Linear Equations

I. Key mathematical terms

Terms	Symbol	Chinese translation
Slope (Gradient)		
Variable		
Coefficient		
Constant		
Intercept		
Perpendicular		
Parallel		
(Cartesian) Coordinate		

II. What is a linear equation?

An equation says that two things are equal. We use the equal sign "=" to state "this equals that". A linear equation is an equation in which the highest power of the variable is always 1. It is also known as a one-degree equation. For example, $ax+by=c$ is a linear equation in two variables. Here, $\boldsymbol{x}$ and $\boldsymbol{y}$ are variables ( ), $\boldsymbol{a}$ and $\boldsymbol{b}$ are coefficients ( ) and $\boldsymbol{c}$ is the constant ( ).

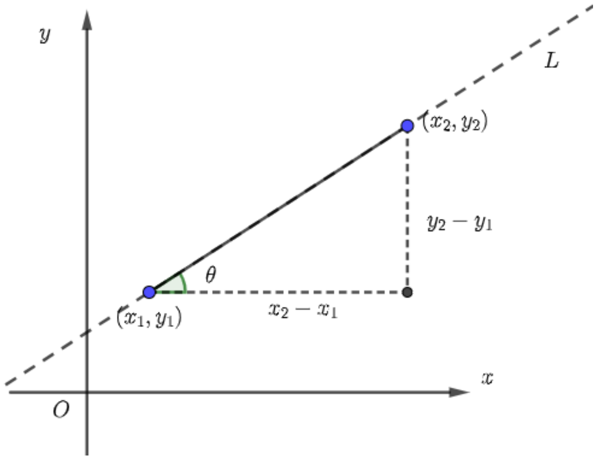
To describe a straight line in the Cartesian coordinate, we need to understand the meaning of slope (gradient).

III. What is the slope? (gradient)

The slope describes the inclination of a straight line. Finding the slope of lines in the coordinate can help us predict whether the lines are parallel, perpendicular, or just intersects at one point.

We can find the slope of a straight line with two distinct points lying on it. The formula for finding the slope of a line calculates the ratio of the "vertical change" to the "horizontal change" between two distinct points on the line.

The slope of a straight line



Slope (Gradient) = $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x} = \dfrac{\text{Change in } y}{\text{Change in } x}$

Example1.

1. Work out the slope of the line joining these pairs of points

   (1) $(4,2),(6,3)$

   (2) $(-1,2),(0,1)$

   (3) $(0,2a),(-10a,0)$

   (4) $(p,p^2),(q,q^2)$

   (5) $(2,3),(5,3)$

   (6) $(-1,3),(-1,7)$

<key>

The slope of a horizontal line is __________________.

The slope of a vertical line __________________.

(Hint: You can work out 1. (5) and 1. (6) to find out.)

2. The line joining $(3,-4),(-g,2g)$ has slope -3. Work out the value of g.

IV. Equations of straight lines

To determine the equation of a line, we can use different types of formulas. These formulas are:

Forms for the equations of straight lines		
The slope-intercept form	$y = mx + b$	***m*** is the slope ***b*** is the ***y***-intercept
The point-slope form	$y - y_0 = m(x - x_0)$	***m*** is the slope (x_0, y_0) is a point on the line
The intercept form	$\frac{x}{a} + \frac{y}{b} = 1$	***a*** is the ***x***-intercept ***b*** is the ***y***-intercept
The general form	$ax + by + c = 0$	***a, b,*** and ***c*** are constants
Vertical line	$x = a$	A vertical line with ***a*** as the ***x***-intercept
Horizontal line	$y = b$	A horizontal line with ***b*** as the ***y***-intercept

The slope-intercept form: $y = mx + b$

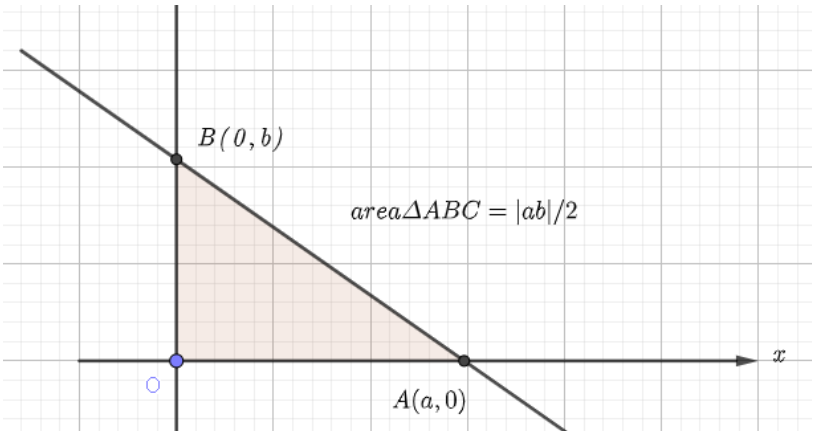
In the slope-intercept form, ***m*** is the slope and ***b*** is the ***y***-intercept. The lines having undefined slopes, cannot be represented by this form. The ***y***-intercept can be found when $x=0$ and $y=b$. The slope-intercept form is one of the most common forms we've learned in junior high school. We can easily read the slope and y-intercept from the equation.

The point-slope form: $y - y_0 = m(x - x_0)$

The point-slope form is useful when we know: One point (x_0, y_0) on the line with Slope ***m***. Same as the slope-intercept form, if the lines have undefined slopes, we cannot use this form.

The intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

In the intercept form, ***a*** is the ***x***-intercept and ***b*** is the ***y***-intercept. We cannot use this form when the ***x***-intercept or ***y***-intercept equals zero. Since the line cuts both coordinates, the area of the right-angle triangle is equal to half of the product of its intercepts. Also, the sign of the intercepts in the equation helps us know the location of the line with respect to the coordinate axes.



The intercept form

The general form: $ax+by+c=0$

The general form is an equation of the form ax+by+c=0. In general form, a or b can be zero, but not both at the same time. The equation is usually written so that a>=0. The graph of the equation is a straight line and every straight line can be represented by this form. (even the vertical line with undefined slope)

Vertical line: $x=a$

To describe a vertical line, we only need to write x=a. A vertical line has an undefined slope.

Horizontal line: $y=b$

To describe a horizontal line, we only need to write y=b. A horizontal line has Slope zero.

Example2.

1. Find the equation of the line with slope 5 that passes through the point (0,7).

2. Find the equation of the line with slope 3 that passes through the point (-2,3).

3. Find the equation of the line with x-intercept -2 and y-intercept 3.

4. Find the equation of the line that passes through (5,3) and (-2,1). (Use the general form to answer the question.)

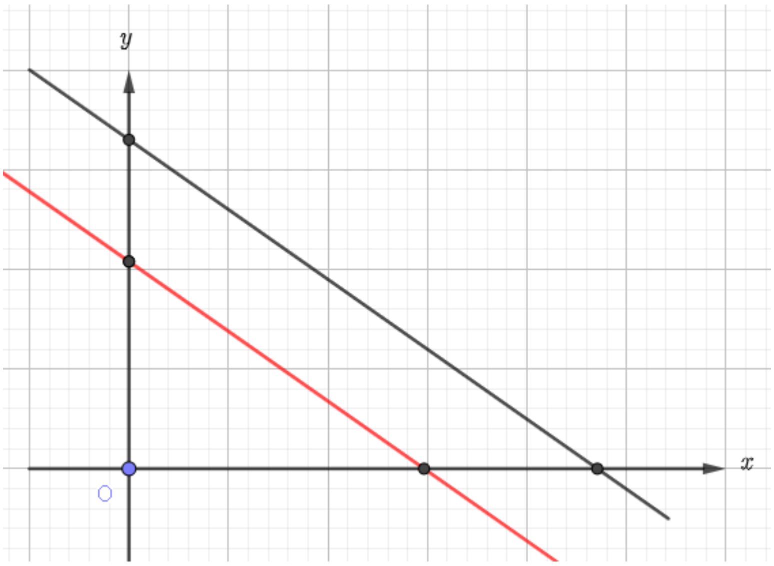
5. Find the equation of the line with an undefined slope that passes through (0,-7).

6. Find the equation of the line with slope 0 that passes through the point (-0.5,1.7).

7. The straight line passes through (7a,5) and (3a,3) has an equation $x+by-12=0$. Find the values a, b.

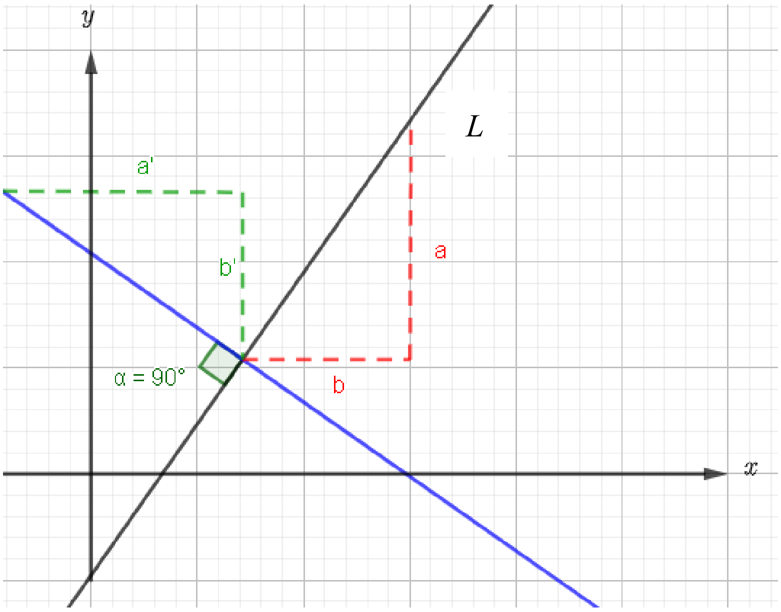
V. Parallel and perpendicular lines

Parallel lines have the same slope.



Parallel lines have the same slope.

Perpendicular lines are right angles to each other. If we know the slope of one line, we can find the slope of the other.



The red triangle and green triangle are congruent.
The slope product of this two triangle equals -1.

Suppose the line L has a slope of m($m \neq 0$), then the line perpendicular to it has a slope of -$1/m$. If two lines are perpendicular, the product of their slope is -1.
<key> The horizontal and vertical lines are perpendicular but their slope's product doesn't exist. (The vertical line has an undefined slope.)

Example3.

1. Work out whether these pairs of lines are parallel, perpendicular, or neither:

(1) $\begin{cases} 3x - y - 2 = 0 \\ x + 3y - 6 = 0 \end{cases}$

(2) $\begin{cases} x + y - 5 = 7 \\ 3x + 3y = 0 \end{cases}$

(3) $\begin{cases} x = 2y \\ x + y = 5 \end{cases}$

(4) $\begin{cases} x = \dfrac{1}{2}y \\ 2x - y + 5 = 0 \end{cases}$

2. Find an equation of the line that passes through the point (3,4) and is perpendicular to the line $3x-5y+1=0$.

3. Find an equation of the line that passes through the point (-3,5) and is vertical to the line $3x-5y+1=0$.

4. The vertices of a quadrilateral ABCD have coordinates A(-1,5), B(7,1), C(5,-3) and D(-3,1). Show that the quadrilateral is a rectangle.

<資料來源>

1. **Slope**
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3. **Pearson Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS Chapter 5**

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