雙語教學主題(國中七年級教材):解一元一次方程式
Topic: solving linear equations in one variable

這個單元常用到的一些用語
The vocabulary we generally use in this topic
Add(addition), subtract(subtraction), multiply(multiplication), divide(division),
plus(+), minus(-), times(x), divided(÷), isolate, equality,
reflective, symmetric, transitive, substitutive, linear equation, variable,
properties of equality (等量公理)

We learned in the last class that equations often have some variables and unknown values. And we also learned that solving an equation is to find out the unknown values of the variables. In this lesson, we are going to solve basic linear equations involving integers, fractions, and decimals.

Before we learn to solve the equations, we need to learn the properties of equality first. These properties of equality help us find the unknown values of the equations.

Properties of equality

Reflexive property a=a Example: 3=__3___	For any number a, a always equals to itself, which is a=a
Symmetric property If a=b, then b=a Example: If 1+6=x, then x=__1+6___	If a equals b, then b equals a If 1+6 equals x, then x equals 1 plus 6
Transitive property If a=b and b=c, then a=c Example: If x=5 and 5=y, then __x=y___	If a equals b and b equals c, then a equals c If x equals 5 and 5 equals y, then x equals y
Substitutive property If a=b, then we can replace a with b Example: If x=5 and y=x+3, then __y=(5)+3__	If x equals 5 and y equals x plus 3, then y equals (5) plus 3(replace x with 5)
For the next four properties of basic	

operation, the quantity of two sides of the equal sign is always the same. Whenever we apply the properties of equality, we have to keep the equal sign true；just like the scale, we have to keep it in balance. So don't forget to do everything on both sides together to keep the equal sign true.	
Addition property If a=b, then a+c=b+c Example: If x-3=5, then x-3+(3)=5+(3)	Add the same quantity on both sides If a equals b, then a plus c equals b plus c for any number c If x minus 3 equals 5, then x minus 3 plus (3) equals 5 plus (3)
Subtraction property If a=b, then a-c=b-c Example: If x+3=5, then x+3-(3)=5-(3)	Subtract the same quantity on both sides If a equals b, then a minus c equals b minus c for any number c If x plus 3 equals 5, then x plus 3 minus (3) equals 5 minus (3)
Multiplication property If a=b, then ac=bc Example: If $\frac{x}{2}=5$, then $\frac{x}{2}(2)=5(2)$	Multiple the same quantity on both sides If a equals b, then a times c equals b times c for any number c If x over 2 equals 5, then x over 2 times 2 equals 5 times 2
Division property If a=b, then $\frac{a}{c}=\frac{b}{c}$, c≠0 Example: If 3x=5, then $\frac{3x}{3}=\frac{5}{3}$	Divided both sides by the same quantity If a equals b, then a divided by c equals b divided by c for any **non-zero** number c If 3 times x equals 5, then 3 times x over 3 equals 5 over 3

We all know the order of basic operations is:
Multiplication and division first, and then addition and subtraction

Multiplication and division have equal priority. If there are multiplication and division involved together, we do it from left to right For example: 3÷2·x We divide 3 by 2 then the result times x Also, addition and subtraction have equal priority. If there are addition and subtraction involved together, we do it from left to right as well

But when we solve the equations, we always need to reverse the process. That means we will handle addition and subtraction first, then multiplication and division

Let's see the examples below:

Ex 1: Find out the value of x in the equation x-3=5 x-3 +3 =5 +3 x=5+3 x=8 Check: When we replace x by 8 in the equation, we have x-3=8-3 =5 The left side 8-3 is 5, obviously the result from the left side is equal to the number on the right side	When we need to find out what the value of x is, we always, I mean always keep x on one side of the equal sign and let the numbers stay on the other side of the equal sign In this equation, x is not isolated, x is with a number on the left side of the equal sign, it's x minus 3 on the left side. We need to get rid of this number by using the inverse operation of **minus**, that is **plus**. When we add 3 on both sides, minus 3 and positive 3 will be canceled and become zero Now x is isolated on the left side of the equation, we can easily find out the value of x by adding 5 and 3, which is 8 So the value of x is 8 We solved the equation.

5=5 So the answer is correct.	
Ex 2: Solve the equation y+2=6 y+2 -2=6 -2 y=4 Check: We replace y=4 on the left side of the equation, we get y+2=4+2=6 The result is equal to the number on the right side which is 6 So y=4 is the solution	We will find out the value of y in this equation. Since y is not alone, we need to isolate y on one side of the equation, so we have to take away plus 2. The inverse operation of plus 2 is minus 2, We now minus 2 on both sides of the equation to keep the equal sign balance. Plus 2 is canceled with minus 2, now y is alone on the left side of the equation. On the right side we have 6 minus 2, we know that is 4 So y equals 4
Ex 3: Find the solution of the equation $\frac{3}{4}$w=6 $\frac{3}{4}$w ($\frac{4}{3}$)=6 ($\frac{4}{3}$)	3 over 4 times w is equal to 6 We want w to be isolated by multiplying the reciprocal of the fraction $\frac{3}{4}$ which is $\frac{4}{3}$ on both sides of the equation 3 quarters times w times 4 thirds is equal to 6 times 4 thirds On the right side, numerator 3 is canceled by denominator 3, and numerator 4 is also canceled by denominator 4, so w is left alone on the left side. On the left side, numerator 6 and

w=2·4 w=8 Check: We replace w by 8 on the left side of the equation $\frac{3}{4}w=\frac{3}{4}(8)=3\cdot 2=6$ The result from the left side of the equation is equal to the number on the right side which is 6 So w=8 is the solution to this equation	denominator 3 have the common factor 3, we divided 3 on both of them, then we get 2 left to times 4 which is 8
Ex 4: Find the value of z in the equation 4z=3 $\frac{4z}{4}=\frac{3}{4}$ $z=\frac{3}{4}$ Check: We replace z on the left side of the equation by $\frac{3}{4}$, we get $4z=4(\frac{3}{4})=3$ 3 is the same of the number on the right side 3=3	4 times z is 3 Since the inverse operation of multiplication is division, so here we only need to divide both sides by 4, then 4 and 4 are canceled and z is left alone. 4 times z divided by 4 is equal to 3 divided by 4 4 divided by 4 is 1, so z equals 3 over 4

So the value of z is $\frac{3}{4}$	

After we learn so many basic properties of operations, we now will further our learning by solving more complexed equations.

Let's start with solving a two-step equation

Ex 5: Solve the linear equation 2x-3=5 2x-3 +3=5 +3 2x=8 $\frac{2x}{2}=\frac{8}{2}$ x=4 Check: We replace x by 4 on the left side of the equation 2x-3=2(4)-3=5 Which is also equal to the number on the right side So x=4 is the solution	As we mentioned before, when we want to undo the operations to get what the value of the variable is, we always handle addition and subtraction first, then multiplication and division. In this example, if we want x to be alone on one side of the equation, we need to let the operation 'times 2' and 'minus 3' disappear. I always say: x 'times 2' means x and 2 are more strongly connected; but x 'minus 3' shows that x and 3 have a weaker link comparing to the multiplication. So when we undo these operations, we start by handling 'minus 3' than 'times 2'. The first step: We add positive 3 on both sides Then 2 times x is left on the left side and 5 plus 3 is 8 on the right side. Now we divide both sides by the common factor of 2 and 8 which is 2 Then x equals 4
Ex 6:	

Find the value of y in the equation $2=3y+5$ $2-5=3y+5-5$ $-3=3y$ $\frac{-3}{3}=\frac{3y}{3}$ $-1=y$ $y=-1$ Check: Let's replace y by -1 on the right side of the equation $3y+5=3(-1)+5=-3+5=2$ which is equal to the number of the left side of the equation so the value of y is -1	 2 equals 3 times y plus 5 We need to isolate y, so we want 'times 3' and 'plus 5' to disappear. We undo 'plus 5' by subtract 5 on both sides 2 minus 5 is equal to 3 times y plus 5 minus 5 Then negative 3 equals 3 times y Divide both sides by 3 negative 3 divided by 3 is equal to 3 times y divided by 3 negative 1 equals y As we mentioned symmetric property above, y is also equal to negative 1
Ex 7: Solve the equation $\frac{x}{2}+\frac{1}{3}=\frac{5}{6}$ $6(\frac{x}{2}+\frac{1}{3})=\frac{5}{6}\cdot 6$ $6\cdot\frac{x}{2}+6\cdot\frac{1}{3}=5$ $3x+2=5$ $3x+2-2=5-2$	We see some fractions in this equation Normally dealing with integers is easier than fractions, so the first step we do is find out the LCM(least common multiple) for the denominators2,3 and 6 and the LCM is 6. We multiply 6 on both sides Then apply the distributive property 6 multiplies all the fractions in parentheses 6 times x over 2 plus 6 times one third is equal to 5, then 3 times x plus 2 equals 5 Subtract 2 on both sides 3 times x plus 2 minus 2 is equal to 5 minus 2

$3x=3$ $X=1$ Check: We replace x by1 on the right side of the equation $\frac{x}{2}+\frac{1}{3}=\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ The result is the same as the number on the right side of the equation $\frac{5}{6}$ So x=1 is the solution	3 times x equals 3 Divided both sides by 3, we get X equals 1
Ex 8: Solve the equation $0.4=3+0.2y$ $0.4(10)=(3+0.2y)(10)$ $4=3(10)+0.2y(10)$ $4=30+2y$ $4-30=30+2y-30$ $-26=2y$ $\frac{-26}{2}=\frac{2y}{2}$ $-13=y$ $y=-13$ Check: We replace y by -13 on the right side of the equation	We see some decimals in this equation Normally dealing with integers is easier than decimals. All the decimals are in the place of tenths, we can multiply 10 on both sides and the decimals will be gone. The first step is we multiply 10 on both sides zero point 4 times 10 is equal to the quantity of 3 plus zero point 2 times y times 10 Apply the distributive property 4 equals 3 times 10 plus zero point 2 times y times 10 4 equals 30 plus 2 times y Subtract 30 on both sides 4 minus 30 is equal to 30 plus 2 times y minus 30 Negative 26 is equal to 2 times y Divide both sides by 2 Negative 26 over 2 is equal to 2 times y over 2, we get Negative 13 equals y The same as y equals negative 13

3+0.2y=3+0.2(-13)=3+(-2.6)=0.4 The same as the number on the left side So y=-13 is the solution	
Ex 9: Solve the equation	
5-3x=4	5 minus 3 times x is 4 No matter what the value of x would be, we can always add the same quantity on both sides of the equation and still keep the balance. It's more complicated to handle negative 3 times x, so I'll show you 2 ways to solve this equation
Method 1:	Method 1:
5-3x +3x=4 +3x	Add 3x on both sides 5 minus 3 times x plus 3 times x is equal to 4 plus 3 times x Then we get
5=4+3x	5 equals 4 plus 3 times x
5 -4=4+3x -4	We subtract 4 on both sides 5 minus 4 is equal to 4 plus 3 times x minus 4, we get
1=3x	1 equals 3 times x
$\frac{1}{3}=\frac{3x}{3}$	Divide both sides by 3 one third is equal to 3 times x over 3
$\frac{1}{3}=x$	one third equals x Also x equals one third
$x=\frac{1}{3}$	
Method 2:	Method 2:
5-3x=4	If we want to keep x on the left side, we need to move the numbers 5 and 3 to the right side. We first subtract 5 on both sides
5-3x -5=4 -5	5 minus 3 times x minus 5 is equal to 4 minus 5, we get
-3x=-1	Negative 3 times x is equal to negative 1 Divide both sides by negative 3

$\frac{-3x}{-3}=\frac{-1}{-3}$ $x=\frac{1}{3}$ Check: We replace x by $\frac{1}{3}$ on the left side of the equation $5-3x=5-3(\frac{1}{3})=5-1=4$ The number on the right side is also 4 So no doubt $x=\frac{1}{3}$ is the solution	Negative 3 times x over negative 3 is equal to negative 1 over negative 3 Then x equals one third

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