

## Topic: Properties of logarithms/Laws of logarithms (數A二上)

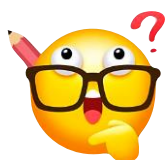
Review

How do we read  $\log 5$  ?

How do we read  $\log_5 125$  ?  $\log_5 125 = ?$

How do we read  $\log_3 \frac{1}{3}$  ?  $\log_3 \frac{1}{3} = ?$

### 1. Change of base formula



I would like to find the answer of  $\log_3 \frac{1}{3}$  by the calculator.  
But there is no the button  $\log_3$ . What can I do ?



Maybe we can use the only button  $\log$  to do something change.

### Investigation

Find the value of the following with a calculator and think about the relationship between the different bases of the logarithm.

a.  $\frac{\log \frac{1}{3}}{\log 3} =$

b.  $\frac{\log 125}{\log 5} =$

So, what do you think?

Watch the clip and fill in the blank.

**Change of base formula**

$\log_b a =$  \_\_\_\_\_



### Exercises

Find the value of the following with a calculator. (Use base 10)

a.  $\log_2 5$

b.  $\log_5 18$

### Challenge

Use the change base formula to find the value of  $\log_2 5 \times \log_5 2$

## 2. Adding logarithms

We can do the operation of logarithms.

Watch the clip from 2:12 and fill in the blank.



$$\log_a p + \log_a q = \underline{\hspace{2cm}}$$

Proof

Let  $m = \log_a p$  and  $n = \log_a q$

The corresponding exponential forms of these two equations are \_\_\_\_\_

Then multiply  $p$  by  $q$  to obtain \_\_\_\_\_

The corresponding logarithmic form of \_\_\_\_\_ is \_\_\_\_\_.

So, \_\_\_\_\_

### Exercises

Find the value of the following

a.  $\log 2 + \log 5$

b.  $\log_6 2 + \log_6 3$

### Challenge

Fill in the blanks

$$\log_a p + \log_a p + \log_a p = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

What have you noticed?

## 3. Subtracting logarithms

$$\log_a p - \log_a q = \underline{\hspace{2cm}}$$

Proof

Let  $m = \log_a p$  and  $n = \log_a q$

The corresponding exponential forms of these two equations are \_\_\_\_\_

Then \_\_\_\_\_ to obtain \_\_\_\_\_

The corresponding logarithmic form of \_\_\_\_\_ is \_\_\_\_\_.

So, \_\_\_\_\_

### Exercises

Find the value of the following

a.  $\log 300 - \log 3$

b.  $\log_6 12 - \log_6 2$

### 4. Powers and coefficients

#### Investigation

Use the definition of logarithms and the properties of exponents to verify (證明)

$$\log_a p^n = n \log_a p$$

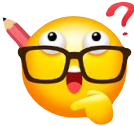

### Exercises


Find the value of the following


a.  $\log_3 9^4$


b.  $2\log 6 + 3\log 5 - \log 45$

Topic: Properties of logarithms/Laws of logarithms (數A 二上) 使用建議

<p>內容</p>	<p>Review          How do we read <math>\log 5</math> ?          How do we read <math>\log_5 125</math> ? <math>\log_5 125 = ?</math>          How do we read <math>\log_3 \frac{1}{3}</math> ? <math>\log_3 \frac{1}{3} = ?</math></p>
<p>使用建議</p>	<p>[教學活動安排]          開場先複習讀法做為暖身且為後續探索換底公式做鋪陳</p> <p>[可參考的英文問句/提問/開場]          Hello everyone. Let's review this . How do we read this expression( <math>\log 5</math> )? (Or how do we this expression?)          How do we read this (<math>\log_5 125</math>)? What is the value?          How do we read this (<math>\log_3 \frac{1}{3}</math>)? What is the value?</p> <p>[參考讀法]  <math>\log 5</math>: Log of five (一定要有 of 才是函數)  <math>\log_5 125</math>: log base five of one hundred and twenty- five or log one hundred and twenty- five of base five.  <math>\log_5 125 = 3</math> log base five of one hundred and twenty- five equals to three.  <math>\log_3 \frac{1}{3}</math>: log base three of one over three or log one over three of base three.  <math>\log_3 \frac{1}{3} = -1</math> log base three one over three equals to negative one.</p>
<p>內容</p>	<p>1. Change of base formula</p> <div style="display: flex; align-items: center; margin-bottom: 20px;">  <div style="margin-left: 20px; border: 1px solid black; padding: 10px; border-radius: 10px; width: fit-content;"> <p>I would like to find the answer of <math>\log_3 \frac{1}{3}</math> by the calculator. But there is no the button <b>log<sub>3</sub></b>. What can I do?</p> </div> </div> <div style="display: flex; align-items: center; margin-bottom: 20px;">  <div style="margin-left: 20px; border: 1px solid black; padding: 10px; border-radius: 10px; width: fit-content;"> <p>Maybe we can use the only button <b>log</b> to do something change.</p> </div> </div> <p><b>Investigation</b>          Find the value of the following with a calculator and think about the relationship between the different bases of the logarithm.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>a. <math>\frac{\log \frac{1}{3}}{\log 3} =</math></p> </div> <div style="text-align: center;"> <p>b. <math>\frac{\log 125}{\log 5} =</math></p> </div> </div>

	So, what do you think?
使用建議	<p>[教學活動安排]</p> <p>透過對話引起學生好奇並讓學生從實際數字例子使用計算機去猜測可能的結果</p> <p>[可參考的英文問句/提問/開場]</p> <p>Today we're going to learn the properties of logarithms. The first one is the change of base formula.</p> <p>Did you have the same question as the dialogue on the worksheet? See the dialogue.</p> <p>Let's investigate and think exactly what the change of base formula is.</p> <p>[參考答案]</p> <p><b>Investigation</b></p> <p>Evaluate the equations with a calculator and think what is the relationship between the different bases of the logarithm.</p> <p>a. <math>\frac{\log \frac{1}{3}}{\log 3} = -1 = \log_3 \frac{1}{3}</math>      b. <math>\frac{\log 125}{\log 5} = 3 = \log_5 125</math></p> <p>So, what do you think?</p> $\log_b a = \frac{\log a}{\log b}$
內容	<p><b>Change of base formula</b></p> <p>Watch the clip and fill in the blank.</p> <p><math>\log_b a = \underline{\hspace{2cm}}</math></p> 
教學建議	<p>[教學活動安排]</p> <p>透過觀看影片讓學生理解此性質並做練習</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's watch the clip to see what is the "change of base formula".</p> <p>Let's watch this short video together to learn about the 'change of base formula'</p> <p>看完影片到 1:38, 00(學生名) Share with us what is the "change of base formula"</p> <p>回饋學生: Well done.</p> <p>接著繼續看影片到 3:48. Let's keep watching this clip to see how to use this property.</p> <p>[參考答案]</p>

內容	<p><b>Exercises</b></p> <p>Find the value of the following with a calculator. (Use base 10)</p> <p>a. <math>\log_2 5</math>                      b. <math>\log_5 18</math></p>
教學建議	<p>[教學活動安排]</p> <p>讓學生練習此性質</p> <p>[可參考的英文問句/提問/開場]</p> <p>Now we can use this property to find the value of logarithms with different bases. See exercises. Now it's your turn to try. We will check the answers after three minutes.</p> <p>[參考答案]</p>
內容	<p><b>Challenge</b></p> <p>Use the change base formula to find the value of <math>\log_2 5 \times \log_5 2</math></p>
教學建議	<p>[教學活動安排]</p> <p>教師可透過這個挑戰題引導學生去看到 <math>\log_a b = \frac{1}{\log_b a}</math></p> <p>亦可播放影片的最後 2 分鐘。</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's do this challenge. Use the property we just learned. In two minutes, we'll check the answer.</p> <p>Do you think this general case <math>\log_a b \times \log_b a = 1</math> is right?</p> <p>Let's watch the last part of the clip.</p> <p>[參考答案]</p>
內容	<p>2. Adding logarithms</p> <p>We can do the operation of logarithms.</p> <p>Watch the clip from 2:12 and fill in the blanks.</p> <p><math>\log_a p + \log_a q = \underline{\hspace{2cm}}</math></p> <p>Proof</p> <div data-bbox="1214 1765 1382 1928" style="float: right;">  </div>

	<p>Let <math>m = \log_a p</math> and <math>n = \log_a q</math></p> <p>The corresponding exponential forms of these two equations are _____</p> <p>Then multiply p by q to obtain _____</p> <p>The corresponding logarithmic form of _____ is _____.</p> <p>So, _____</p>
教學建議	<p>[教學活動安排]</p> <p>透過觀看影片讓學生理解此性質並做練習</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's move to another property of logarithms. It is "Adding logarithms". What and why? Let's watch the clip from 2:12 and fill in the blanks.</p> <p>[參考答案]</p> <p>2. Adding logarithms We can do the operation of logarithms. Watch the clip from 2:12 and fill in the blank.</p> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; display: inline-block;"> <math>\log_a p + \log_a q = \log_a pq</math> </div>  <p>Proof Let <math>m = \log_a p</math> and <math>n = \log_a q</math> The corresponding exponential forms of these two equations are <math>p = a^m</math>, <math>q = a^n</math> Then multiply p by q to obtain <math>p \times q = a^m \times a^n = a^{m+n}</math> The corresponding logarithmic form of <math>pq = a^{m+n}</math> is <math>m+n = \log_a pq</math> So, <math>\log_a p + \log_a q = \log_a pq</math></p>
內容	<p><u>Exercise</u></p> <p>Find the value of the following</p> <p>a. <math>\log 2 + \log 5</math></p> <p>b. <math>\log_6 2 + \log_6 3</math></p>
教學建議	<p>[教學活動安排]</p> <p>讓學生練習此性質</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's do the exercise to become more familiar with this property.</p> <p>It's your turn to try. We will check the answers after three minutes.</p> <p>[參考答案]</p>
內容	

	<p><b>Challenge</b></p> <p>Fill in the blanks</p> $\log_a p + \log_a p + \log_a p = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ <p>What did you notice?</p>
<p>教學建議</p>	<p>[教學活動安排]</p> <p>希望透過這個挑戰讓學生可以發現 <math>n \log_a p = \log_a p^n</math></p> <p>[可參考的英文問句/提問/開場]</p> <p>What if we add the same logarithms? Try to do the challenge.</p> <p>[參考答案]</p>
<p>內容</p>	<p>3. Subtracting logarithms</p> $\log_a p - \log_a q = \underline{\hspace{2cm}}$ <p>Proof</p> <p>Let <math>m = \log_a p</math> and <math>n = \log_a q</math></p> <p>The corresponding exponential forms of these two equations are <math>\underline{\hspace{2cm}}</math></p> <p>Then <math>\underline{\hspace{2cm}}</math> to obtain <math>\underline{\hspace{2cm}}</math></p> <p>The corresponding logarithmic form of <math>\underline{\hspace{2cm}}</math> is <math>\underline{\hspace{2cm}}</math>.</p> <p>So, <math>\underline{\hspace{2cm}}</math></p>
<p>教學建議</p>	<p>[教學活動安排]</p> <p>讓學生去嘗試同理可得減法的性質</p> <p>[可參考的英文問句/提問/開場]</p> <p>When we add logarithms, we do multiplication. So when we subtract logarithms we do? Any volunteer? Yes, we do division.</p> <p>Let's fill in the blanks.</p> <p>[參考答案]</p> <p>3. Subtracting logarithms</p> <div style="border: 1px solid gray; padding: 5px; width: fit-content; margin: 10px auto;"> <math>\log_a p - \log_a q = \log_a \frac{p}{q}</math> </div> <p>Proof</p> <p>Let <math>m = \log_a p</math> and <math>n = \log_a q</math></p> <p>The corresponding exponential forms of these two equations are <math>p = a^m, q = a^n</math></p> <p>Then <math>p</math> divided by <math>q</math> to obtain <math>\frac{p}{q} = a^{m-n}</math></p> <p>The corresponding logarithmic form of <math>\frac{p}{q} = a^{m-n}</math> is <math>m - n = \log_a \frac{p}{q}</math></p> <p>So, <math>\log_a p - \log_a q = \log_a \frac{p}{q}</math></p>
<p>內容</p>	<p><b>Exercise</b></p>



	<p>Find the value of the following</p> <p>a. <math>\log 300 - \log 3</math></p> <p>b. <math>\log_6 12 - \log_6 2</math></p>
教學建議	<p>[教學活動安排]</p> <p>讓學生練習此性質</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's do the exercise to become familiar with this property.</p> <p>It's your turn to try. We will check the answers after three minutes.</p> <p>[參考答案]</p>
內容	<p>4. Powers and coefficients</p> <p><b><u>Investigation</u></b></p> <p>Use the definition of logarithms and the properties of exponents to verify (證明) <math>\log_a p^n = n \log_a p</math></p>
教學建議	<p>[教學活動安排]</p> <p>此性質在前面已提過一次，這次強調次方可以提成係數，並讓學生練習利用定義及指數的性質去證明。(老師可以視情況直接證明)</p> <p>[可參考的英文問句/提問/開場]</p> <p>We learned this property at the challenge problem. But now I change the way I look at it. We can see the power becomes the coefficient. I want you to try to use the definition of logarithms and the properties of exponents to verify this property. Give it a try.</p> <p>[參考答案]</p>
內容	<p><b><u>Exercise</u></b></p> <p>Find the value of the following</p> <p>a. <math>\log_3 9^4</math></p> <p>b. <math>2\log 6 + 3\log 5 - \log 45</math></p>

教學建議	<p>[教學活動安排]</p> <p>讓學生練習此性質</p> <p>[可參考的英文問句/提問/開場]</p> <p>Let's do the exercise to become familiar with this property.</p> <p>It's your turn to try. We will check the answers after three minutes.</p> <p>[參考答案]</p>

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