sum of "5 + 10."

When you need the final answer, press the "=" button to get the sum of "20."

◇ 相乘

Multiply numbers using the "x" key.

For example, to multiply 6 by 5, press "6," "x," "5," and then "=". The final answer will read "30."

Physical calculators often use "x" as a multiplication key.

Multiply in a series by pressing "x" followed by a number.

For example, if your calculator says "6 x 5," press "x," "2," and then "=" to get a final answer of "60."

◇ 指數

Calculate the exponent of a number by pressing "x^y."

The exponent (or power) of a number refers to how many times it's multiplied by itself.

The exponent key takes the first number (x) and multiplies it by itself a specified number of times as determined by "y."

For example, "2^6" is 2 to the power of 6, which is equal to "2 x 2 x 2 x 2 x 2 x 2."

This can be calculated with the power button using this order of operations: press "2," press "x^y," press "6," and press "=". The final answer is "64."

Any number (x) to the power of 2 is called x squared, while any number (x) to the power of 3 is called x cubed.

### 求第 $n$ 項 $a_n$

已知一個等比數列的首項為 6，公比為 2，求此等比數列的第 5 項。

翻譯示例：

Example 11: Find the  $n$ th term  $a_n$ .

Given a geometric sequence with the first term 6 and the common ratio 2.

Find the fifth term of this geometric sequence.

Solution:

$$a_1 = 6, r = 2, \text{ and } n = 5$$

Plug in numbers into the formula  $a_n = a_1 \times r^{n-1}$ ,

$$a_5 = 6 \times 2^{5-1} = 6 \times 16 = 96$$

The sixth term of this geometric sequence is 96.

### 應用 $a_n = a_1 \times r^{n-1}$

已知一個等比數列的首項為 2，公比為  $\frac{3}{2}$ ，求  $\frac{81}{8}$  是此數列的第幾項？

翻譯示例：

Example 12: Apply  $a_n = a_1 \times r^{n-1}$ .

Given a geometric sequence with the first term 2 and the common ratio  $\frac{3}{2}$ .

Which term is  $\frac{81}{8}$ ?

Solution:

$$a_1 = 2 \text{ and } r = \frac{3}{2}.$$

Suppose the  $n$  term is  $\frac{81}{8}$ .

Plug in numbers into  $a_n = a_1 \times r^{n-1}$ .

$$a_n = \frac{81}{8} = 2 \times \left(\frac{3}{2}\right)^{n-1}$$

$$\left(\frac{3}{2}\right)^{n-1} = \frac{81}{8} \times \frac{1}{2} = \frac{81}{16} = \left(\frac{3}{2}\right)^4$$

$$n-1 = 4 \text{ and } n = 5$$

So,  $\frac{81}{8}$  is the 5<sup>th</sup> term of the geometric sequence.

## 參考資料來源

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2. IB Maths SL Book Oxford  
Chapter 6 Patterns, sequences, and series
3. Holt McDougal Larson Algebra 2  
Chapter 7 Sequences and Series
4. [Number Sequences - Square, Cube, and Fibonacci \(mathsisfun.com\)](https://www.mathsisfun.com/numberpatterns.html)  
<https://www.mathsisfun.com/numberpatterns.html>
5. [How to Use a Calculator \(with Pictures\) - wikiHow](#)

☆老師們可以自己從中選擇以做出适合自己學生程度的學習單或是在課堂中適時補充這些英文。

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