指數律
Laws of Exponent

Class:_____   Name: _____

1. $a^m \times a^n$ [Product of Powers]

Product of Powers	Product as Repeated Multiplication	Single Power
$5^4 \times 5^3$	$(5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5)$	5^7
$(-2)^2 \times (-2)^3$		
$(\frac{1}{3})^4 \times (\frac{1}{3})^2$		

※ Exponent Law for Product of Powers : $a^m \times a^n = a^{m+n}$

2. Fill in the blanks.
   (1) $10^7 \times 10^6 = 10^{\square}$

   (2) $(-\frac{1}{3})^3 \times (-\frac{1}{3})^9 = (-\frac{1}{3})^{\square}$

3. $a^m \div a^n$ [Quotient of Powers]

Quotient of Powers	Quotient as Repeated Multiplication	Single Power
$8^7 \div 8^3$	$\dfrac{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8}$	8^4
$4^5 \div 4^2$		
$(-9)^6 \div (-9)^5$		

※ Exponent Law for Quotient of Powers : $a^m \div a^n = a^{m-n}$

4. Fill in the blanks.
   (1) $5^{12} \div 5^7 = 5^{\square}$

   (2) $(-13)^9 \div (-13)^4 = (-13)^{\square}$

5. $(a^m)^n$ [Power of Powers]

Power of Powers	As Repeated Multiplication	Factors	Single Power
$(6^2)^3$	$6^2 \times 6^2 \times 6^2$	$(6\times6)\times(6\times6)\times(6\times6)$	6^6
$(7^4)^2$			
$(2^3)^3$			

※ Exponent Law for Power of a Power : $(a^m)^n = a^{m \times n}$

6. Fill in the blanks.

(1) $(11^3)^5 = 11^\square$

(2) $(8^6)^4 = 8^\square$

7. $(a \times b)^m$ [Power of Product]

Power of Product	As Repeated Multiplication	Product of Powers
$(6\times4)^2$	$(6\times4)\times(6\times4)=(6\times6)\times(4\times4)$	$6^2\times4^2$
$(2\times9)^3$		
$(7\times11)^2$		

※ Exponent Law for Power of a Product : $(a \times b)^m = a^m \times b^m$

8. Fill in the blanks.

(1) $(2\times5)^5 = 2^\square \times 5^\square$

(2) $(14\times10)^7 = 14^\square \times 10^\square$

一、設計理念：

1. 學生已學過分數的四則運算及指數的表示法，本章節指數律可涉及底數為分數。
2. 由於學習內容以計算為主，故學習單皆以英文呈現，並先附上一題範例方便學生了解所要操作的動作。
3. 語言部分仍須考量學生認知負荷，適時藉由中文輔助學生學習。
4. 學生於指數律操作容易記錯公式或錯誤使用公式，故學習單主要透過反覆操作讓學生理解指數律的規則。
5. 通常 5^{4+3} 的英文會使用"five to the power of the quantity of four plus three"，但在課程中我們會同時將過程寫在黑板上，所以有時會省略"quantity"一字，也不會造成學生理解錯誤。加 quantity 一字的目的是為了讓學生知道我們應該要先做指數部分。
6. 以下提供四條指數律的英文參考說法。

算式	名稱	公式規則
$a^m \times a^n$	Product of Powers	To multiply powers with the same base, add their exponents.
$a^m \div a^n$	Quotient of Powers	To divide powers with the same base, subtract their exponents.
$(a^m)^n$	Power of Powers	To raise a power to a power, multiply the exponents.
$(a \times b)^m$	Power of Product	To raise a product to a power, raise each factor by the given exponent.

二、英文詞彙：

中文	英文
指數	exponent
底數	base
次方	power
指數律	exponent law
乘法	multiplication
乘積	product
商數	quotient
加	plus
減	minus
乘	multiply
除	divide

三、數學英文用法：

數學表示法	英文
5^{4+3}	five to the power of the quantity of four plus three
8^{7-3}	eight to the power of the quantity of seven minus three
$(6^2)^3$	six to the power of two quantity to the power of three
$6^{2 \times 3}$	six to the power of the quantity of two times three
$(6 \times 4)^2$	six times four quantity to the power of two

四、教學參考範例：

1	我們再來複習一次指數的意思。What does 3^5 mean? It means 3 is multiplied by itself 5 times, so 3^5 is equal to 3 times 3 times 3 times 3 times 3.
2	接下來，我們想對指數做一些運算，看看運算時有沒有什麼特別的性質，因為指數的形式是省略乘法，所以我們先從乘法開始看。Let's see the first one, product of powers. We want to multiply two exponents. For example, what is 5 to the power of 4 times 5 to the power of 3. You can calculate the value directly, like 5 to the power of 4. It means 5 is multiplied by itself 4 times, so it is equal to 625. In the same way, 5 to the power of 3 is 125. Then the final answer is 625 times 125, but this is not the way we want to use in today's class.
3	按照題目引導我們的方法，第一個步驟是先將兩個指數寫成連續乘法的型式，也就是把指數的意義寫出來。So, 5 to the power of 4 is 5 times 5 times 5 times 5, and 5 to the power of 3 is 5 times 5 times 5.
4	我們知道如果都是在做乘法時，其實括號可以省略。Then, how many 5s multiply together? 7. How do we express the final answer? It is equal to 5 to the power of 7. We do not need to calculate the exact value. We can write down the answer in exponent form directly.
5	Let's do some practice. You can try the rest by yourself. What is -2 quantity to the power of 2 times -2 quantity to the power of 3, and what is $\dfrac{1}{3}$ to the power of 4 times $\dfrac{1}{3}$ to the power of 2? Follow the steps above and try to rewrite them in the same way.

五、引入語言使用建議：

第 1 段	複習指數的概念，先用中文喚起學生指數的概念，再利用英文舉指數的實例，讓學生逐步進入英語的使用。
第 2 段	為了讓學生較容易接下來的授課目標，先用中文概述接下來想做的事情，再切換成英文講述。由於後半段僅是涉及到直接計算，對學生已熟悉，且老師同時會在黑板上操作，故語言理解上認知負荷低，故皆採英文。
第 3 段	前半段先用中文引導，後半段再以英文進行實際操作，而操作僅是換成實際數字的乘法，英文相對簡單，故轉換為英文。
第 4 段	用中文解釋「幾的幾次方」的英文要怎麼說，讓學生熟悉套用模式，方便往後課程大量使用。
第 5 段	讓學生能將練習表格中剩下來的兩題，藉由實際朗讀題目讓學生了解接下來自己需要練習的題目，由於認知上不難故採全英文。

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